"AMSB, R-symmetry and Yukawa textures"

Outline

1. Soft supersymmetry breaking
2. Exact results for β-functions
3. Anomaly Mediated Supersymmetry Breaking
4. $\mathcal{R}$-symmetry and the tachyonic sleptons

Work done with Ian Jack, Austin Pickering

N = 1 SUPERSYMMETRY

A general $N = 1$ theory is described by the superpotential:

$$W(\Phi) = \frac{1}{6}Y^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}\mu^{ij}\Phi_i\Phi_j$$

The corresponding Lagrangian is:

$$L_{\rm SUSY} = L_G + L_M$$

where

$$L_G = -\frac{1}{4}G^{\mu\nu}G_{\mu\nu} + i\lambda^\alpha\sigma^\mu_{\alpha\dot\alpha}D_\mu\overline{\lambda}^{\dot\alpha} + \frac{1}{2}D^2$$

and

$$\begin{aligned} L_M &= i\psi\sigma.D\overline{\psi} + |D_\mu\phi|^2 + F^iF_i + F^iW_i + F_iW^i \\ &- \frac{1}{2}W_{ij}\psi^i\psi^j - \frac{1}{2}W^{ij}\psi_i\psi_j \\ &+ g(R^a)^i{}_j\left[D^a\phi_i\phi^j - \sqrt{2}\phi_i\lambda^a\psi^j - \sqrt{2}\phi^j\overline{\psi}_i\overline{\lambda}^a\right] \end{aligned}$$

where $W^i \equiv \frac{\partial W}{\partial\phi_i}$ etc.

SOFT SUPERSYMMETRY BREAKING

The following terms are usually added to break supersymmetry:

$$\begin{aligned} L^{(1)}_{\rm SOFT} &= (m^2)^j{}_i \phi^i \phi_j \\ &+ \left(\tfrac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \tfrac{1}{2} b^{ij} \phi_i \phi_j + \tfrac{1}{2} M \lambda\lambda + \text{h.c.}\right). \end{aligned}$$

but there is no reason not to have these as well (unless there are gauge singlet fields):

$$L^{(2)}_{\rm SOFT} = \tfrac{1}{2} r_i^{jk} \phi^i \phi_j \phi_k + \tfrac{1}{2} {m_F}^{ij} \psi_i \psi_j + {m_A}^{ia} \psi_i \lambda_a + \text{h.c.}$$

None of these terms introduce quadratic divergences so we say they preserve naturalness. But as usual we'll ignore $L^{(2)}$.

THE SUPERSYMMETRIC STANDARD MODEL

The superpotential is (keeping only 3rd generation Yukawas):

$$W = \lambda_t H_2 Q \bar{t} + \lambda_b H_1 Q \bar{b} + \lambda_\tau H_1 L \bar{\tau} + \mu H_1 H_2$$

giving

$$\begin{aligned} L^{(1)}_{\mathrm{SOFT}} &= \sum_\phi m^2_\phi \phi^* \phi + \left[m^2_3 H_1 H_2 + \sum_{i=1}^{3} \tfrac{1}{2} M_i \lambda_i \lambda_i + \mathrm{h.c.} \right] \\ &+ \left[A_t \lambda_t H_2 Q \bar{t} + A_b \lambda_b H_1 Q \bar{b} + A_\tau \lambda_\tau H_1 L \bar{\tau} + \mathrm{h.c.} \right] \end{aligned}$$

and (but again we won't be using it)

$$\begin{aligned} L^{(2)}_{\mathrm{SOFT}} &= m_4 \psi_{H_1} \psi_{H_2} + m_9 \lambda_t H^*_1 Q \bar{t} \\ &+ m_7 \lambda_b H^*_2 Q \bar{b} + m_5 \lambda_\tau H^*_2 L \bar{\tau} + \mathrm{h.c.} \end{aligned}$$

The β-FUNCTIONS

- THE YUKAWA β-function

$$\beta_Y^{ijk} = Y^{p(ij}\gamma^{k)}{}_p = Y^{ijp}\gamma^k{}_p + (k \leftrightarrow i) + (k \leftrightarrow j),$$

- THE MASS β-function

$$\beta_\mu^{ij} = \mu^{p(i}\gamma^{j)}{}_p$$

$\gamma^i{}_j(g, Y, Y^*)$ is the anomalous dimension of the chiral multiplet.

These results are a consequence of the non-renormalisation theorem, and hold if we use MS or $\overline{\mathrm{MS}}$. They are preserved by a redefinition $g \to g + ag^3 + \cdots$ but not in general by an analogous redefinition of Y.

At one loop we have that

$$16\pi^2\gamma^{(1)i}{}_j = P^i{}_j = \frac{1}{2}Y^{ikl}Y_{jkl} - 2g^2C(R)^i{}_j.$$

THE GAUGE β-FUNCTION

$$\beta_g^{\mathrm{NSVZ}} = \frac{g^3}{16\pi^2}\left[\frac{Q - 2r^{-1}\mathrm{tr}[\gamma C(R)]}{1 - 2g^2 C(G)(16\pi^2)^{-1}}\right],$$

where $Q = T(R) - 3C(G) = N_f - 3N_c$ for SQCD.

For the case $\gamma = 0$ (no chiral superfields) we have

$$\beta_g^{\mathrm{NSVZ}} = -3\frac{g^3 C(G)}{16\pi^2} - 6\frac{g^5 C(G)^2}{(16\pi^2)^2} - 12\frac{g^7 C(G)^3}{(16\pi^2)^3} + \cdots$$

while

$$\beta_g^{\mathrm{DRED}} = -3\frac{g^3 C(G)}{16\pi^2} - 6\frac{g^5 C(G)^2}{(16\pi^2)^2} - 21\frac{g^7 C(G)^3}{(16\pi^2)^3} + \cdots$$

We can reconcile these results with a redefinition of g:

$$\delta g = \tfrac{3}{2} g^5 C(G)^2 (16\pi^2)^{-2}.$$

THE SOFT β-FUNCTIONS

All the soft β-functions can be expressed exactly in terms of the functions β_g and γ of the unbroken theory.

$$\begin{aligned}
\beta_h^{ijk} &= \gamma^{(i}{}_l h^{jk)l} - 2\gamma_1^{(i}{}_l Y^{jk)l} \\
\beta_b^{ij} &= \gamma^{(i}{}_l b^{j)l} - 2\gamma_1^{(i}{}_l \mu^{j)l} \\
\beta_M &= 2\mathcal{O}\left(\frac{\beta_g}{g}\right)
\end{aligned}$$

where

$$\mathcal{O} = \left(Mg^2\frac{\partial}{\partial g^2} - h^{lmn}\frac{\partial}{\partial Y^{lmn}}\right)$$

and

$$(\gamma_1)^i{}_j = \mathcal{O}\gamma^i{}_j .$$

THE SOFT SCALAR MASS β-FUNCTION

The β-function for the scalar m^2 is more subtle:

$$(\beta_{m^2})^i{}_j = \left[\Delta + X(g, Y, Y^*, h, h^*, m, M)\frac{\partial}{\partial g}\right]\gamma^i{}_j$$

where

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2M\overline{M}g^2\frac{\partial}{\partial g^2} + \tilde{Y}_{lmn}\frac{\partial}{\partial Y_{lmn}} + \tilde{Y}^{lmn}\frac{\partial}{\partial Y^{lmn}},$$

$Y_{lmn} = (Y^{lmn})^*$, and

$$\tilde{Y}^{ijk} = (m^2)^i{}_l Y^{ljk} + (m^2)^j{}_l Y^{ilk} + (m^2)^k{}_l Y^{ijl}.$$

Here

$$X_{\rm NSVZ} = -2\frac{g^3}{16\pi^2}\frac{r^{-1}{\rm tr}[m^2C(R)] - MM^*C(G)}{1 - 2C(G)g^2(16\pi^2)^{-1}}.$$

THE AMSB SOLUTION

Remarkably the β-function equations can be integrated:

$$\begin{aligned} M &= m_0\beta_g/g \\ h &= -m_0\beta_Y \\ m^2 &= \tfrac{1}{2}m_0 m_0^* \mu\frac{d}{d\mu}\gamma \\ b &= -m_0\beta_\mu \end{aligned}$$

To verify that these results are RG invariant, you need the exact result for X. The solution for M, h and m^2 are obtained if the only source of breaking is a vev for an auxiliary field in the supergravity multiplet itself: the AMSB scenario (m_0 is the gravitino mass). For b, the "natural" result would be of order $m_0\mu$, this is linked to the so-called μ-problem.

The Gaugino masses

In the AMSB scenario:

$$M_i = m_0 \frac{\beta_i}{g_i} = m_0 b_i \frac{\alpha_i}{4\pi}$$

With $b_i = (\frac{33}{5}, 1, -3)$ this gives

$$M_1 : M_2 : M_3 = 0.3 : 0.1 : 1,$$

to be compared with the usual assumption that $M_1 = M_2 = M_3$ at gauge unification, which gives

$$M_1 : M_2 : M_3 = 0.14 : 0.28 : 1,$$

Thus in the AMSB scenario, there is likely to be an approximately degenerate triplet of light winos (a chargino and a neutralino). The characteristic phenomenology has been explored in a number of papers.

The Slepton Mass Problem

The first generation has negligible Yukawa couplings so

$$\begin{aligned} 4\pi\gamma_{e^c} &= -\tfrac{6}{5}\alpha_1 \\ 4\pi\gamma_E &= -\tfrac{3}{2}\alpha_2 - \tfrac{3}{10}\alpha_1 \end{aligned}$$

which gives

$$\begin{aligned} 4\pi m_{e^c}^2 &= -\tfrac{6}{10}|m_0|^2\beta_{\alpha_1} \\ 4\pi m_E^2 &= -|m_0|^2\left(\tfrac{3}{4}\beta_{\alpha_2} + \tfrac{3}{20}\beta_{\alpha_1}\right) \end{aligned}$$

The squarks are saved by asymptotic freedom, i.e. because $\beta_{\alpha_3} < 0$.

First solution explored in detail was to add a universal soft breaking:

$$m^2 = \frac{1}{2}|m_0|^2\mu\frac{d}{d\mu}\gamma + \overline{m}_0^2$$

Defect: no longer RG invariant: we've shown that one solution is to use Fayet-Iliopoulos D-terms, and an extra U_1, but here we consider an alternative based on $\mathcal{R}$-symmetry.

AMSB and $\mathcal{R}$-SYMMETRY

The following generalisation of the m^2_{AMSB} solution is RG invariant:

$$(m^2)^i{}_j = m^2_{AMSB} + \overline{m}_0^2(\gamma^i{}_j + (1 - \frac{3}{2}\mathcal{R}^i)\delta^i{}_j)$$

as long as

$$\begin{aligned}(\mathcal{R}^i + \mathcal{R}^j + \mathcal{R}^k)Y_{ijk} &= 2Y_{ijk} \\ 2\,\mathrm{Tr}\left[(1 - \frac{3}{2})\mathcal{R}C(R)\right] + Q &= 0\end{aligned}$$

This corresponds precisely to requiring the $\mathcal{R}$'s to be the charges of an anomaly-free $\mathcal{R}$-symmetry.

$\mathcal{R}$ AND THE MSSM

In the MSSM, requiring that there be a generation-independent $\mathcal{R}$-charge assignment consistent with the superpotential and with cancellation of $\mathcal{R}(SU_3)^2$, $\mathcal{R}(SU_2)^2$ and $\mathcal{R}(U_1)^2$ mixed anomalies leads to....

NO SOLUTION

So we have to distinguish between the generations. This leads in general to FCNC problems, but we can avoid these if we

(a) Assign identical $\mathcal{R}$-charges to the first two generations

(b) Choose $\mathcal{R}$-charges so that only the third generation appears in the superpotential.

If then we also impose cancellation of the $\mathcal{R}^2U_1$ anomaly, then we can choose the leptonic $\mathcal{R}$-charges $L_{1,3}$ and $e_{1,3}$ arbitrarily and the others $(q_{1,3}, u_{1,3}, d_{1,3}, h_1, h_2)$ are given in terms of these four.

YUKAWA TEXTURES

Let us suppose that the the light quark and lepton masses originate from higher-dimension terms in the effective field theory of the form (for the up-type quarks)

$$H_2 Q_i u_j (\theta/M_U)^{a_{ij}} \quad \text{or} \quad H_2 Q_i u_j (\overline{\theta}/M_U)^{\overline{a}_{ij}}$$

where $\theta, \overline{\theta}$ is a pair of MSSM singlet fields with $\mathcal{R}$-charges $\pm q_\theta$ that get equal vacuum expectation values.

The result is Yukawa textures of the form

$$\Delta_u = \begin{pmatrix} \epsilon^{\sigma|\kappa|} & \epsilon^{\sigma|\kappa|} & \epsilon^{\sigma|\delta_q|} \\ \epsilon^{\sigma|\kappa|} & \epsilon^{\sigma|\kappa|} & \epsilon^{\sigma|\delta_q|} \\ \epsilon^{\sigma|\kappa+\delta_q|} & \epsilon^{\sigma|\kappa+\delta_q|} & 1 \end{pmatrix}$$

$$\Delta_d = \begin{pmatrix} \epsilon^{\sigma|\kappa|} & \epsilon^{\sigma|\kappa|} & \epsilon^{\sigma|\delta_q|} \\ \epsilon^{\sigma|\kappa|} & \epsilon^{\sigma|\kappa|} & \epsilon^{\sigma|\delta_q|} \\ \epsilon^{\sigma|\kappa-\delta_q|} & \epsilon^{\sigma|\kappa-\delta_q|} & 1 \end{pmatrix}$$

for the up and down quarks, and

$$\Delta_L = \begin{pmatrix} \epsilon^{\sigma|\kappa+3|} & \epsilon^{\sigma|\kappa+3|} & \epsilon^{\sigma|\delta_L|} \\ \epsilon^{\sigma|\kappa+3|} & \epsilon^{\sigma|\kappa+3|} & \epsilon^{\sigma|\delta_L|} \\ \epsilon^{\sigma|\kappa+3-\delta_L|} & \epsilon^{\sigma|\kappa+3-\delta_L|} & 1 \end{pmatrix}$$

for the leptons, where κ, δ_L, δ_q are functions of the leptonic charges, $\epsilon = \left|\frac{<\theta>}{M_U}\right|$ and $\sigma = (|q_\theta|)^{-1}$.

If we hypothesise that $\Delta_L = \Delta_d$ then we are led to an almost unique solution for the textures such that

$$\Delta_u = \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon \\ \epsilon^4 & \epsilon^4 & \epsilon \\ \epsilon^5 & \epsilon^5 & 1 \end{pmatrix}, \Delta_d = \Delta_L = \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon \\ \epsilon^4 & \epsilon^4 & \epsilon \\ \epsilon^3 & \epsilon^3 & 1 \end{pmatrix}.$$

and the fermionic $\mathcal{R}$-charges of the particles are as follows:

q_3	l_3	u_3	d_3	e_3
$\frac{e}{6} - \frac{2}{9}$	$-\frac{e}{2} - \frac{1}{6}$	$-\frac{2e}{3} - \frac{29}{18}$	$\frac{e}{3} + \frac{1}{18}$	e

q_1	l_1	u_1	d_1	e_1
$\frac{e}{6} - \frac{43}{72}$	$-\frac{e}{2} + \frac{5}{24}$	$-\frac{2e}{3} + \frac{19}{72}$	$\frac{e}{3} - \frac{77}{72}$	$e + \frac{9}{8}$

H_1	H_2
$-\frac{e}{2} - \frac{5}{6}$	$\frac{e}{2} + \frac{5}{6}$

Table 1: The fermionic $\mathcal{R}$-charges

The $\mathcal{R}$-contributions to the slepton masses are all positive as long as

$$-\frac{1}{3} < e < \frac{1}{3} \quad \text{and} \quad \overline{m}_0^2 < 0.$$

The framework is consistent with reasonable-looking Yukawa matrices, e.g:

$$\lambda_u \propto \begin{pmatrix} -0.28\epsilon^4 & 1.3\epsilon^4 & 0.4\epsilon \\ -0.32\epsilon^4 & 1.45\epsilon^4 & 1.36\epsilon \\ -0.36\epsilon^5 & 1.67\epsilon^5 & 1 \end{pmatrix}$$

$$\lambda_d \propto \begin{pmatrix} -1.75\epsilon^4 & 1.99\epsilon^4 & 0.25\epsilon \\ -3.01\epsilon^4 & 2.53\epsilon^4 & 1.18\epsilon \\ 0.26\epsilon^3 & -0.48\epsilon^3 & 0.95 \end{pmatrix}$$

with $\epsilon = 0.25$. These lead to correct quark masses and CKM matrix (neglecting CP-violation).

FCNC: With a typical spectrum we find, rotating the squarks to the quark-diagonal mass basis that (for example):

$$\delta_{12}^{LL} = (m^2)_{ds}^{LL}/(m_{\tilde{g}}^2) \approx 10^{-2}$$

for $m_{\tilde{g}} = 1\mathrm{TeV}$. $\mu \to e\gamma$ and $b \to s\gamma$ are also adequately suppressed.

On the next transparency we give the mass spectrum. Interesting features include the large $\tilde{\tau}/\tilde{e}$ splitting and the fact that the $\tilde{\nu}_\tau$ is the LSP in some regions.

$\tan\beta(sgn\mu)$	2(+)	2(−)	5(+)	5(+)	10(+)
e	$-1/9$	$-1/9$	$-1/9$	$-2/9$	$-2/9$
$\overline{m}_0^2(\mathrm{TeV}^2)$	-0.1	-0.1	-0.1	-0.25	-0.2
$\tilde{t}_1$	652	615	567	302	404
$\tilde{t}_2$	882	908	876	879	875
$\tilde{b}_1$	865	865	843	853	843
$\tilde{b}_2$	977	977	974	1009	987
$\tilde{\tau}_1$	94	87	75	136	86
$\tilde{\tau}_2$	110	116	127	289	251
$\tilde{u}_L$	918	918	917	880	892
$\tilde{u}_R$	997	997	997	1084	1057
$\tilde{d}_L$	920	920	921	884	896
$\tilde{d}_R$	887	887	887	776	814
$\tilde{e}_L$	260	260	261	473	418
$\tilde{e}_R$	423	423	423	664	590
$\tilde{\nu}_\tau$	83	83	73	277	234
$\tilde{\nu}_e$	251	251	249	467	410
h	96	105	119	114	124
H	598	598	585	121	308
A	593	593	584	110	307
$H^\pm$	599	599	590	137	318
$\tilde{\chi}_1^\pm$	98	116	104	101	106
$\tilde{\chi}_2^\pm$	628	625	663	449	530
$\tilde{\chi}_1$	98	115	103	99	103
$\tilde{\chi}_2$	364	372	367	357	365
$\tilde{\chi}_3$	619	620	662	446	532
$\tilde{\chi}_4$	637	628	672	470	544
$\tilde{g}$	1008	1008	1008	1008	1008

Mass Sum Rules

The following sum rules for the physical masses are independent of the $\mathcal{R}$-assignments:

$$
\begin{aligned}
m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2} + m^2_{\tilde{b}_1} + m^2_{\tilde{b}_2} - 2m^2_t - 2.75 m^2_{\tilde{g}} &= 0.92\overline{m}^2_0 \\
m^2_{\tilde{\tau}_1} + m^2_{\tilde{\tau}_2} + m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2} - 2m^2_t - 1.14 m^2_{\tilde{g}} &= 0.96\overline{m}^2_0 \\
m^2_{\tilde{e}_L} + m^2_{\tilde{e}_R} + m^2_{\tilde{u}_L} + m^2_{\tilde{u}_R} - 1.70 m^2_{\tilde{g}} &= -3.56\overline{m}^2_0 \\
m^2_{\tilde{u}_R} + m^2_{\tilde{d}_R} + m^2_{\tilde{u}_L} + m^2_{\tilde{d}_L} - 3.51 m^2_{\tilde{g}} &= 0.91\overline{m}^2_0 \\
m^2_A - 2\sec 2\beta \left(m^2_{\tilde{\tau}_1} + m^2_{\tilde{\tau}_2} - 2m^2_\tau \right) - 0.49 m^2_{\tilde{g}} &= 1.05\overline{m}^2_0
\end{aligned}
$$

where the masses are in TeV^2.

Conclusions/In progress

- The μ-problem. A version of either model with extra states with non-trivial MSSM quantum numbers might be interesting.

- Neutrino masses, and leptonic flavor violation

- Finite temperatures, Charge/colour breaking vacua...