Leading electroweak logarithms at one loop

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- Relevance of leading electroweak logarithms
- Origin of leading electroweak logarithms
- Strategy of calculation
- General results for
  double and single logarithms
- Results for specific processes

Introduction

leading electroweak corrections at LEP energies

- leading electromagnetic logarithms from ISR and FSR: $\mathcal{O}(10\%)$
- running of α: $\sim 6\%$
- corrections $\propto m_{\mathrm{t}}^2/M_{\mathrm{W}}^2$: $\sim 3\%$

leading electroweak corrections at TeV energies

- running of electroweak couplings
- leading electroweak logarithms from soft and collinear electroweak gauge bosons, W and Z can be treated exclusively

typical size of double logarithms

$$\frac{\alpha}{4\pi s_{\mathrm{W}}^2}\log^2\frac{s}{M_{\mathrm{W}}^2} = 6.6\%\ (24\%)\ @\,1\,\mathrm{TeV}\ (10\,\mathrm{TeV})$$

typical size of single logarithms

$$\frac{\alpha}{4\pi s_{\mathrm{W}}^2}\log\frac{s}{M_{\mathrm{W}}^2} = 1.3\%\ (2.5\%)\ @\,1\,\mathrm{TeV}\ (10\,\mathrm{TeV})$$

single logaritms can be enhanced by numerical factors, may be dominant at 1 TeV

Existing results

universal double logarithms

- resummation to all orders
  Ciafaloni and Comelli 1999;
  Fadin, Lipatov, Martin and Melles 1999;
  Kühn, Penin and Smirnov 1999

- explicit two-loop calculations
  Melles 2000;
  Beenakker and Werthenbach 2000;
  Hori, Kawamuri and Kodaira 2000

single logarithms

explicit one-loop calculations

- $e^+e^- \to W^+W^-$
  Beenakker et al. 1993

- $e^+e^- \to f\bar{f}$
  Kühn and Penin 1999;
  Beccaria et al. 2000

subleading two-loop logarithms

- $e^+e^- \to f\bar{f}$
  Kühn, Penin and Smirnov 2000

Aim of this work

extraction of all double and single leading-logarithmic electroweak corrections

- in the complete electroweak Standard Model
- for general processes
- at the one-loop level
- for non-mass-suppressed amplitudes
- in the high-energy limit
  $r_{kl} = (p_k + p_l)^2 \gg M_\mathrm{W}^2$

derivation of simple “factorization” formulas

Strategy of calculation

separate UV-singular loop contributions

using dimensional regularization with regularization scale $\mu^2 = s$

$\Rightarrow$ all large logarithms originate from small-mass limit

$\Rightarrow$ extract mass-singular logarithms in small-mass expansion

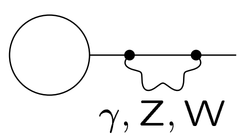
avoid masses in denominators $\Rightarrow$

- 't Hooft–Feynman gauge
- equivalence theorem
  for longitudinal gauge bosons
  $\varepsilon_\mu = k_\mu / M_\mathrm{W} + \mathcal{O}(M_\mathrm{W}/k_0)$

Origin of leading electroweak logarithms

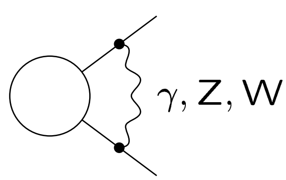
A) parameter renormalization at scale $M_\mathrm{W}^2 \ll s$
(gauge-invariant,
renormalization-scheme-dependent)
$\Rightarrow$ running of electroweak couplings from
M_W to $\sqrt{s}$
single logarithms

B) field renormalization of external lines

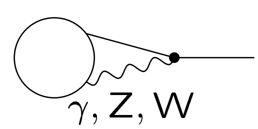


single logarithms

C) diagrams with soft and/or collinear
gauge bosons coupled to external lines



double logarithms

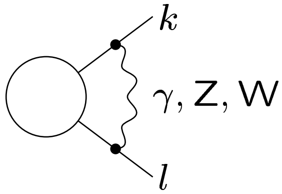


single logarithms

scalar or fermion exchange gives no
logarithms (mass suppressed)

(B+C gauge-invariant)

Soft-collinear contributions

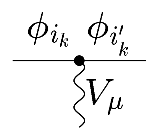


$k, l = 1, \ldots n$ external legs

eikonal approximation ⇒

$$\delta\mathcal{M}^{i_1\ldots i_n} = \frac{\alpha}{4\pi}\frac{1}{2}\sum_{k=1}^{n}\sum_{l\neq k}\sum_{V=\gamma,Z,W^\pm} I^V_{i'_k i_k}(k) I^V_{i'_l i_l}(l)\mathcal{M}_0^{i_1\ldots i'_k\ldots i'_l\ldots i_n}$$
$$\times\left[\log^2\frac{|r_{kl}|}{M_V^2} - \delta_{V\gamma}\log^2\frac{m_k^2}{\lambda^2}\right]$$

with gauge coupling matrices $I^V_{i'_k i_k}$



split logarithms in universal and angular-dependent part

$$\log^2\frac{|r_{kl}|}{M_V^2} \approx \log^2\frac{s}{M_V^2} + 2\log\frac{s}{M_V^2}\log\frac{|r_{kl}|}{s}$$

leading + subleading
soft-collinear logarithms

subleading soft-collinear logarithms

- angular-dependent
- depend on SU(2) × U(1) rotated Born matrix elements since $I^{W^\pm}_{i'_k i_k}$ non-diagonal (e.g. $\mathrm{e} \to \nu_\mathrm{e}$)

Subleading soft-collinear logarithms

angular-dependent double logarithms

sum over pairs of external legs

$$\delta^{\mathrm{SSC}}\mathcal{M}^{i_1\ldots i_n} = \frac{\alpha}{4\pi}\sum_{k=1}^{n}\sum_{l<k}\sum_{V=\gamma,Z,W}\delta^{V,\mathrm{SSC}}_{i'_k i_k i'_l i_l}(k,l)\mathcal{M}_0^{i_1..i'_k..i'_l..i_n}$$

with

$$\delta^{\gamma,\mathrm{SSC}}_{i'_k i_k i'_l i_l}(k,l) = 2\left[\log\frac{s}{M_\mathrm{W}^2} + \log\frac{M_\mathrm{W}^2}{\lambda^2}\right]\log\frac{|r_{kl}|}{s} I^{\gamma}_{i'_k i_k}(k) I^{\gamma}_{i'_k i_k}(l)$$

$$\delta^{\mathrm{Z},\mathrm{SSC}}_{i'_k i_k i'_l i_l}(k,l) = 2\log\frac{s}{M_\mathrm{W}^2}\log\frac{|r_{kl}|}{s} I^{Z}_{i'_k i_k}(k) I^{Z}_{i'_l i_l}(l)$$

$$\delta^{\mathrm{W},\mathrm{SSC}}_{i'_k i_k i'_l i_l}(k,l) = 2\log\frac{s}{M_\mathrm{W}^2}\log\frac{|r_{kl}|}{s}\sum_{\sigma=\pm} I^{\sigma}_{i'_k i_k}(k) I^{-\sigma}_{i'_l i_l}(l)$$

$I^{\sigma}_{i'_k i_k}$ non-diagonal

$\Rightarrow$ $\delta^{\mathrm{SSC}}\mathcal{M}$ depends on $\mathrm{SU}(2)\times\mathrm{U}(1)$ rotated Born matrix elements

Leading soft-collinear logarithms

double (Sudakov) logarithms

global SU(2) × U(1) invariance

$$0 = \frac{\delta}{\delta\theta^V}\mathcal{M}_0^{i_1\ldots i_n} \sim \mathrm{i}e\sum_k I^V_{i'_k i_k}(k)\mathcal{M}_0^{i_1\ldots i'_k\ldots i_n}$$

yields single sum over external legs

$$\delta^{\mathrm{LSC}}\mathcal{M}^{i_1\ldots i_n} = \frac{\alpha}{4\pi}\sum_{k=1}^{n}\delta^{\mathrm{LSC}}_{i'_k i_k}(k)\mathcal{M}_0^{i_1..i'_k..i_n}$$

$$\delta^{\mathrm{LSC}}(k) = -\frac{1}{2}\left[C^{\mathrm{ew}}(k)\log^2\frac{s}{M_\mathrm{W}^2} + Q_k^2 L^{\mathrm{em}}(s,\lambda^2,m_k^2)\right]$$

with *electroweak Casimir operator*

$$C^{\mathrm{ew}} = \sum_{V=\gamma,Z,W}(I^V)^2 = \frac{1}{c_\mathrm{w}^2}\left(\frac{Y}{2}\right)^2 + \frac{1}{s_\mathrm{w}^2}C^{\mathrm{SU(2)}}$$

(non-diagonal for external γ, Z)

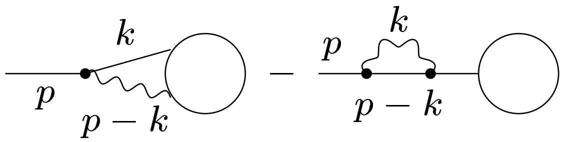
and the purely electromagnetic logarithms

$$L^{\mathrm{em}}(s,\lambda^2,m_k^2) = 2\log\frac{s}{M_\mathrm{W}^2}\log\frac{M_\mathrm{W}^2}{\lambda^2} + \log^2\frac{M_\mathrm{W}^2}{\lambda^2} - \log^2\frac{m_k^2}{\lambda^2}$$

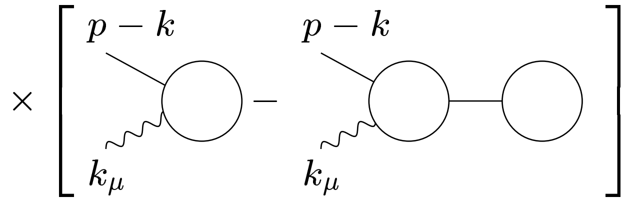
resulting from mass gap between λ and M_W

results agree with literature

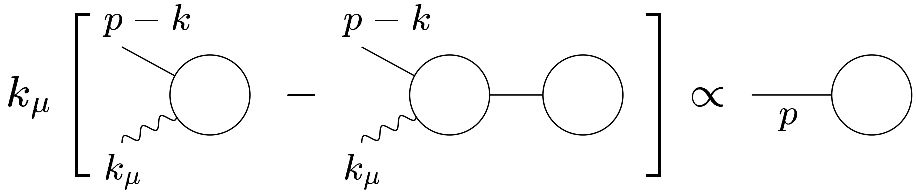
collinear limit:



$$\sim \int \mathrm{d}^4k \frac{f(x)k_\mu}{(k^2 - M_V^2)[(p-k)^2 - m^2]}$$

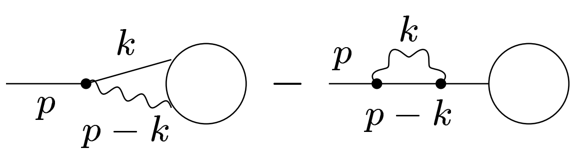


Ward identiy from BRS invariance



$$= \mathcal{M}_0$$

⇒ collinear diagrams factorize into collinear factor times Born matrix element



$$\sim \delta^{\mathrm{coll}} \mathcal{M}_0$$

combine with external-line renormalization

$$\delta^{\mathrm{C}} \mathcal{M}^{i_1 \ldots i_n} = \frac{\alpha}{4\pi} \sum_{k=1}^{n} \delta^{\mathrm{C}}_{i'_k i_k}(\varphi_k) \mathcal{M}_0^{i_1 \ldots i'_k \ldots i_n}$$

with

$$\delta^{\mathrm{C}}_{i'_k i_k}(\varphi_k) = \delta^{\mathrm{coll}}_{i'_k i_k}(\varphi_k) + \frac{1}{2} \delta Z^{\varphi_k}_{i'_k i_k}\Big|_{\mu^2 = s}$$

fermions:

$$\delta^{\mathrm{C}}(f_\kappa) = \left\{\frac{3}{2}C^{\mathrm{ew}}(f_\kappa) - \frac{1}{8s_{\mathrm{w}}^2}\left[(1+\delta_{\kappa\mathrm{R}})\frac{m_f^2}{M_{\mathrm{W}}^2} + \delta_{\kappa\mathrm{L}}\frac{m_{f'}^2}{M_{\mathrm{W}}^2}\right]\right\}\log\frac{s}{M_{\mathrm{W}}^2} + Q_f^2 l^{\mathrm{em}}(m_f^2)$$

with the purely electromagnetic logarithms

$$l^{\mathrm{em}}(m_k^2) = \frac{1}{2}\log\frac{M_{\mathrm{W}}^2}{m_k^2} + \log\frac{M_{\mathrm{W}}^2}{\lambda^2}$$

transverse W^+ bosons:

$$\delta^{\mathrm{C}}(\mathrm{W}_{\mathrm{T}}) = \frac{1}{2}b_{WW}^{\mathrm{ew}}\log\frac{s}{M_{\mathrm{W}}^2} + Q_{\mathrm{W}}^2 l^{\mathrm{em}}(M_{\mathrm{W}}^2)$$

transverse neutral gauge bosons (mixing):

$$\delta^{\mathrm{C}}_{V'V}(V_{\mathrm{T}}) = \frac{1}{2}\left[b_{AZ}^{\mathrm{ew}}\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix} + b_{V'V}^{\mathrm{ew}}\right]\log\frac{s}{M_{\mathrm{W}}^2} - \frac{1}{2}\Delta\alpha(M_{\mathrm{W}}^2)\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$$

with the coefficients of the one-loop β-functions

$$b_{ab}^{\mathrm{ew}} = \frac{11}{3}C_{ab}^{\mathrm{ew}} - \frac{1}{3}\mathrm{Tr}\left\{I^a(\phi)I^b(\phi)\right\} - \frac{2}{3}\sum_{f,i}N_{\mathrm{C}}^f\sum_{\lambda}\mathrm{Tr}\left\{I^a(f^\lambda)I^b(f^\lambda)\right\}$$

and the electromagnetic running of α up to the electroweak scale

$$\Delta\alpha(M_{\mathrm{W}}^2) = \frac{4}{3}\sum_{f,i,\sigma\neq t}N_{\mathrm{C}}^f Q_{f_\sigma}^2\log\frac{M_{\mathrm{W}}^2}{m_{f_{\sigma,i}}^2}$$

$f = q, l, \quad i = 1,2,3, \quad \lambda = \mathrm{L},\mathrm{R}, \quad \sigma = \pm$ (isospin)

Results for coll. and soft single logarithms II

longitudinal gauge bosons:

$$\delta^{\mathrm{C}}(\mathrm{W_L}) = \left[2C^{\mathrm{ew}}(\Phi) - \frac{N_C^t}{4s_{\mathrm{w}}^2}\frac{m_t^2}{M_{\mathrm{W}}^2}\right]\log\frac{s}{M_{\mathrm{W}}^2} + Q_{\mathrm{W}}^2 l^{\mathrm{em}}(M_{\mathrm{W}}^2)$$

$$\delta^{\mathrm{C}}(\mathrm{Z_L}) = \left[2C^{\mathrm{ew}}(\Phi) - \frac{N_C^t}{4s_{\mathrm{w}}^2}\frac{m_t^2}{M_{\mathrm{W}}^2}\right]\log\frac{s}{M_{\mathrm{W}}^2}$$

Higgs bosons:

$$\delta^{\mathrm{C}}(H) = \left[2C^{\mathrm{ew}}(\Phi) - \frac{N_C^t}{4s_{\mathrm{w}}^2}\frac{m_t^2}{M_{\mathrm{W}}^2}\right]\log\frac{s}{M_{\mathrm{W}}^2}$$

- collinear logarithms of longitudinal gauge bosons are as expected from the tree level equivalence theorem

- Yukawa contributions in $\delta^{\mathrm{C}}(\mathrm{t})$, $\delta^{\mathrm{C}}(\mathrm{b})$, $\delta^{\mathrm{C}}(\mathrm{W_L})$, $\delta^{\mathrm{C}}(\mathrm{Z_L})$, $\delta^{\mathrm{C}}(\mathrm{H})$

Renormalization-group logarithms

result directly from Born matrix element

$$\delta^{\mathrm{RG}}\mathcal{M} = \frac{\delta\mathcal{M}_0}{\delta e}\delta e + \frac{\delta\mathcal{M}_0}{\delta c_{\mathrm{w}}}\delta c_{\mathrm{w}}\bigg|_{\mu^2=s}$$

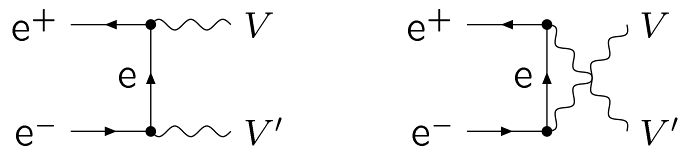
on-shell scheme:

$$\frac{\delta c_{\mathrm{w}}^2}{c_{\mathrm{w}}^2} = \frac{\alpha}{4\pi}\frac{s_{\mathrm{w}}}{c_{\mathrm{w}}}b_{AZ}^{\mathrm{ew}}\log\frac{s}{M_{\mathrm{W}}^2}$$

$$\frac{\delta e}{e} = \frac{\alpha}{4\pi}\Bigg(\underbrace{-\frac{1}{2}b_{AA}^{\mathrm{ew}}\log\frac{s}{M_{\mathrm{W}}^2}}_{\text{running from } M_{\mathrm{W}}^2\to s} + \underbrace{\frac{1}{2}\Delta\alpha(M_{\mathrm{W}}^2)}_{0\to M_{\mathrm{W}}^2}\Bigg)$$

Example: $\mathrm{e}^+\mathrm{e}^- \to V_1 V_2$

lowest order



$$M_{0,\mathrm{e}^+_\kappa \mathrm{e}^-_\kappa \to V^1_\mathrm{T} V^2_\mathrm{T}} = e^2 I^{V^1}(\mathrm{e}^-_\kappa) I^{V^2}(\mathrm{e}^-_\kappa) \left[\frac{A_t}{t} + \frac{A_u}{u}\right]$$

correction factor

$$\delta\mathcal{M}(\mathrm{e}^+_\kappa \mathrm{e}^-_\kappa \to V^1_\mathrm{T} V^2_\mathrm{T}) = \delta^{\kappa\mathrm{T}}_{\mathrm{e}^+\mathrm{e}^- \to VV'} \mathcal{M}_0(\mathrm{e}^+_\kappa \mathrm{e}^-_\kappa \to V^1_\mathrm{T} V^2_\mathrm{T})$$

numerical results for electroweak part

$$\delta^{\mathrm{RT}}_{\mathrm{e}^+\mathrm{e}^- \to \gamma\gamma} = -1.29L + 0.22\, l_\mathrm{C} + 3.67\, l_\mathrm{RG}$$
$$\delta^{\mathrm{RT}}_{\mathrm{e}^+\mathrm{e}^- \to \gamma\mathrm{Z}} = -1.29L - 11.3\, l_\mathrm{C} + 15.1\, l_\mathrm{RG}$$
$$\delta^{\mathrm{RT}}_{\mathrm{e}^+\mathrm{e}^- \to \mathrm{ZZ}} = -1.29L - 22.8\, l_\mathrm{C} + 26.6\, l_\mathrm{RG}$$

$$\delta^{\mathrm{LT}}_{\mathrm{e}^+\mathrm{e}^- \to \gamma\gamma} = -8.15L + 9.00 F_1(t) l + +7.36\, l_\mathrm{C} + 3.67\, l_\mathrm{RG}$$
$$\delta^{\mathrm{LT}}_{\mathrm{e}^+\mathrm{e}^- \to \gamma\mathrm{Z}} = -12.2L + [17.0 F_1(t) - 8.09 F_2(t)]\, l + 28.1\, l_\mathrm{C} - 17.1\, l_\mathrm{RG}$$
$$\delta^{\mathrm{LT}}_{\mathrm{e}^+\mathrm{e}^- \to \mathrm{ZZ}} = -16.2L + [25.1 F_1(t) - 45.4 F_2(t)]\, l + 48.9\, l_\mathrm{C} - 37.9\, l_\mathrm{RG}$$

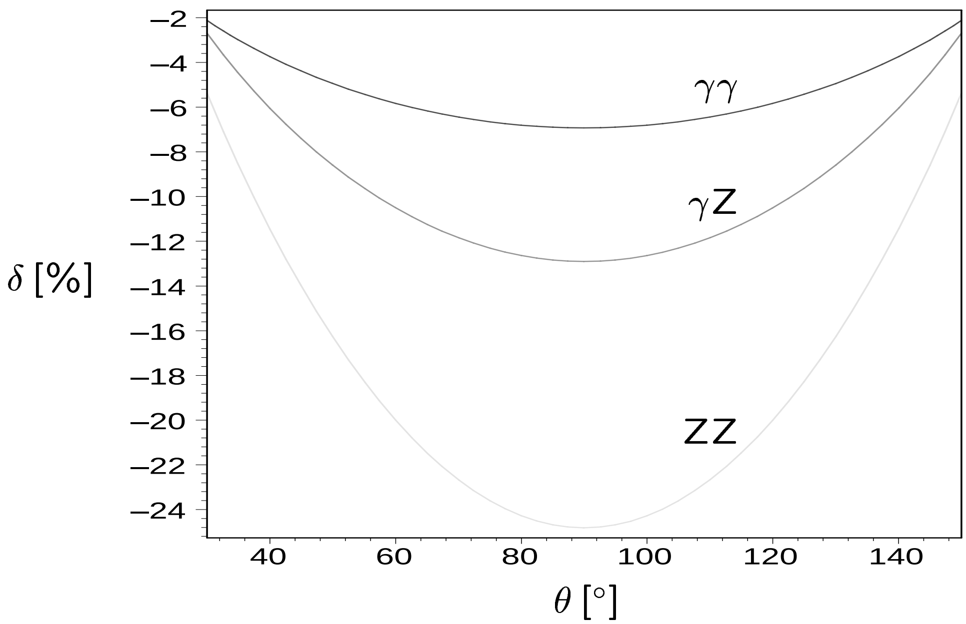
with

$$L = \frac{\alpha}{4\pi} \log^2 \frac{s}{M_\mathrm{W}^2} \approx 6.6\% \text{ @ } 1\,\mathrm{TeV}$$
$$l = l_\mathrm{C} = l_\mathrm{RG} = \frac{\alpha}{4\pi} \log \frac{s}{M_\mathrm{W}^2} \approx 1.3\% \text{ @ } 1\,\mathrm{TeV}$$
$$F_1(t) = \frac{u}{s} \log \frac{|t|}{s} + \frac{t}{s} \log \frac{|u|}{s} \approx 1.11 \text{ @ } 60^\circ$$
$$F_2(t) = \frac{t}{s} \log \frac{|t|}{s} + \frac{u}{s} \log \frac{|u|}{s} \approx 0.56 \text{ @ } 60^\circ$$

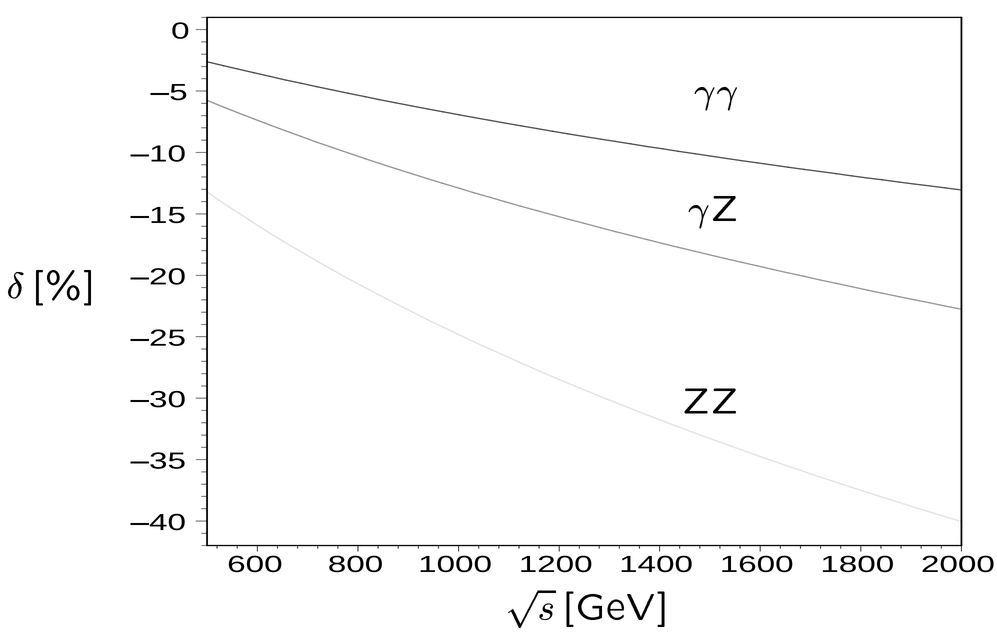
Numerical results for $e^+e^- \to V_1V_2$

electroweak corrections to $e_L^+e_L^- \to \gamma\gamma, \gamma Z, ZZ$

$\sqrt{s} = 1\,\text{TeV}$



$\theta = 90^\circ$



Example: $e^+e^- \to f\bar{f}$

correction factor $(\kappa, \kappa' = \mathrm{R}, \mathrm{L})$

$$\delta\mathcal{M}(e^+_\kappa e^-_\kappa \to f_{\kappa'}\bar{f}_{\kappa'}) = \delta^{\kappa\kappa'}_{e^+e^-\to f\bar{f}}\mathcal{M}_0(e^+_\kappa e^-_\kappa \to f_{\kappa'}\bar{f}_{\kappa'})$$

analytic results agree with literature

numerical results for electroweak part

$$\delta^{\mathrm{RR}}_{e^+e^-\to\mu^+\mu^-} = -2.58L - \left(5.15\log\frac{t}{u}\right)l + 7.73 l_{\mathrm{C}} + 8.80\, l_{\mathrm{RG}}$$
$$\delta^{\mathrm{RL}}_{e^+e^-\to\mu^+\mu^-} = -4.96L - \left(2.58\log\frac{t}{u}\right)l + 14.9 l_{\mathrm{C}} + 8.80\, l_{\mathrm{RG}}$$
$$\delta^{\mathrm{LL}}_{e^+e^-\to\mu^+\mu^-} = -7.35L + \left(-5.76\log\frac{t}{u} + 13.9\log\frac{|t|}{s}\right)l + 22.1 l_{\mathrm{C}} - 9.03\, l_{\mathrm{RG}}$$

$$\delta^{\mathrm{RR}}_{e^+e^-\to t\bar{t}} = -1.86L - 3.43\left(\log\frac{t}{u}\right)l + 5.58 l_{\mathrm{C}} - 10.6\, l_{\mathrm{Yuk}} + 8.80\, l_{\mathrm{RG}}$$
$$\delta^{\mathrm{RL}}_{e^+e^-\to t\bar{t}} = -4.68L - 0.86\left(\log\frac{t}{u}\right)l + 14.0 l_{\mathrm{C}} - 5.30\, l_{\mathrm{Yuk}} + 8.80\, l_{\mathrm{RG}}$$
$$\delta^{\mathrm{LR}}_{e^+e^-\to t\bar{t}} = -4.25L - 1.72\left(\log\frac{t}{u}\right)l + 12.7 l_{\mathrm{C}} - 10.6\, l_{\mathrm{Yuk}} + 8.80\, l_{\mathrm{RG}}$$
$$\delta^{\mathrm{LL}}_{e^+e^-\to t\bar{t}} = -7.07L - \left(4.90\log\frac{t}{u} - 16.3\log\frac{|t|}{s}\right)l + 21.2 l_{\mathrm{C}} - 5.30\, l_{\mathrm{Yuk}} - 12.2\, l_{\mathrm{RG}}$$

with

$$L = \frac{\alpha}{4\pi}\log^2\frac{s}{M_{\mathrm{W}}^2} \approx 6.6\% \text{ @ 1 TeV}$$
$$l = l_{\mathrm{C}} = l_{\mathrm{Yuk}} = l_{\mathrm{RG}} = \frac{\alpha}{4\pi}\log\frac{s}{M_{\mathrm{W}}^2} \approx 1.3\% \text{ @ 1 TeV}$$
$$\log\frac{t}{u} \approx -1.1 \text{ @ } 60^\circ, \qquad \log\frac{|t|}{s} \approx -1.4 \text{ @ } 60^\circ$$

Example: $e^+e^- \to W^+W^-$

lowest order

$$\mathcal{M}_{0,e_R^+e_R^-\to W_L^+W_L^-} = \frac{e^2}{4s_W^2c_W^2}\frac{A_s}{s-M_Z^2}$$

$$\mathcal{M}_{0,e_L^+e_L^-\to W_L^+W_L^-} = \frac{e^2}{2c_W^2}\frac{A_s}{s-M_Z^2}$$

$$\mathcal{M}_{0,e_L^+e_L^-\to W_T^+W_T^-} = \frac{e^2}{2s_W^2}\frac{A_t}{t}$$

correction factor ($\kappa = \mathrm{R,L}$ and $\lambda = \mathrm{L,T}$)

$$\delta\mathcal{M}(e_\kappa^+e_\kappa^- \to W_\lambda^+W_\lambda^-) = \delta^{\kappa\lambda}_{e^+e^-\to W^+W^-}\,\mathcal{M}_0(e_\kappa^+e_\kappa^- \to W_\lambda^+W_\lambda^-)$$

analytic results agree with literature

numerical results for electroweak part

$$\delta^{LL}_{e^+e^-\to W^+W^-} = -7.35L - \left(5.76\log\frac{t}{u} + 13.9\log\frac{|t|}{s}\right)l + 25.7\,l_C - 31.8\,l_{Yuk} - 9.03\,l_{RG}$$

$$\delta^{RL}_{e^+e^-\to W^+W^-} = -4.96L - \left(2.58\log\frac{t}{u}\right)l + 18.6\,l_C - 31.8\,l_{Yuk} + 8.80\,l_{RG}$$

$$\delta^{LT}_{e^+e^-\to W^+W^-} = -12.6L - 8.95\left[\log\frac{t}{u} + \left(1-\frac{t}{u}\right)\log\frac{|t|}{s}\right]l + 25.2\,l_C - 14.2\,l_{RG}$$

with

$$\log^2\frac{s}{M_W^2} = \frac{\alpha}{4\pi}\log^2\frac{s}{M_W^2} \approx 6.6\% \text{ @ 1 TeV}$$

$$\log\frac{s}{M_W^2} = l_C = l_{Yuk} = l_{RG} = \frac{\alpha}{4\pi}\log\frac{s}{M_W^2} \approx 1.3\% \text{ @ 1 TeV}$$

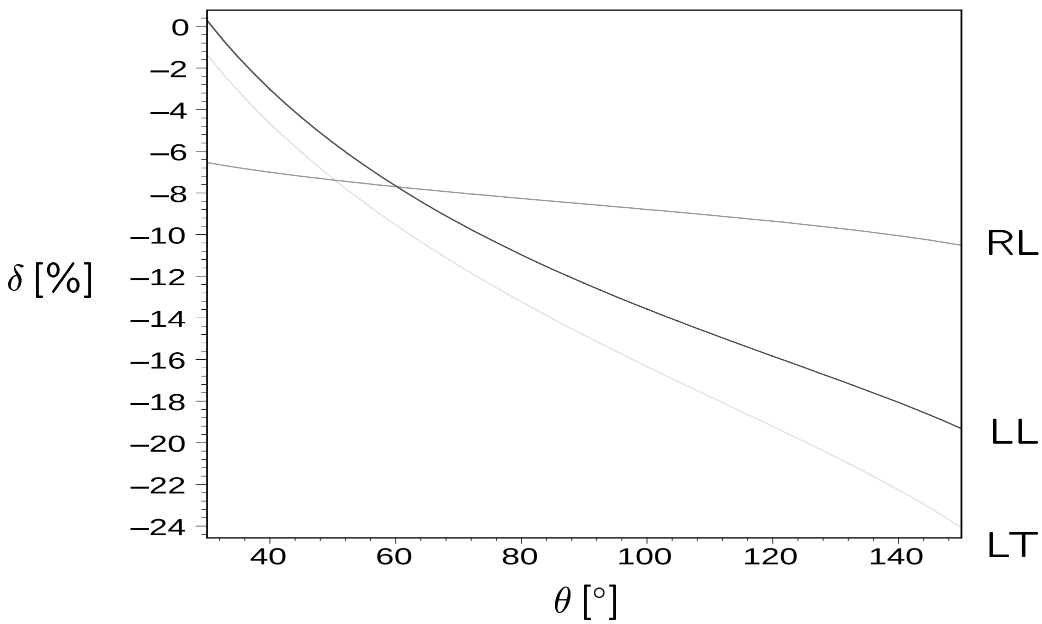
$$\log\frac{t}{u} \approx -1.1 \text{ @ } 60^\circ, \qquad \log\frac{|t|}{s} \approx -1.4 \text{ @ } 60^\circ$$

$$\left[\log\frac{t}{u} + \left(1-\frac{t}{u}\right)\log\frac{|t|}{s}\right] \approx -2.0 \text{ @ } 60^\circ$$

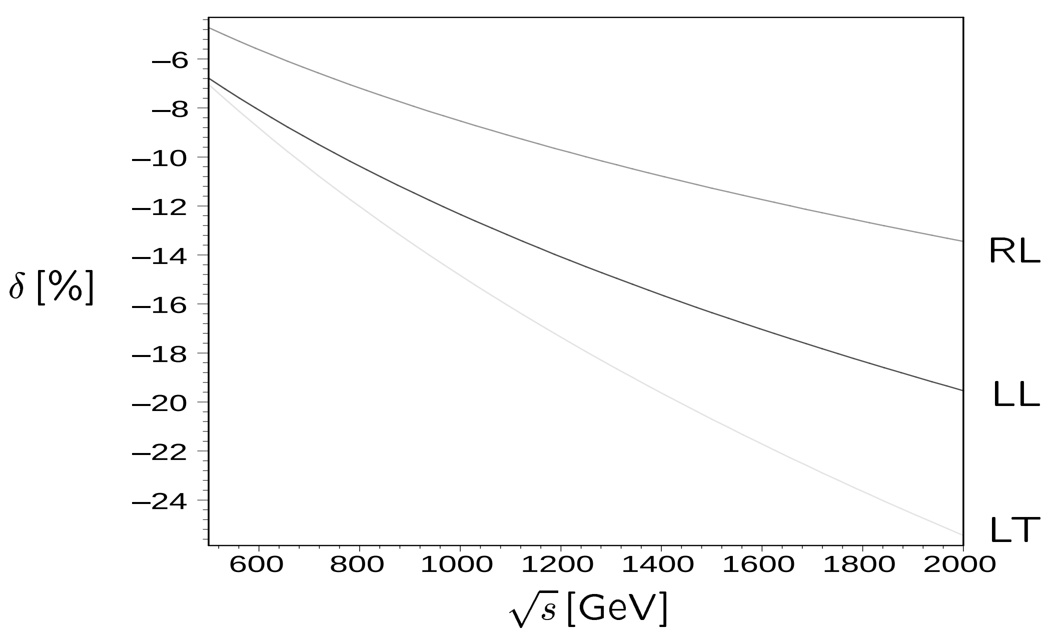
Numerical results for $e^+e^- \to W^+W^-$

electroweak corrections to $e^+_\kappa e^-_\kappa \to W^+_\lambda W^-_\lambda$

$\sqrt{s} = 1$ TeV



$\theta = 90°$



Conclusion

soft-collinear double logarithms known

method for extraction of subleading double logarithms and single logarithms developed

⇒ methods for the extraction of all electroweak one-loop logarithmic corrections at high energies exist

single and leading soft-collinear double logarithms

- associated with external lines
- factorize lowest-order matrix element
- involve mixing between photon and Z-boson matrix elements

subleading soft-collinear logarithms

- associated with pairs of external lines
- involve mixing between matrix elements of $SU(2) \times U(1)$ partners of external particles

longitudinal gauge bosons behave as Goldstone bosons