$|V_{ub}|$ From $b \to s\gamma$ and Semileptonic Decays

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Cabibbo-Kobayashi-Maskawa Matrix

$$\begin{pmatrix} \bar{u} & \bar{c} & \bar{t} \end{pmatrix} \gamma^\mu V_{\rm CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} W_\mu + {\rm h.c.}$$

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

How can we measure $|V_{ub}|$?

Need to look at $b \to u$ transitions

- Exclusive semileptonic decays: $B \to \rho \ell \bar{\nu}$, $B \to \pi \ell \bar{\nu}$.
- Inclusive semileptonic decays: $B \to X_u \ell \bar{\nu}$.

Unable to calculate (perturbatively) hadronic dynamics
$\Rightarrow$ LARGE MODEL DEPENDENCE

e.g., a recent analysis of CLEO using $B \to \rho \ell \bar{\nu}$ gives

$$|V_{ub}| = [3.25 \pm 0.14({\rm stat.})^{+0.21}_{-0.29}({\rm syst.}) \pm 0.55({\rm model})] \times 10^{-3}$$

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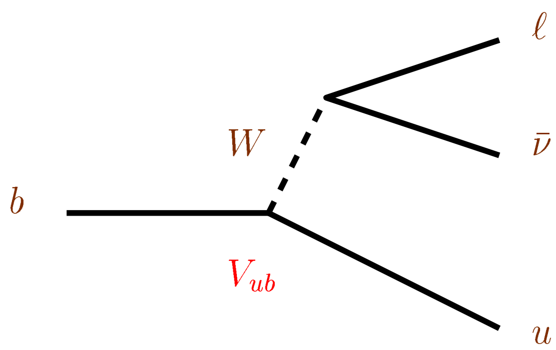
$|V_{ub}|$ From Inclusive Decays

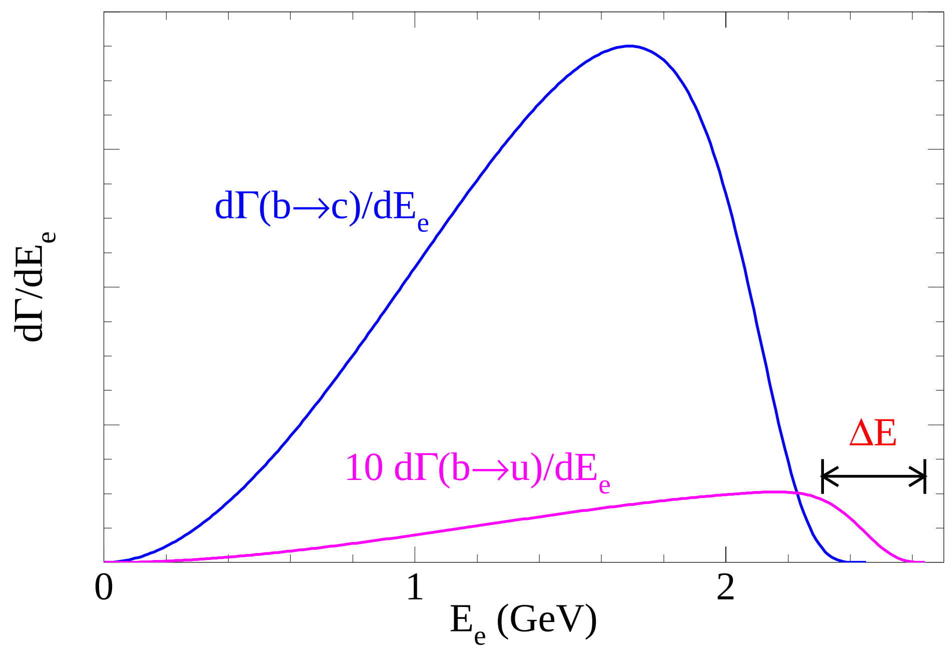
To get $|V_{ub}|$, would like to measure $b \to u\ell\bar{\nu}$ total rate



Problem: Large background from $b \to c\ell\bar{\nu}$

To remove background, need to cut

i.e., cut on electron spectrum



New small mass scale $\Delta E \approx 300$ MeV $\Rightarrow \log \frac{m_b}{\Delta E}$

Calculating The Electron Spectrum

Inclusive $b \to u\ell\bar{\nu}$ calculation begins with effective Hamiltonian:

$$\begin{aligned} H_{eff} &= \frac{-4G_F}{\sqrt{2}} V_{ub} (\bar{u}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu P_L \nu_\ell) \\ &= \frac{-4G_F}{\sqrt{2}} V_{ub} J_\mu J_\ell^\mu \end{aligned}$$

Differential decay distribution:

$$d\Gamma \propto |V_{ub}|^2 W_{\alpha\beta} L^{\alpha\beta}$$

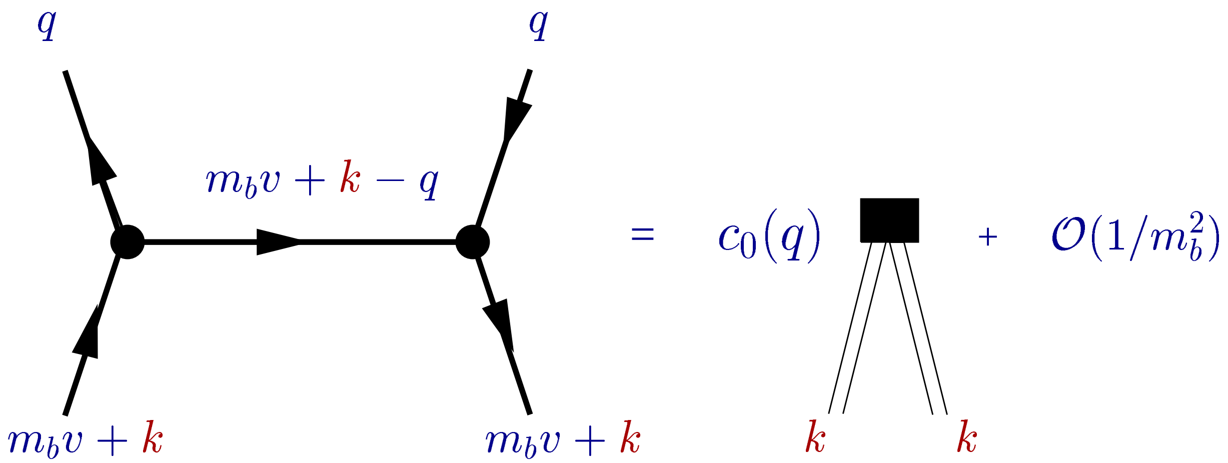
where $L^{\alpha\beta}$ is leptonic tensor, $W_{\alpha\beta}$ is hadronic tensor

$W_{\alpha\beta}$ related by Optical Theorem to imaginary part of time ordered product of currents

$$\begin{aligned} W_{\alpha\beta} &= -\frac{1}{\pi} \mathrm{Im}\, T_{\alpha\beta} \\ T_{\alpha\beta} &= -\frac{i}{2M_B} \int d^4x e^{-iq\cdot x} \langle B | T(J_\alpha^\dagger(x) J_\beta(0)) | B \rangle \end{aligned}$$

The Operator Product Expansion

$T_{\alpha\beta}$ can be calculated using OPE, with
Wilson coefficients calculated in perturbation theory



Tree level $\propto$ **1**

First non-perturbative corrections $\propto (\lambda_1, \lambda_2)$

$$\lambda_1 = \frac{\langle B|\bar{h}_v (iD)^2 h_v|B\rangle}{2M_B} = -0.13 \pm -0.06$$

$$\lambda_2 = \frac{\langle B|\bar{h}_v g\sigma_{\mu\nu} G^{\mu\nu}|B\rangle}{12M_B} = 0.12$$

Non-perturbative Corrections

OPE: expansion in $\frac{\Lambda}{m_b}$, $\frac{\Lambda}{m_b - 2E_\ell}$

Problem as $E_\ell \to \frac{m_b}{2}$, higher dimensional operators not suppressed

Expansion becomes singular

$$\frac{d\Gamma}{dx} \sim \frac{5\lambda_1 + 33\lambda_2}{3m_b^2}\theta(1-x) - \frac{\lambda_1 + 33\lambda_2}{6m_b^2}\delta(1-x) - \frac{\lambda_1}{6m_b^2}\delta'(1-x)$$

where $x = 2E_\ell/m_b$

Breakdown of OPE manifests as appearance of (derivatives of) δ-functions

Need to resum most singular terms $\delta^{(n)}(1-x)$!

Can be resummed into universal structure function, $f(k_+)$. Rate now is convolution

$$\frac{d\Gamma}{dE_\ell} = \int dk_+ \, f(k_+) \frac{d\Gamma_p}{dE_\ell}(m_b^*)$$

Only first 3 moments of $f(k_+)$ known

$\Rightarrow$ MODEL DEPENDENCE

Perturbative Corrections

Rate near endpoint ($x \to 1$)

$$\frac{d\Gamma}{dx} \propto 1 - \frac{2\alpha_s}{3\pi}\left[\log^2(1-x) + \frac{31}{6}\log(1-x) + \pi^2 + \frac{5}{4}\right]$$

As $x \to 1$, logs become large $\Rightarrow$ need to resum

To resum, use Infrared Factorization

Same techniques used for DIS, Drell-Yan, etc

Resum logs for both $b \to u\ell\bar{\nu}$ and $b \to s\gamma$

In endpoint region

- Light quark shot out with large energy, small invariant mass
- Jet of particles produced through collinear radiation
- Constituents of jet can interact with each other and b quark through soft gluons
- Soft gluons do not care if $b \to u\ell\bar{\nu}$ or $b \to s\gamma$
- Hard gluon exchange disallowed

Three configurations of momenta:

$$\text{Hard } (H): \quad k_+ \sim k_- \sim k_t = O(m_b)$$

$$\text{Jet } (J): \quad k_+ = O[m_b(1-x)],\ k_- = O(m_b),$$
$$k_t = O(m_b\sqrt{1-x})$$

$$\text{Soft } (S): \quad k_+ \sim k_- \sim k_t = O[m_b(1-x)]$$

Rate can be written in factorized form (as $x \to 1$)

$$\frac{d\Gamma}{dx} \sim \int dz S(z,\mu) J(z,\mu) H(\mu)$$

where μ is factorization scale

S same for $b \to u\ell\bar{\nu}$ and $b \to s\gamma$, H and J depend on process

Rate factorizes after taking moments

$$\begin{aligned} M_N^\gamma &= \int_o^{M_B/m_b} dx\, x^{n-1} \frac{1}{\Gamma_o^\gamma}\frac{d\Gamma^\gamma}{dx} = S_N J_N^\gamma H_N^\gamma \\ M_N^{sl} &= -\int_o^{M_B/m_b} dx\, x^{n-1} \frac{1}{\Gamma_o^\gamma}\frac{d}{dx}\frac{d\Gamma^\gamma}{dx} \\ &= \int dx_\nu S_N J_N^{sl}(x_\nu) H_N^{sl}(x_\nu) \end{aligned}$$

$S_N = f_n \sigma_N$ where f_N are moments of shape function

$$\begin{aligned} M_N^\gamma &= f_N \sigma_N J_N^\gamma H_N^\gamma \\ M_N^{sl} &= \int dx_\nu f_N \sigma_N J_N^{sl}(x_\nu) H_N^{sl}(x_\nu) \end{aligned}$$

Important fact: after resumming perturbative corrections

$$\sigma_N J_N^{sl} = \sigma_N J_N^\gamma \exp[g_{sl}(\alpha_s \log N)]$$

We can now combine above results

$$\begin{aligned} M_N^{sl} &= \int dx_\nu f_N \sigma_N J_N^{sl}(x_\nu) H_N^{sl}(x_\nu) \\ &= \int dx_\nu f_N \sigma_N J_N^\gamma \exp[g_{sl}(\alpha_s \log N)] H_N^{sl}(x_\nu) \\ &= \int dx_\nu M_N^\gamma \exp[g_{sl}(\alpha_s \log N)] H_N^{sl}(x_\nu) \end{aligned}$$

Going back to x space, RHS is convolution of $b \to s\gamma$ rate with known function. This means

$$\boxed{\frac{|V_{ub}|^2}{|V_{ts}^* V_{tb}|^2} = \frac{\int \Gamma(b \to u \ell \nu)}{\int \int \frac{d\Gamma^\gamma}{dx^\gamma} * K(x^\gamma, \alpha_s)}}$$

Uncertainties

Corrections to $|V_{ub}|^2$ are $O(\Lambda/m_b, \alpha_s(1-x), (1-x)^3)$

For cut on electron energy $x_{\rm cut} = 2E_e^{\rm cut}/m_b \approx 0.87$, these corrections are all less than 10%.

Parton-Hadron Duality

- Hard to quantify
- Expect power suppressed corrections
- If spectrum dominated by single resonances expect big duality errors $(B \to \pi\ell\bar{\nu},\ B \to \rho\ell\bar{\nu})$
- Similar strategy for V_{ub} possible using hadronic invariant mass

$$\frac{d\Gamma}{dE_e} : \quad \delta\Gamma((M_B^2 - M_D^2)/(2M_B) < E_\ell < M_B/2) \quad \sim 10\%$$

$$\frac{d\Gamma}{ds_H} : \quad \delta\Gamma(0 < s_H < M_D^2) \quad \sim 40 - 80\%$$

$$\frac{d\Gamma}{dq^2} : \quad \delta\Gamma((M_B - M_D)^2 < q^2 < M_B^2) \quad \sim 20\%$$

Conclusions

- Can remove model dependence from $b \to u\ell\bar{\nu}$ by combining with $b \to s\gamma$
- Model independent extraction of $|V_{ub}|$ can be made with error less than 10%, modulo parton-hadron violations
- Should not trust <u>any</u> extraction until convergence of measurements
- Comparing different extractions may teach us about parton-hadron duality