Four-fermion production in $\mathrm{e^+e^-}$ annihilation *

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*mainly based on Nucl. Phys. **B560** (1999) 33; Phys. Lett. **B475** (2000) 127; hep-ph/9912447; hep-ph/0006307, to appear in Nucl. Phys. **B**.

1 Introduction

Recent progress for $\mathrm{e^+e^-} \to \mathrm{WW} \to 4f$

...experimentally by LEP2:

- W-pair cross section σ_{WW} reaches 1% accuracy
- W-boson mass M_{W} exceeds $50\,\mathrm{MeV}$ accuracy
- analysis of $4f+\gamma$ final states
  $\to$ first direct bounds on quartic gauge-boson couplings (QGCs)

OPAL '99, L3 '00

...theoretically:

- inclusion of $\mathcal{O}(\alpha)$ correction in predictions
  (Beenakker/Berends/Chapovsky, YFSWW, RacoonWW)
- full matrix-element calculations for $4f+\gamma$ production
  (Jegerlehner/Kolodziej, GRACE, PHEGAS, WRAP, RacoonWW)
- various numerical studies
  (corrections, γ recombination, QGCs, etc.)

$\Rightarrow$ Issue of this talk is

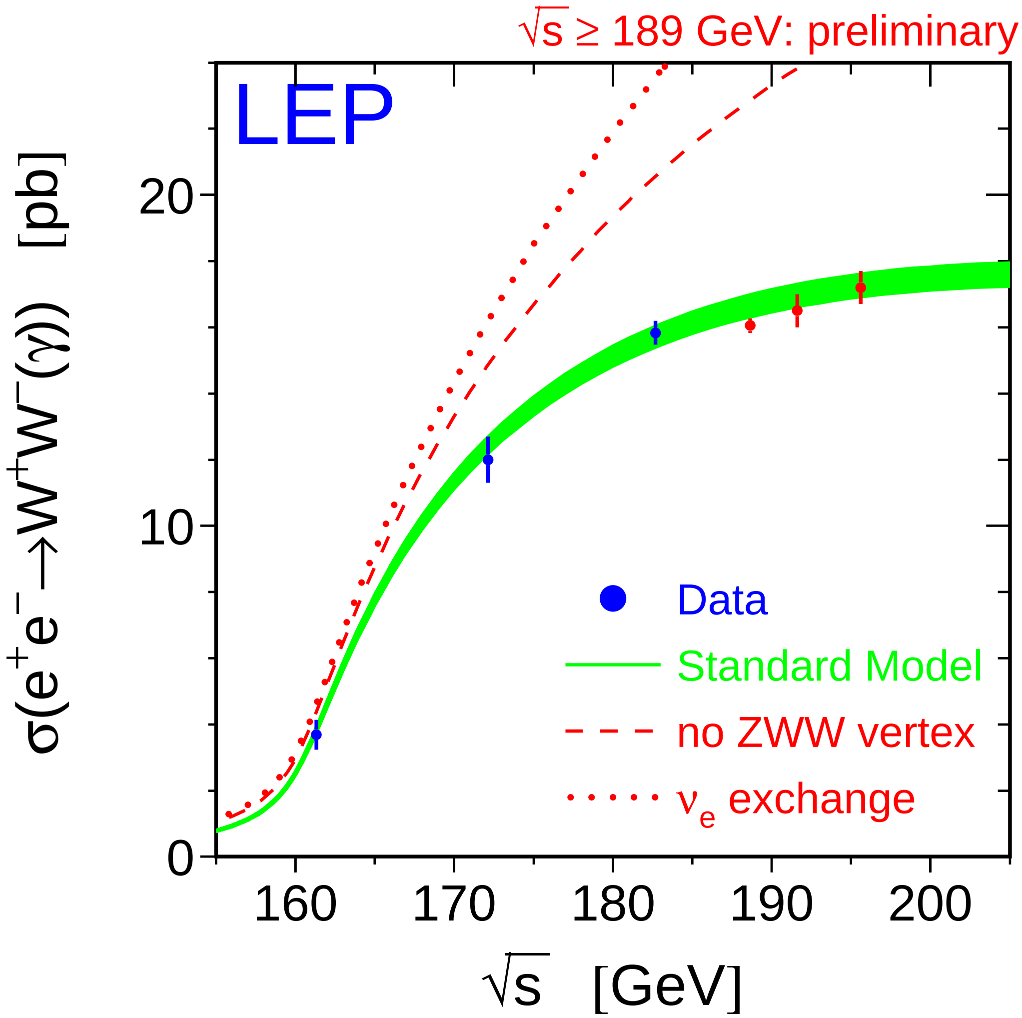
- to briefly review the state of the art for $\mathrm{ee}{\to}\mathrm{WW}{\to}4f(+\gamma)$
- to explain the strategy of the RacoonWW approach
- to present numerical results

Not considered in the talk:

Single-W and Z-pair production ($\to$ talks of G. Passarino, B.F.L. Ward)

Total WW cross section at LEP2 — status of 1999

LEPEWWG '99



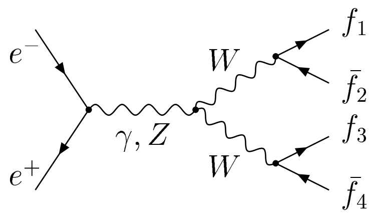
GENTLE prediction (Bardin et al.)
with theoretical uncertainty of $\sim \pm 2\%$

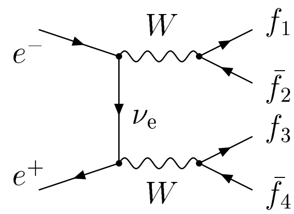
↑

non-leading electroweak corrections

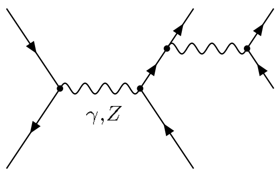
2 Lowest-order predictions for ee→WW→4f(+γ)

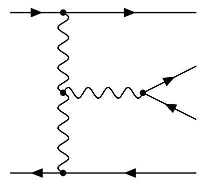
Three "signal diagrams": two resonant W's

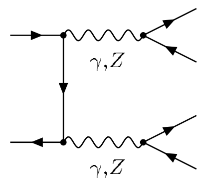




"Background diagrams": at most one resonant W







etc.

Number of background diagrams:

$$
\begin{array}{lc}
\mathrm{e}^+\mathrm{e}^- \to \mu^-\bar{\nu}_\mu\tau^+\nu_\tau : & 6 \\
\vdots & \vdots \\
\mathrm{e}^+\mathrm{e}^- \to \mathrm{e}^-\bar{\nu}_\mathrm{e}\mathrm{e}^+\nu_\mathrm{e} : & 53
\end{array}
$$

Impact of background diagrams $\sim \frac{\Gamma_\mathrm{W}}{M_\mathrm{W}} \sim 3\%$, but

- systematic enhancement effects (e.g. by $\mathrm{e}^\pm$ in final state)
- reduction by cuts possible

Full inclusion of background diagrams necessary !

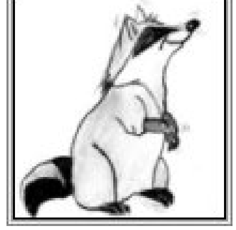
Inflation of expense for $e^+e^- \to 4f + \gamma$:

	$e^+e^- \to 4f$	$e^+e^- \to 4f+\gamma$
# diagrams: (lowest order)	9–144	14–1008
dimension of phase space:	8	11
programs for		
single final states:	ERATO, GENTLE, HIGGSPV, KORALW, WTO, WWF, WWGENPV	WRAP, GRACE, CompHEP, "JK", PHEGAS
all final states:	ALPHA, CompHEP, Excalibur, grc4f, WPHACT, RacoonWW	RacoonWW

Main complications:

- fast evaluation of diagrams
- extremely rich peaking structure of integrand
- IR- and collinear singularities in $4f + \gamma$
- gauge invariance in presence of unstable particles

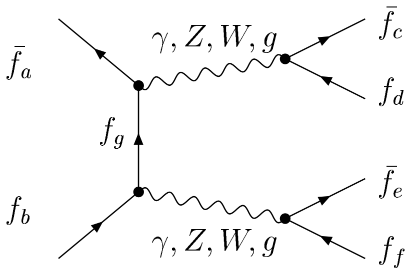
$e^+e^- \to 4f + \gamma$ with RacoonWW

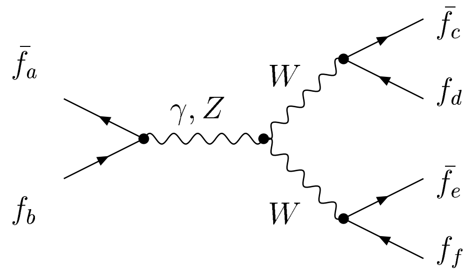


Denner, S.D., Roth, Wackeroth '99

- "Generic" calculation of amplitudes:

  Amplitude built from two basic sets of graphs:





  Evaluation with Weyl–van der Waerden spinors

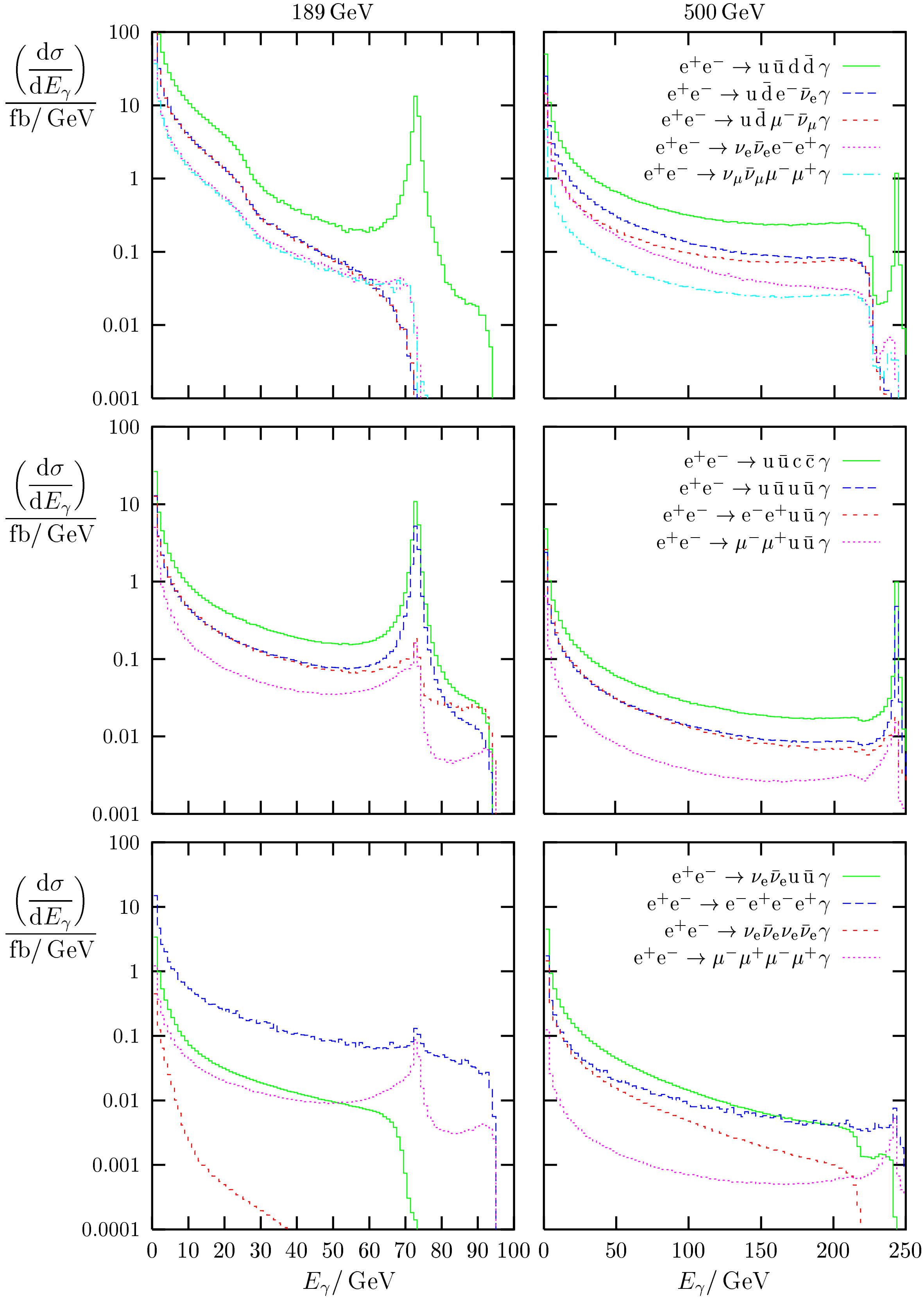
  $\to$ two compact generic functions
  for all final states and polarizations !

- Phase-space integration:
  - "multi-channel" Monte Carlo integration (Berends, Daverveldt, Kleiss '85; Hilgart, Kleiss, Le Diberder '93)
    $\to$ 14–928 different channels (=parametrizations)
  - "adaptive weight optimization" (Kleiss, Pittau '94)
    $\to$ weighted events

- Introduction of decay widths
  optional: constant/running width, complex mass
  $\to$ important for controlling gauge invariance

γ-energy spectra with RacoonWW

Denner, S.D., Roth, Wackeroth '99



Comparison to related work

Other MC generators for $4f + \gamma$ final states:

WRAP (Montagna et al.)

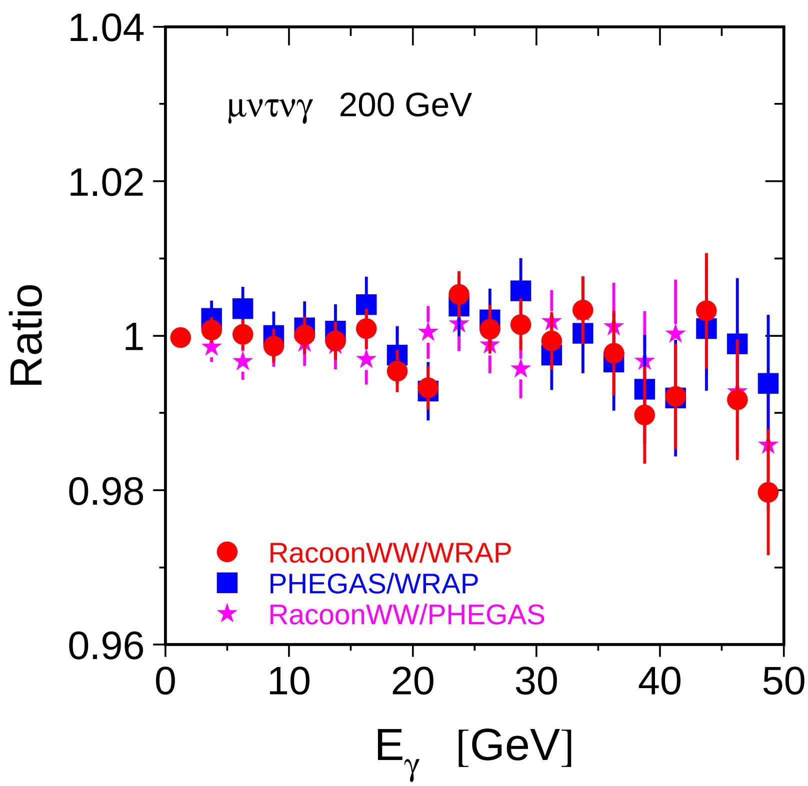
PHEGAS (Papadopoulos)

$\rightarrow$ Perfect technical agreement

for cross sections and distributions of selected final states

A typical result for an E_γ spectrum:

LEP2 MC workshop report
hep-ph/0005309



3 $\mathcal{O}(\alpha)$ corrections to $\mathrm{e^+e^-}\to\mathrm{WW}\to 4f$

3.1 General considerations

Virtual corrections — $\mathrm{e^+e^-} \to 4f$ at one loop

Problems in full calculation:

- enormous technical complications:
  $\mathcal{O}(10^3$–$10^4)$ diagrams per $4f$ state, many $4f$ states, numerical instabilities, etc.
- no convincing solution to maintain gauge invariance in practice

Possible solution: double-pole approximation (DPA)

$\Rightarrow$ Take only diagrams for $\mathrm{e^+e^-} \to \mathrm{WW} \to 4f$
with two nearly resonant W bosons !

- Considerable reduction in number of Feynman graphs
- On-shell production and decay of W's as building block
- Generic calculation for all final states possible
- Expected uncertainty not too close to threshold:

$$\sim \mathcal{O}\left(\frac{\Gamma_{\mathrm{W}}}{M_{\mathrm{W}}}\frac{\alpha}{\pi}\ln(\cdots)\right) \lesssim 0.5\%$$

Real corrections — $\mathrm{e^+e^-} \to 4f + \gamma$ at tree level

Full calculations for all final states exist.

$\Rightarrow$ DPA for real corrections can be controlled.

Note: error estimate of DPA for real corrections
not obvious for $E_\gamma \sim \Gamma_{\mathrm{W}}$!

3.2 The RacoonWW approach

Denner, S.D., Roth, Wackeroth '99

$e^+e^- \to 4f$ at tree level

Full matrix-element calculation with massless fermions

$\mathcal{O}(\alpha)$ radiative corrections (RCs) to $e^+e^- \to WW \to 4f$

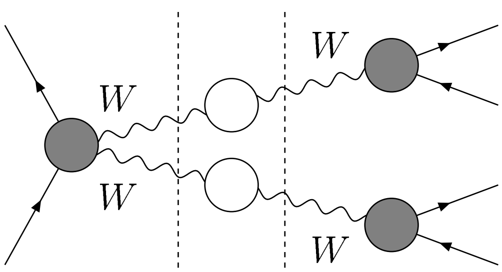
virtual

real

$e^+e^- \to 4f+\gamma$ at tree level:
Full matrix-element calculation

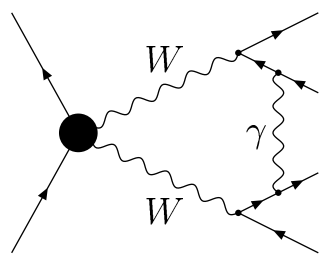
$e^+e^- \to WW \to 4f$ at one loop:
Double-pole approximation

Factorizable corrections



Building blocks:
– RCs to on-shell production
– RCs to on-shell decay
But: respect spin correlations!

Non-factorizable corrections



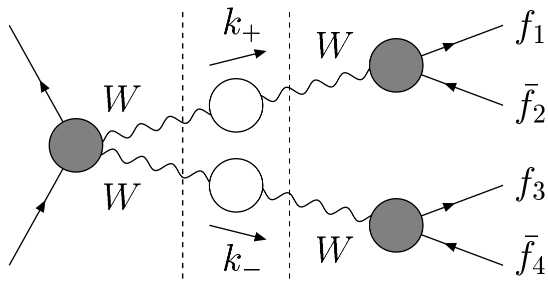
etc.

Photon exchange between production and decay subprocesses ($E_\gamma \lesssim \Gamma_W$)

Beyond $\mathcal{O}(\alpha)$

Soft-γ exponentiation and leading-log ISR up to $\mathcal{O}(\alpha^3)$

Virtual factorizable corrections



One-loop contribution to matrix element in DPA:

$$\mathcal{M}^{e^+e^-\to WW\to 4f}_{\text{virt,fact,DPA}} = \frac{R(M_W^2, M_W^2)}{(k_+^2 - M_W^2 + iM_W\Gamma_W)(k_-^2 - M_W^2 + iM_W\Gamma_W)}$$

with the gauge-independent residue

$$R(M_W^2, M_W^2) = \sum_{W-\text{pols}} \Big(\delta\mathcal{M}^{e^+e^-\to W^+W^-} \mathcal{M}^{W^+\to f_1\bar{f}_2}_{\text{Born}} \mathcal{M}^{W^-\to f_3\bar{f}_4}_{\text{Born}} + \mathcal{M}^{e^+e^-\to W^+W^-}_{\text{Born}} \delta\mathcal{M}^{W^+\to f_1\bar{f}_2} \mathcal{M}^{W^-\to f_3\bar{f}_4}_{\text{Born}} + \mathcal{M}^{e^+e^-\to W^+W^-}_{\text{Born}} \mathcal{M}^{W^+\to f_1\bar{f}_2}_{\text{Born}} \delta\mathcal{M}^{W^-\to f_3\bar{f}_4} \Big)$$

One-loop corrections to subprocesses known:

$\delta\mathcal{M}^{e^+e^-\to W^+W^-}$ — Böhm et al. '88; Fleischer, Jegerlehner, Zralek '89

$\delta\mathcal{M}^{W\to f\bar{f}'}$ — Bardin, S. Riemann, T. Riemann '86; Jegerlehner '86; Denner, Sack '90

Full result for $\mathcal{M}^{e^+e^-\to WW\to 4f}_{\text{virt,fact,DPA}}$ in two ways: (Denner, S.D., Roth, Wackeroth '99)

- combination of known results (Böhm et al., Denner/Sack)
- completely new and independent calculation of all loops

$\Rightarrow$ Perfect numerical agreement

Virtual non-factorizable corrections

Melnikov, Yakovlev '96
Beenakker, Berends, Chapovsky '97
Denner, S.D., Roth '97

One-loop contribution to matrix element in DPA:

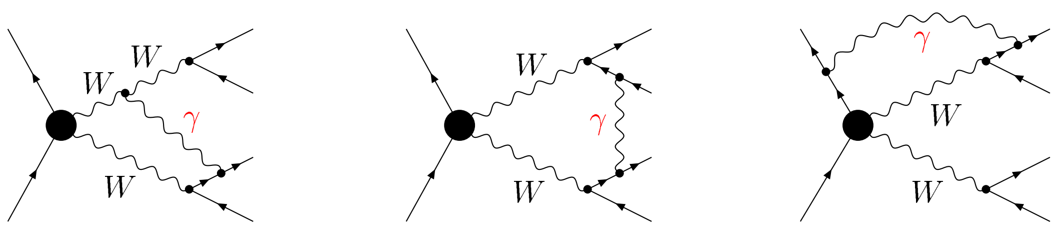
$$\mathcal{M}^{e^+e^-\to WW\to 4f}_{\text{virt,nonfact,DPA}} = \delta\mathcal{M}^{e^+e^-\to 4f}\Big|_{\text{doubly-resonant part}} - \mathcal{M}^{e^+e^-\to WW\to 4f}_{\text{virt,fact,DPA}}$$

$$= \mathcal{M}^{e^+e^-\to WW\to 4f}_{\text{Born,DPA}}\ \delta_{\text{virt,nonfact,DPA}}$$

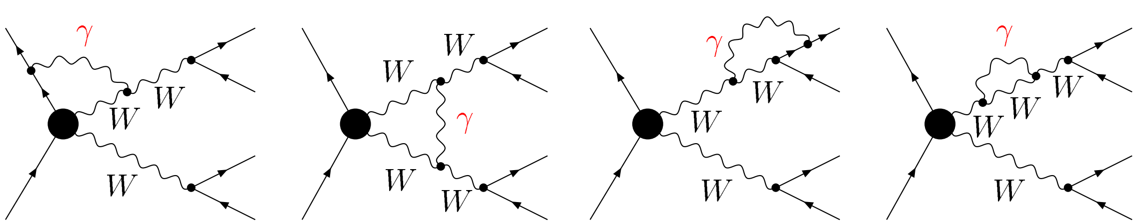
Features of the correction factor $\delta_{\text{virt,nonfact,DPA}}$:

- gauge invariance by definition
- no factorization of Breit–Wigner-type resonances
  $\Rightarrow$ non-trivial dependence on off-shellness $k_\pm^2$ of W bosons
- free of mass singularities
- compensates IR singularities of W bosons in $\mathcal{M}^{e^+e^-\to WW\to 4f}_{\text{virt,fact,DPA}}$

Manifestly non-factorizable diagrams:



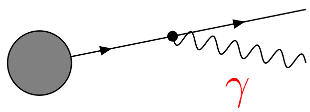
Diagrams contributing to factorizable and non-factorizable RCs:



Combination of virtual and real corrections

Corrections involve singularities:

Typical situation:



} Emission angle or $E_\gamma \to 0$

$\Rightarrow$ IR- and mass singularies proportional to $|\mathcal{M}_{\mathrm{Born}}|^2$

Note: connection with virtual corrections
$\to$ cancellations

$\Rightarrow$ Careful treatment necessary, since

- virtual corrections related to $\mathcal{M}_{\mathrm{Born,DPA}}^{\mathrm{e^+e^-\to WW\to 4}f}$
- real corrections related to $\mathcal{M}_{\mathrm{Born}}^{\mathrm{e^+e^-\to 4}f}$

Application of two methods:

1. "phase-space slicing"
   $\to$ separation of singularities by cuts
2. "dipole subtraction formalism":
   - QCD with massless partons — Catani, Seymour '96
   - γ radiation off massive fermions — S.D. '99, Roth '99

   $\to$ separation with subtraction function

"Master formula":

$$
\begin{aligned}
\int d\sigma = \frac{1}{2s}\Bigg\{ \int d\Phi_{4f} \Bigg[& |\mathcal{M}_{\mathrm{Born}}^{e^+e^-\to 4f}|^2 \\
\text{non-singular} \Bigg\{ & + 2\,\mathrm{Re}\Big((\mathcal{M}_{\mathrm{Born,DPA}}^{e^+e^-\to WW\to 4f})^* \,\delta\mathcal{M}_{\mathrm{virt,fact,DPA}}^{e^+e^-\to WW\to 4f} \\
& \qquad + |\mathcal{M}_{\mathrm{Born,DPA}}^{e^+e^-\to WW\to 4f}|^2 \,\delta_{\mathrm{virt,nonfact,DPA}} \Big) \\
& + |\mathcal{M}_{\mathrm{Born,DPA}}^{e^+e^-\to WW\to 4f}|^2 \,\delta_{\mathrm{sub},1}^{4f} \\
\text{explicit mass singularities} \Big\{ & + |\mathcal{M}_{\mathrm{Born}}^{e^+e^-\to 4f}|^2 \otimes \delta_{\mathrm{sub},2}^{4f} \Bigg] \\
& + \int d\Phi_{4f\gamma} \underbrace{\Bigg[|\mathcal{M}_{\mathrm{Born}}^{e^+e^-\to 4f\gamma}|^2 - |\mathcal{M}_{\mathrm{Born}}^{e^+e^-\to 4f}|^2 \,\delta_{\mathrm{sub}}^{4f\gamma} \Bigg]}_{\text{non-singular}} \Bigg\}
\end{aligned}
$$

Singular structures:

- $\delta_{\mathrm{sub}}^{4f\gamma}$: IR and collinear poles in $4f\gamma$ phase space
  (no regulators: $m_\gamma = m_f = 0$)

- $\delta_{\mathrm{sub},1}^{4f}$: IR and mass singularities
  $\alpha \ln(m_\gamma)\ln(m_f)$, $\alpha\ln(m_\gamma)$, $\alpha\ln^2(m_f)$, $\alpha\ln(m_f)$
  related to $4f$ kinematics
  $\rightarrow$ full compensation by virtual counterparts

- $\delta_{\mathrm{sub},2}^{4f}$: physical ISR singularities $\alpha\ln(m_e/E)$
  associated with $e \to e\gamma$ splitting function
  $\rightarrow$ full Born matrix element kept !

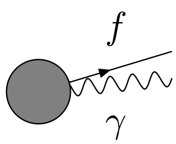
Mass singularities versus photon recombination

Recombination of soft and collinear γ's with fermions

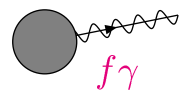
$\rightarrow$ compensation of final-state singularities

Possible procedure:

select fermion with smallest invariant mass $(p_f + p_\gamma)^2$, then:



$\}$ $(p_f + p_\gamma)^2 < M_{\text{rec}}^2 \Rightarrow$



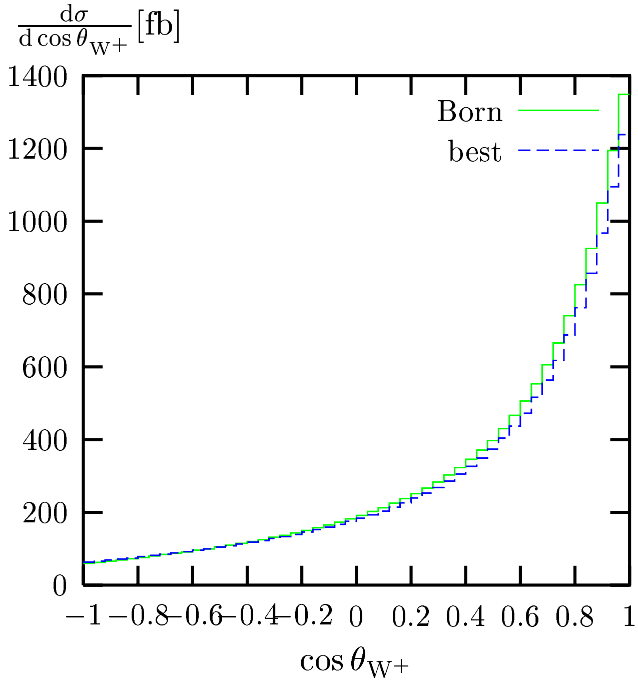
$p_{f\gamma} = p_f + p_\gamma$

$\rightarrow$ Qualitative simulation of imperfect detector resolution !

4 Numerical results of RacoonWW

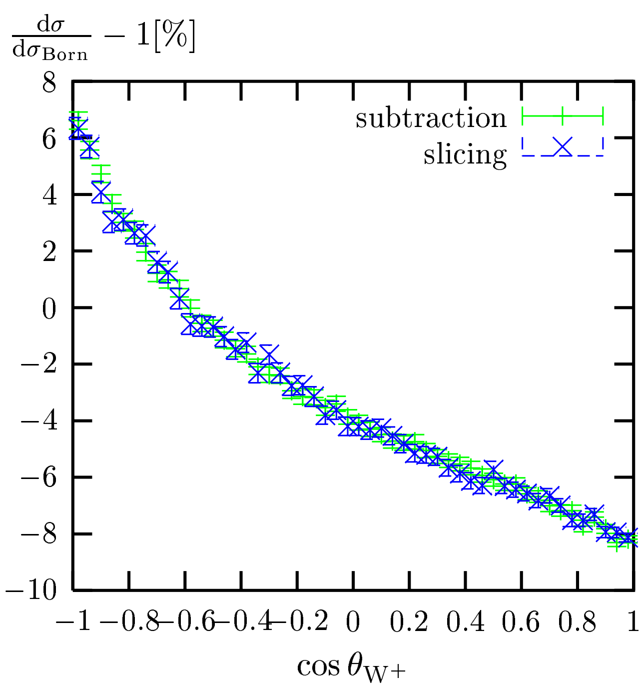
4.1 WW cross section and distributions

W-production angle distribution:



$e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu$

$\sqrt{s} = 200\,\mathrm{GeV}$

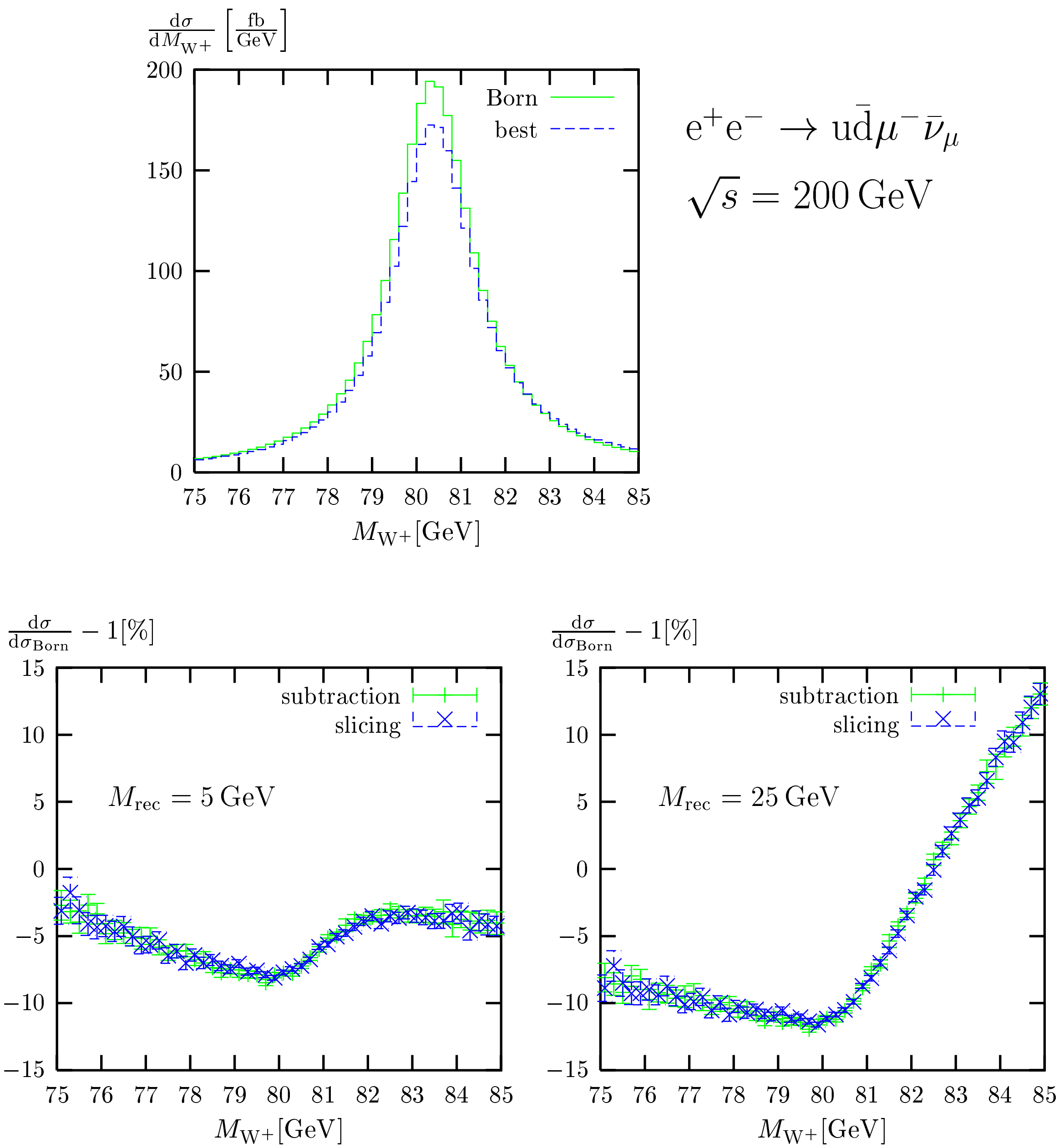


$M_{\mathrm{rec}} = 25\,\mathrm{GeV}$

Only weak

M_{rec} dependence

W^+ invariant-mass distribution:



⇒ Strong sensitivity to photon recombination procedure !

Corrections induce $\Delta M_W \sim +30\,\mathrm{MeV}$ for $M_{rec} = 25\,\mathrm{GeV}$

Impact of non-leading electroweak corrections ?

$\to$ Compare full RCs with improved Born approximation (IBA) !

IBA includes:

- leading universal effects: $\Delta\alpha$, $\Delta\rho$
- Coulomb singularity at WW threshold
- leading-log ISR
  $\to$ scale dependence via $\alpha^n \ln^n(m_e^2/Q^2)$

Old studies for on-shell process $\mathrm{ee} \to \mathrm{WW}$:

For integrated cross-sections:

$$\delta - \delta_{\mathrm{IBA}} \quad \sim \quad \begin{cases} 1\text{–}2\% & \text{at LEP2} \\ 10\text{–}20\% & \text{in TeV range} \end{cases}$$

Böhm, Denner, S.D. '92
CERN report 96-01
Denner, S.D. '97
Kuroda, Kuss, Schildknecht '97
Jadach et al. '98

But effects much larger in distributions !

New study: IBA option in RacoonWW

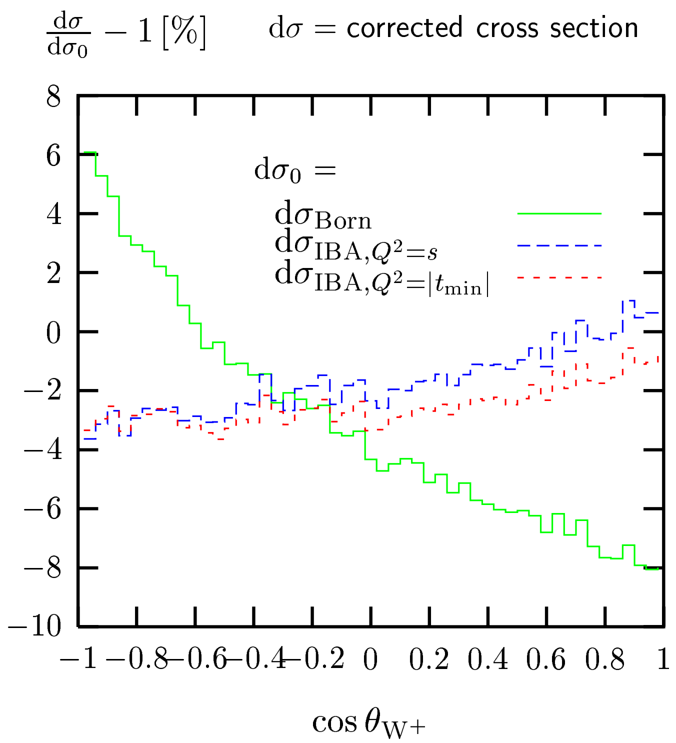
Total $\mathrm{e^+e^-} \to \mathrm{u\bar{d}}\mu^-\bar{\nu}_\mu$ cross section:

preliminary

$\delta - \delta_{\mathrm{IBA}}$	$\sqrt{s} = 200\,\mathrm{GeV}$	$\sqrt{s} = 500\,\mathrm{GeV}$
$Q^2 = s$	$-0.64(5)\%$	$-3.02(8)\%$
$Q^2 = \lvert t_{\mathrm{min}}\rvert$	$-1.81(5)\%$	$-1.16(8)\%$

Non-leading electroweak corrections to distributions

W production angle:



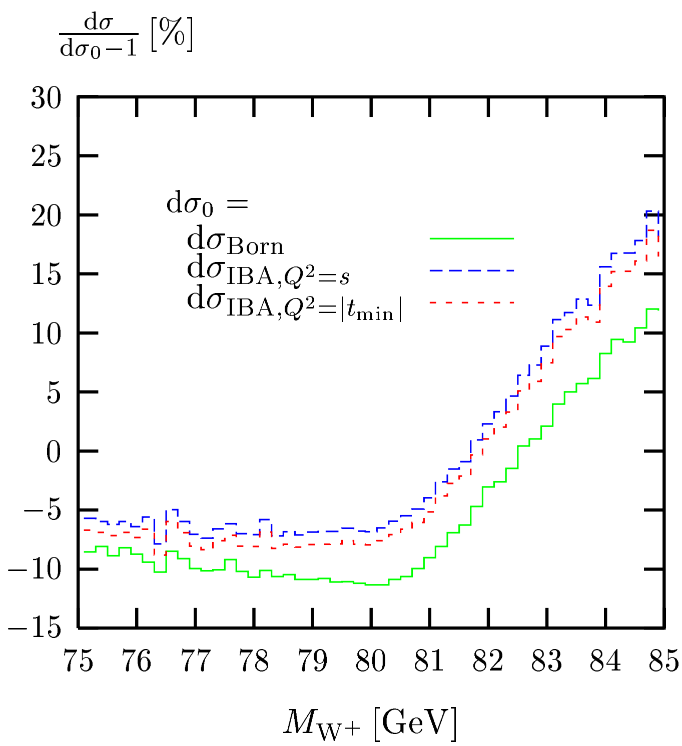
RacoonWW
preliminary

$e^+e^- \to u\bar{d}\mu^-\bar{\nu}_\mu$

$\sqrt{s} = 200\,\mathrm{GeV}$

IBA best for small θ_{W^+}

W invariant mass:

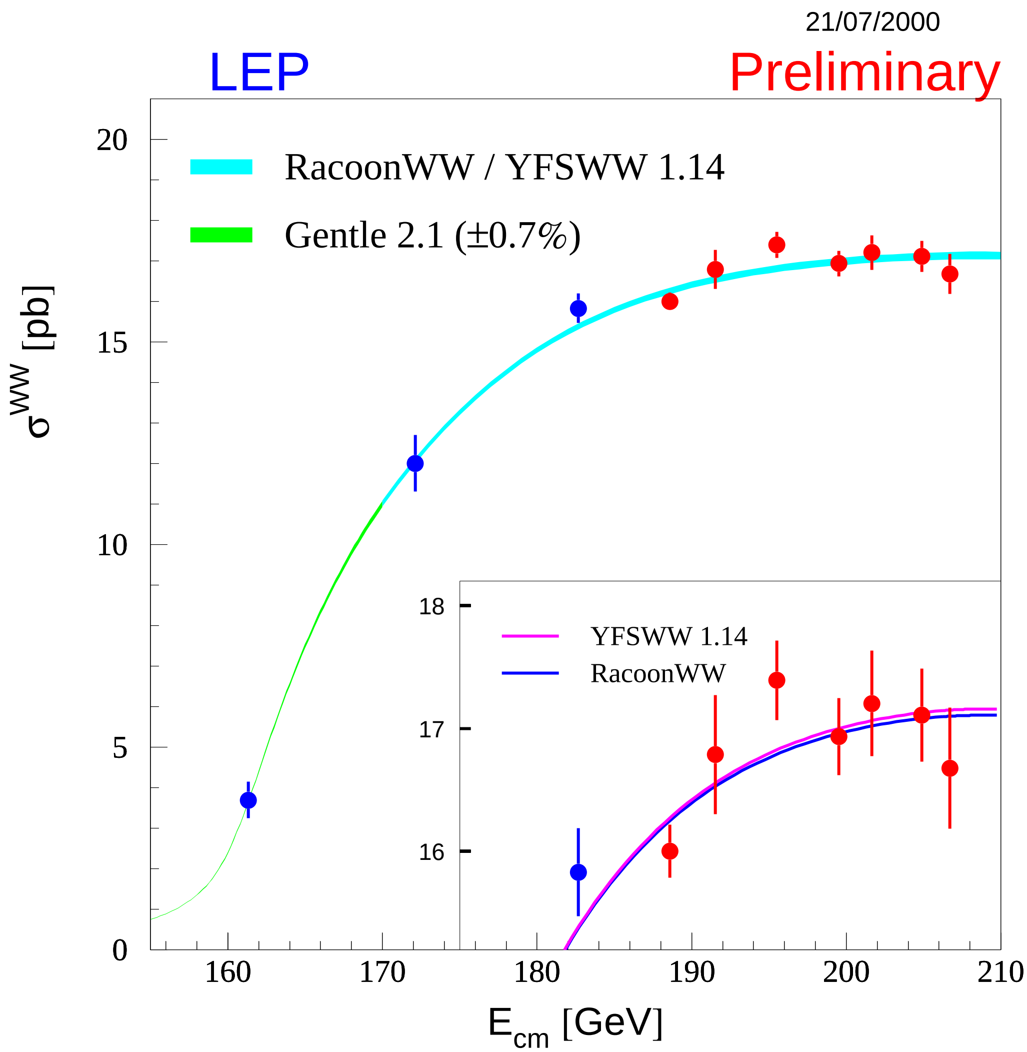


$M_{\mathrm{rec}} = 25\,\mathrm{GeV}$

IBA does not account for line-shape distortion (importance of FSR!)

Total WW cross section at LEP2:

LEPEWWG
Summer 2000



$\Rightarrow$ Evidence for non-universal electroweak corrections !

(New GENTLE results “tuned” to RacoonWW and YFSWW.)

4.2 Comparison of RacoonWW to related work

↪ Details in LEP2 MC workshop report hep-ph/0005309

Beenakker/Berends/Chapovsky:

- semianalytical DPA for virtual and real $\mathcal{O}(\alpha)$ corrections
- idealized situations (e.g. no photon recombination)
- numerical results for $\mathrm{e^+e^-} \to \mathrm{WW} \to 4$ leptons

→ Agreement of total cross sections within some 0.1%

YFSWW3 (Jadach et al.) → talk of B.F.L. Ward

Monte Carlo event generator with

- $\mathcal{O}(\alpha)$ correction to $\mathrm{ee} \to \mathrm{WW}$ with YFS exponentiation
- Elwk. corrections to decays via BR's, FSR with PHOTOS
- no full non-factorizable corrections

→ For LEP2 energies agreement of

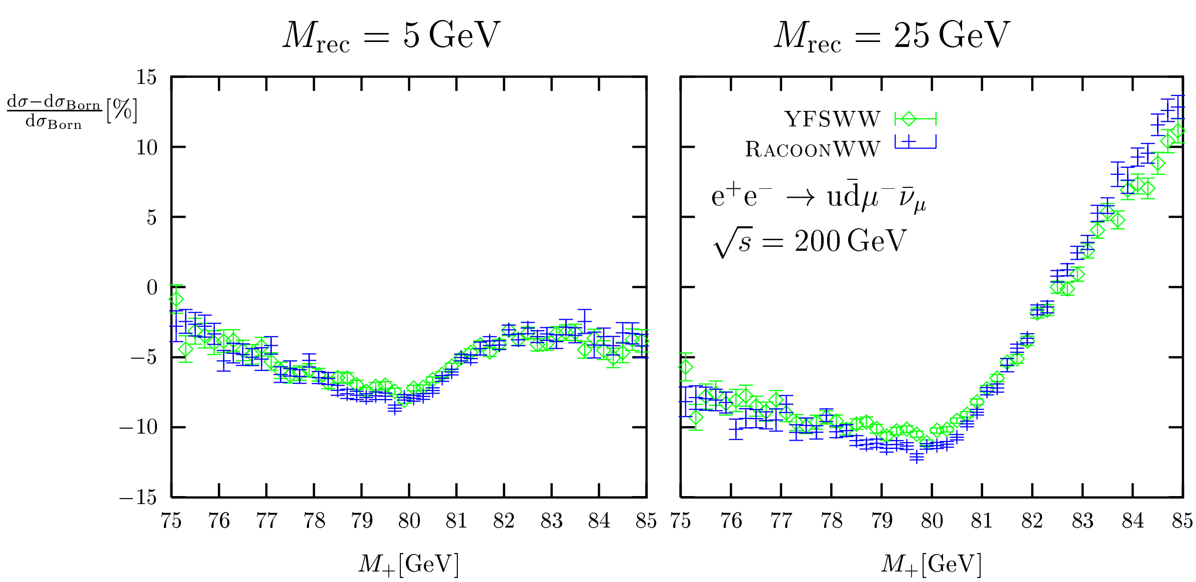
  - total cross sections within 0.3%
  - angular and invariant-mass distributions within 1%
  - photon spectra within 15–20%

RacoonWW versus YFSWW3 results

Total cross sections:

no cuts		$\sigma_{\rm tot}$[fb]	
final state	program	$\sqrt{s} = 200$ GeV	$\sqrt{s} = 500$ GeV
$\nu_\mu \mu^+ \tau^- \bar{\nu}_\tau$	YFSWW3	199.995(62)	89.607(32)
	RacoonWW	199.551(46)	90.018(27)
	(Y–R)/Y	0.22(4)%	−0.46(5)%
$\mathrm{u\bar{d}}\mu^-\bar{\nu}_\mu$	YFSWW3	622.71(19)	279.086(97)
	RacoonWW	621.06(14)	280.149(86)
	(Y–R)/Y	0.27(4)%	−0.38(5)%
$\mathrm{u\bar{d}s\bar{c}}$	YFSWW3	1937.40(61)	868.14(31)
	RacoonWW	1932.20(44)	871.66(27)
	(Y–R)/Y	0.27(4)%	−0.41(5)%

W line shape and photon recombination:



5 Summary

Event generator RacoonWW:

- lowest order: full amplitude for $\mathrm{e^+e^-} \to 4f$
- real corr's: full amplitude for $\mathrm{e^+e^-} \to 4f + \gamma$
  and ISR beyond $\mathcal{O}(\alpha)$
- virtual corr's: consistent DPA for $\mathrm{e^+e^-} \to \mathrm{WW} \to 4f$

$\Rightarrow$ First event generator with complete DPA for $\mathcal{O}(\alpha)$ corr's

State-of-the-art for $\mathrm{e^+e^-} \to \mathrm{WW} \to 4f$:

- Theoretical uncertainty for WW cross section
  $\lesssim 0.5\%$ (0.7%) for $500\,\mathrm{GeV} > \sqrt{s} > 180\,\mathrm{GeV}$ $(170\,\mathrm{GeV})$
- DPA not applicable near WW threshold
  $\to$ use IBA for $\sqrt{s} < 170\,\mathrm{GeV}$
- agreement between RacoonWW and YFSWW3 at LEP2 energies
  - within $\sim 0.3\%$ for total cross sections
  - within $\sim 1\%$ in angular and invariant-mass distributions

Physical insights:

- LEP2 data on WW cross section
  $\to$ significance of non-universal weak corrections
- importance of photon recombination in W reconstruction at LEP2 accuracy