# Do non-trivial ultraviolet-stable fixed points exist in SUSY gauge theories?

Stephen P. Martin
(Northern Illinois University and Fermilab)

work (to appear) with James D. Wells

- IR-stable vs. UV-stable fixed points in SUSY gauge theories.
- Four-loop order $\beta$ functions (NSVZ and DRED)
- At three and four loop order, many realistic models have UV-stable fixed points at intermediate coupling:
  - ⋆ Minimal Missing Partner $SU(5)$
  - ⋆ SUSY QCD with $N_f > 3N_C$ flavors
  - ⋆ The MSSM with many vectorlike flavors
- Implications of UV-stable fixed points (IF they exist!)
- Constraints on UV-stable fixed points? Anomalous dimensions of gauge-invariant operators and unitarity.
- The large-$N_f$ limit: an all-orders test?

SUSY gauge theories with positive one-loop $\beta$ function are usually assumed to have a Landau pole in the UV. However, perturbative corrections to the $\beta$ function beyond two loops are typically large and negative.

Can the full non-perturbative $\beta$ function have a non-trivial ultraviolet-stable fixed point?

Consider a SUSY Yang-Mills theory with a simple gauge group and no superpotential.

$$\frac{dg}{dt} = \beta = g \sum_{n=1}^{\infty} b^{(n)} \left( \frac{g^2}{16\pi^2} \right)^n .$$

The $b^{(n)}$ coefficients are scheme-dependent for $n \geq 3$, and are known in the dimensional reduction (DRED) and Novikov-Shifman-Vainshtein-Zakharov (NSVZ) schemes to four-loop order.

Jack, Jones, North 9609325
Jack, Jones, Pickering 9805482

Notation: Introduce group theory invariants

$C_G$ = Casimir invariant of the gauge group (Dynkin index of the adjoint).

$S_n = \sum_R I_R C_R^n$

(Here $I_R$ = Dynkin index and $C_R$ = Casimir invariant of the rep $R$.)

In the NSVZ scheme:

$$\frac{dg}{dt} = \beta = g \sum_{n=1}^{\infty} b^{(n)} \left(\frac{g^2}{16\pi^2}\right)^n .$$

$$\begin{aligned}
b^{(1)} &= S_0 - 3C_G \\
b^{(2)} &= 4S_1 + 2C_G b^{(1)} \\
b^{(3)} &= 4C_G^2 S_0 + 20C_G S_1 - 12C_G^3 - 8S_2 - 4S_0 S_1; \\
b^{(4)} &= 32S_3 + 48[\zeta(3) - 1]S_1^2 + 76C_G^2 S_1 \\
&\quad +8C_G^3 S_0 + 8S_0 S_2 \\
&\quad -[8 + 48\zeta(3)]C_G S_0 S_1 - 40C_G S_2 \\
&\quad -24C_G^4 - 4S_0^2 S_1
\end{aligned}$$

Notes:

- 2 loop contribution to $\beta_g$ is positive if 1 loop is.
- 3 and 4 loop terms have large negative contributions.
- Larger and/or more numerous chiral superfields imply larger $S_i$.
- $b^{(3)}$ and $b^{(4)}$ are different but similar in DRED.

DISCLAIMER:

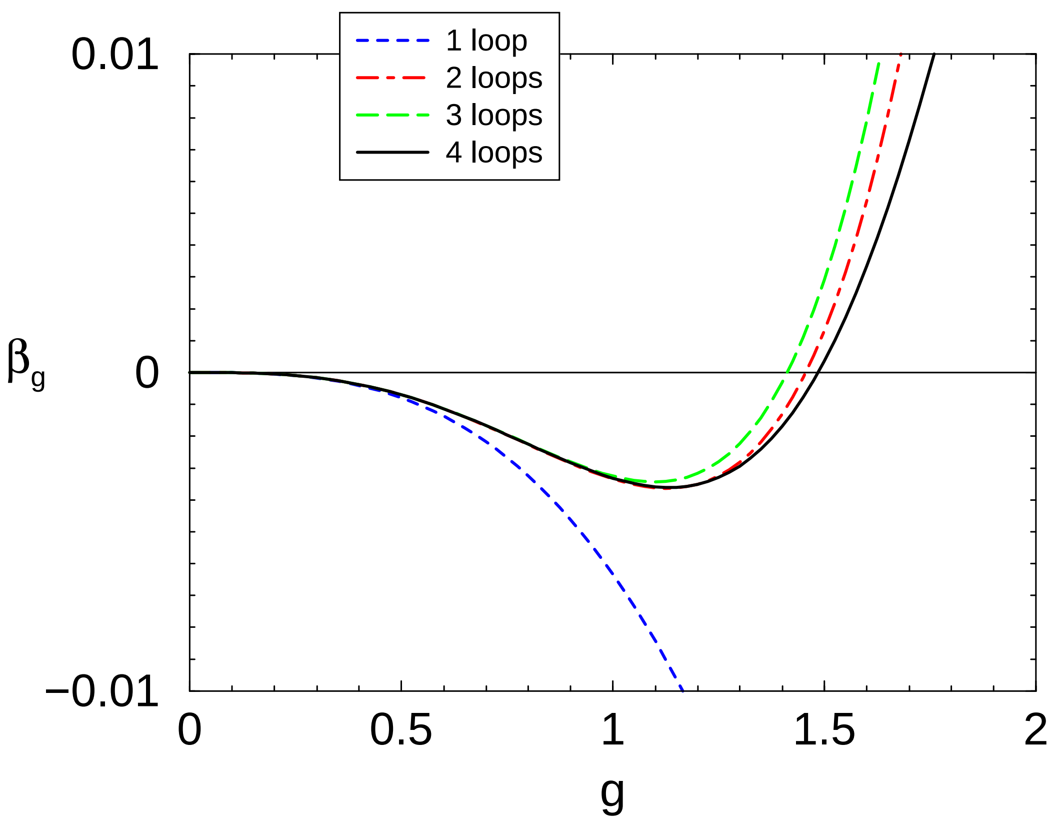
Some of the following discussion involves finite orders in perturbation theory, in a coupling constant regime in which perturbation theory shows no signs of convergence. Results should be considered as "suggestive hints", rather than "evidence" of anything. No warranty, expressed or implied, guarantees that the finite order $\beta$ functions have any validity. The FDA has not evaluated these claims. Your mileage may vary. Past performance is not an indicator of future results. The customer assumes all responsibility.

IR-stable fixed points: the superconformal window.

For SUSYQCD with $N_c$ colors and $N_f$ flavors, duality arguments imply that there is an IR-stable fixed point if $3N_c/2 < N_f < 3N_c$.

Seiberg 1994

Corresponds to $b^{(1)} < 0$ and $b^{(2)} > 0$. Only seen reliably in perturbation theory if $N_f$ is slightly less than $3N_c$. For example, with $N_c = 4$ and $N_f = 11$, the above NSVZ $\beta$ function gives:




The point $g = g_* \neq 0$ where $\beta = 0$ is an IR-stable fixed point, since $\partial\beta/\partial g > 0$ at the fixed point.

The known superconformal IR-stable fixed points require $b^{(1)} < 0$, so that the $\beta$ function is negative near $g = 0$.

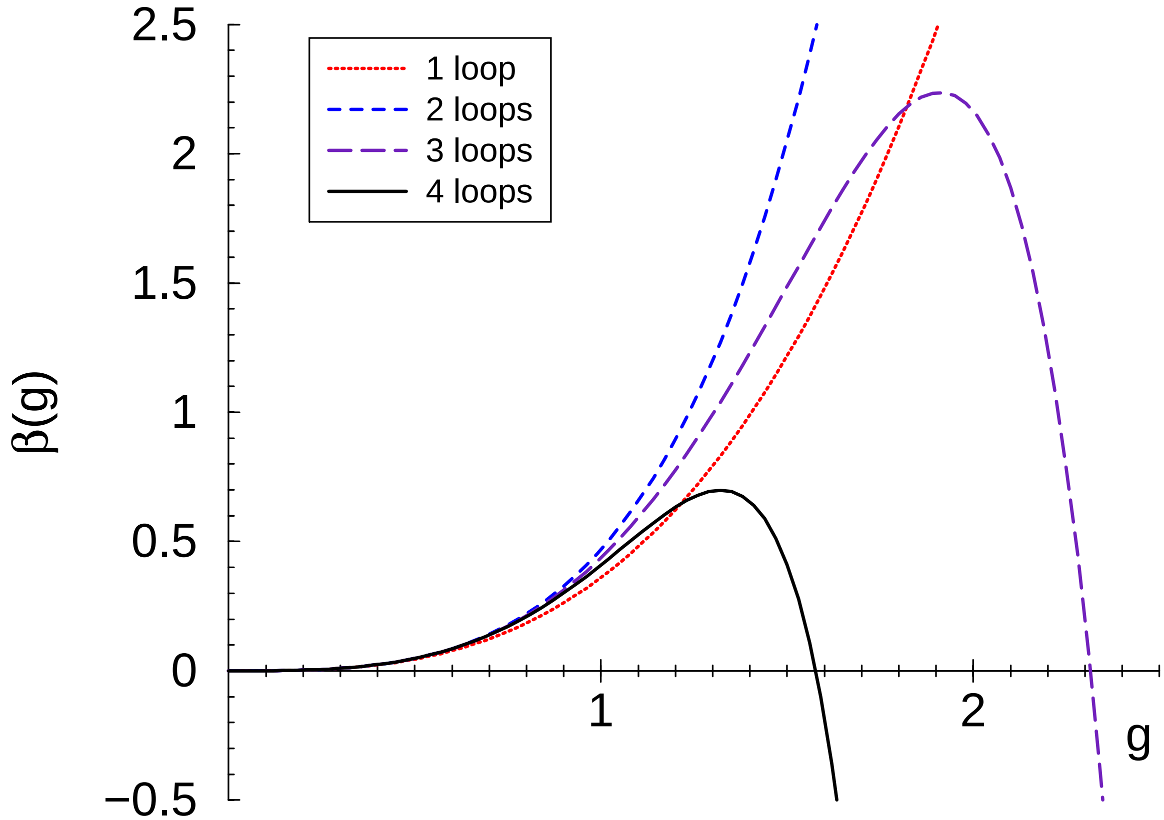
What happens if $b^{(1)} > 0$ ?

Conventional Wisdom: “The theory is not asymptotically free. A Landau pole develops in the UV.” The two-loop $\beta$ function supports this.

However, higher loop corrections to the $\beta$ function can be large and negative. In many theories, both the 3 and 4 loop $\beta$ functions have a non-trivial UV-stable fixed point. For example:

Consider the Minimal Missing Partner $SU(5)$ model.

Reps $=$ $3 \times (\overline{\mathbf{5}} + \mathbf{10})$, $\mathbf{5} + \overline{\mathbf{5}}$, $\mathbf{24}$, $\mathbf{50} + \overline{\mathbf{50}}$, $\mathbf{75}$.




The point $g = g_* \neq 0$ where $\beta = 0$ is a UV-stable fixed point, since $\partial\beta/\partial g < 0$ at the fixed point.

Is this UV-fixed point real?

No finite order calculation can prove that a fixed point really exists, or doesn't exist.

The crucial question is simply whether the full nonperturbative $\beta$ function behaves qualitatively like the 1 and 2 loop approximations, *or* like the 3 and 4 loop approximations.

We are aware of no existing argument or calculation which definitively answers this question.

Notes:

- Disclaimer applies: Perturbation theory is in bad shape. The 1,2,3, and 4 loop contributions to the $\beta$ function are all comparable near the fixed point.
  Caution: The function $1/(1+x)$ has no positive roots, even though its perturbative approximation $1-x+x^2-x^3$ does.
  However, terms in the loop expansion are not just alternating: 1 and 2 loops are positive, 3 and 4 loops are negative.

- Knowing the 5 loop contribution would not settle anything, regardless of its sign. (However, it would be another suggestive hint if it were negative.)

- The "exact" NSVZ $\beta$ function has a pole at $g^2 = 8\pi^2/C_G$:

$$\beta_{\text{NSVZ}} = \frac{g^3}{16\pi^2}\left[\frac{b^{(1)} + 2\sum_R I_R \gamma_r}{1 - C_G g^2/8\pi^2}\right]$$

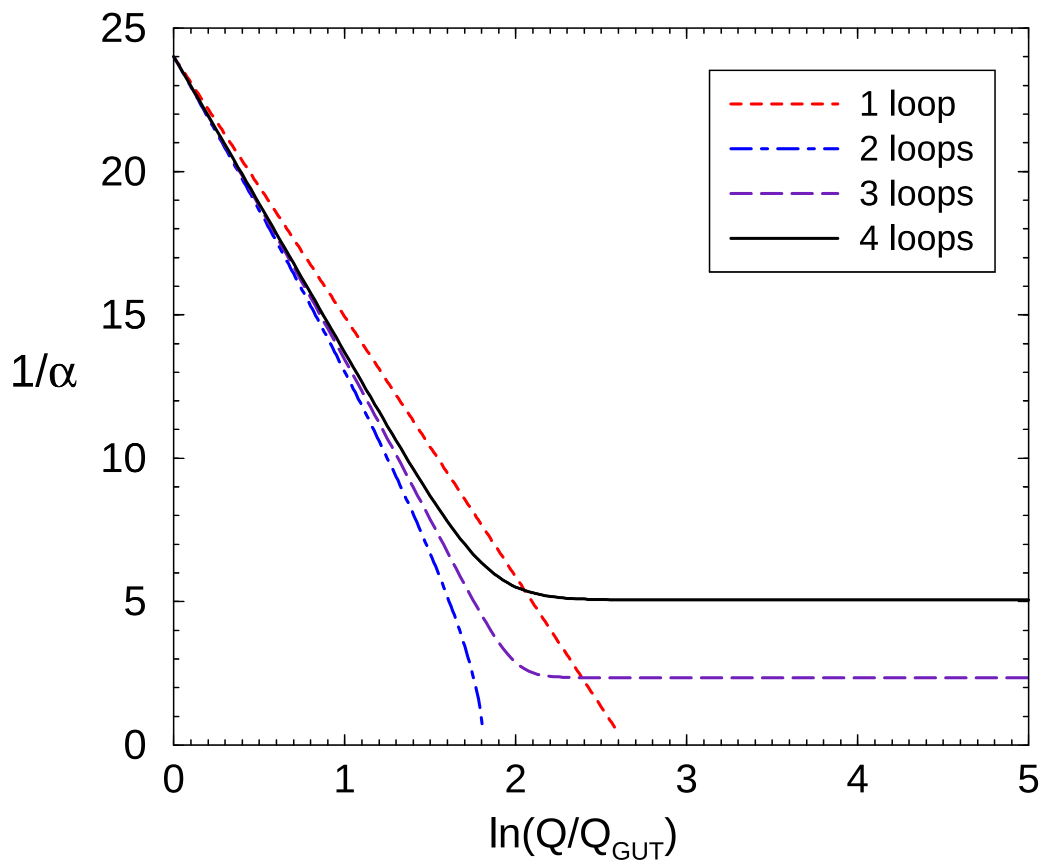
  However, this appears to occur at much stronger coupling than the putative fixed point.

- DRED results are qualitatively similar.

- Because of $b^{(1)} > b^{(2)}$, it is hard to imagine that a non-trivial UV-stable fixed point can occur in the realm of reliable perturbation theory. (But that doesn't mean they don't exist!)

- If the UV fixed point exists, it is reached below the Planck scale...

Running of NSVZ inverse gauge coupling in Minimal Missing Missing Partner SU(5) (with all Yukawa couplings turned off):

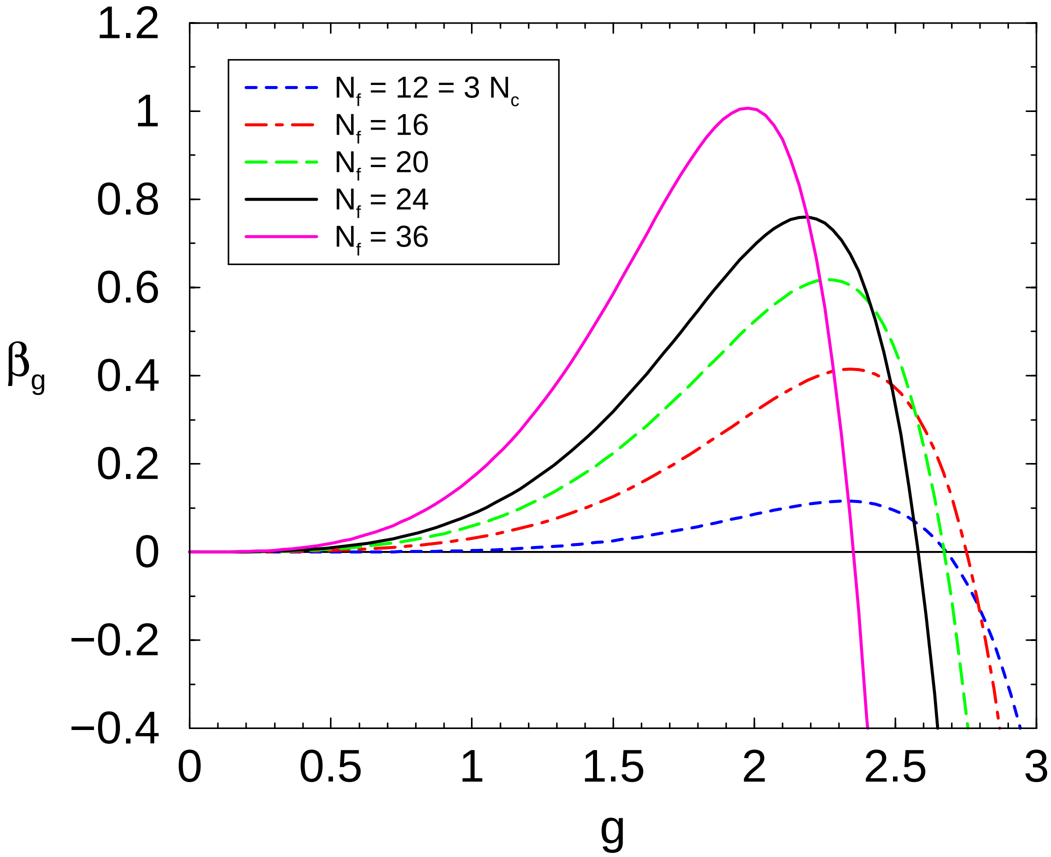



The Planck scale corresponds to roughly $\ln(Q/Q_{\rm GUT}) \approx 4.5$

The general idea of a UV fixed point could imply an intermediate (semi-perturbative) gauge coupling near the string scale. Can this be tied into a way of avoiding the dilaton runaway problem?

SUSY QCD with $N_f \geq 3N_c$.

For $N_c = 4$, four-loop NSVZ $\beta$ functions:




The 3 loop NSVZ beta function has a UV fixed point for all $N_f \geq 20$.

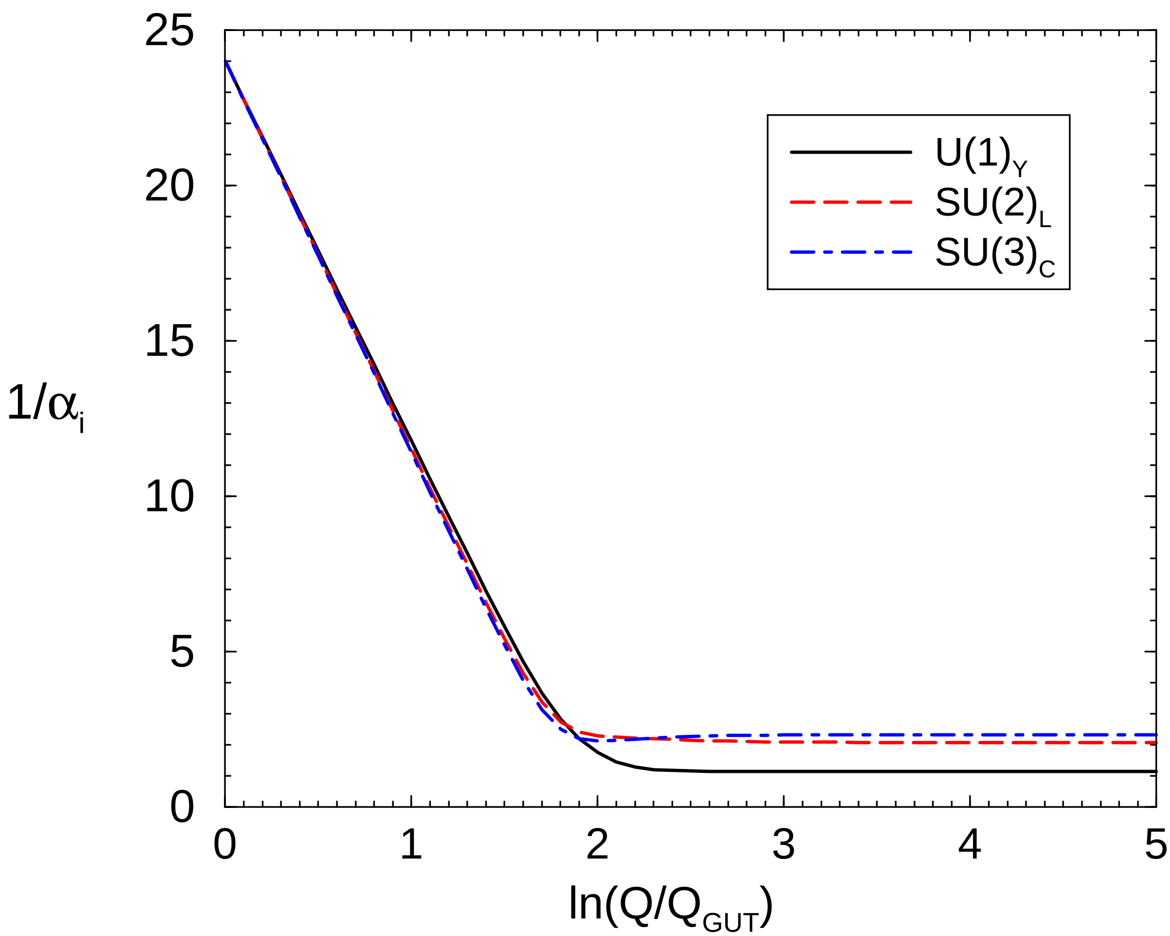
The 4 loop NSVZ beta function has a UV fixed point for all $N_f \geq 12$.

For larger $N_f$, the putative UV fixed point appears to move to weaker coupling, but this is deceptive because the effective expansion parameter is really $g^2 N_f$.

MSSM + many vectorlike fields above the apparent unification scale.

Extra Reps = $[14 \times (\mathbf{3}, \mathbf{2}, 1/6) + 14 \times (\overline{\mathbf{3}}, \mathbf{1}, 1/3) + 13 \times (\overline{\mathbf{3}}, \mathbf{1}, -2/3) + 9 \times (\mathbf{1}, \mathbf{2}, -1/2) + 9 \times (\mathbf{1}, \mathbf{1}, 1)]$+conjugate of $SU(3)_C \times SU(2)_L \times U(1)_Y$

Inverse gauge couplings $1/\alpha_{1,2,3}$ at 4 loops in the NSVZ scheme:




Other types of quasi-fixed point behavior are possible.

Suppose a non-trivial UV-stable fixed point exists.

What are the consequences?

Near a fixed point, soft supersymmetry breaking parameters will see critical behavior; power law running!

(Similar to superconformal IR fixed point idea of Nelson and Strassler.)

**Remarkably, it is possible to make an exact statement about gaugino mass running near the UV fixed point, without relying on perturbation theory. (Just have to assume that a UV fixed point exists.)**

Exact relation between $\beta$ functions for gaugino mass $M$ and gauge coupling $g$:

Hisano, Shifman 9705417
Jack, Jones 9709364

$$\beta_M = 2Mg^2 \frac{\partial}{\partial g^2}(\beta_g/g)$$

Since $\beta_g = 0$ near the fixed point:

$$\frac{dM}{dt} = \beta_M = -KM$$

where

$$K = -\frac{\partial \beta}{\partial g} > 0$$

is a positive constant in the UV fixed point regime.

The critical exponent $K$ is scheme-independent.

The scale dependence of the gaugino mass is therefore:

$$M(Q) = \left(\frac{Q_0}{Q}\right)^K M_0$$

So the gaugino mass $M$ runs according to a power law in the fixed point region.

By supposition, we are near a UV-stable fixed point region, so $\partial\beta/\partial g$ must be negative, so $K$ must be positive.

(If the 3 and 4 loop approximations could be trusted, then $K$ would be large, of order 10 or so in the Minimal Missing Partner $SU(5)$ model. However, we shall return to this point. . . )

This presents a possible solution to the SUSY flavor problem. A small gaugino mass near the Planck scale will grow very large as we move to lower scales within the UV fixed point region, and can "wash out" flavor violating coupling. (This requires that the flavor violating couplings themselves have smaller critical exponents.)

## Generalizes to theories with several gauge couplings:

Suppose that $g_i$ are simultaneously attracted to a UV-stable fixed point. Then the matrix

$$K_{ij} \equiv -\frac{\partial \beta_i}{\partial g_j}$$

must have positive eigenvalues in the fixed point regime. It follows that the exact RG evolution near the fixed point is

$$M_j(Q) = \sum_n (Q_0/Q)^{k_n} \frac{x_j^{(n)}}{g_j}$$

where $x_j^{(n)}$ are the eigenvectors of $K_{ij}$ with eigenvalues $k_n$. For $Q \ll Q_0$, the dominant contributions to each gaugino mass come from the largest eigenvalue.

Scalar mass$^2$ running is more elusive.

The RG equations for a scalar $m^2$ will have the form:

$$\frac{dm^2}{dt} = -aM^2 - cm^2$$

where $a$ and $c$ are unknown but constant near the fixed point. The solution for $m^2$ is:

$$m^2(Q) = (Q_0/Q)^{2K}\,\overline{M}_0^2 + (Q_0/Q)^c\,(m_0^2 - \overline{M}_0^2)$$

where $\overline{M}_0^2 = aM_0^2/(2K - c)$. One way to get a viable model with positive mass$^2$ and a solution to the SUSY flavor problem is to assume $a > 0$ and $2K > c$.

Constraints from scaling dimensions of gauge-invariant operators.

At a conformal fixed point, physical gauge-invariant operators must have scaling dimension $D \geq 1$.

The anomalous dimension of chiral superfields will typically be positive. At a zero of the NSVZ $\beta$ function, one has:

$$2 \sum_R I_R \gamma_R = b^{(1)} > 0$$

If all chiral superfields are in a single rep or its conjugate, then certainly $\gamma_R > 0$, and each chiral superfield $\Phi$ will have scaling dimension $D = 1 + \gamma_R > 1$. So composite gauge-invariant operators of degree $n$ have $D > n$. More generally, we expect this to be true with $\gamma_R$ positive for all reps $R$.

However, the anomalous dimension of the gaugino bilinear field $\lambda\lambda$ is related to the gaugino mass $\beta$ function, and so necessarily negative:

$$\gamma_{\lambda\lambda} = \frac{\beta_M}{M} = \frac{\partial \beta_g}{\partial g} < 0$$

So we may need to require

$$D_{\lambda\lambda} = 3 + \gamma_{\lambda\lambda} \geq 1$$

This limits the critical exponent for the gaugino mass:

$$K = -\frac{\partial \beta_g}{\partial g} \leq 2.$$

It is often said that superconformal invariance also requires the scaling dimension of gauge-invariant fields to satisfy

$$D \geq \frac{3}{2}|R|$$

where $R$ is the $R$-symmetry charge of the field. If so, then for the gaugino bilinear field one has $R = 2$, so presumably

$$D_{\lambda\lambda} = 3 - \frac{\partial \beta_g}{\partial g} \geq 3,$$

so that

$$\frac{\partial \beta_g}{\partial g} = 0.$$

However, we have found unresolved issues with the bound $D \geq 3|R|/2$ in general (not just in theories with a UV fixed point). In addition, it is not completely clear that the UV fixed point quantum theories necessarily must be superconformal, or exactly how the bound applies.

If it proves necessary to impose this requirement, then one can still have a UV-stable fixed point with both

$$\beta = 0 \text{ and } \partial\beta/\partial g = 0$$

at the UV fixed point $g = g_*$, so that

$$\beta_g \sim (g^2 - g_*^2)^n; \qquad n > 1.$$

This would correspond to non-perturbative effects "smoothing" the approach to the fixed point. If so, then the power-law running of the gaugino mass near the fixed point could be lost ($K = 0$).

## The large $N_f$ limit: an all-orders test?

Ferreira, Jack, Jones, and North have computed the beta functions for SUSY Yang Mills theory, to all loop orders, and leading order in $1/N_f$, in both the NSVZ and DRED schemes.

This calculation has a finite radius of convergence in $g^2 N_f$.

We find that the resulting NSVZ $\beta$ function ALWAYS has a root corresponding to a UV-stable fixed point within this radius of convergence if $b^{(1)} > 0$.

However, the DRED $\beta$ function NEVER does.

Resolution: the putative NSVZ fixed point occurs at a point where the unknown $1/N_f^2$ corrections are expected to be too large to neglect, despite the fact that the calculation is within the radius of convergence. The DRED result is similarly not trustworthy there.

Furthermore, the numerically largest negative contributions at 4 loop order do not show up at all in the large $N_f$ expansion.

We conclude that if a UV fixed point exists, one need not expect to find it reflected in the large-$N_f$ (or, more generally, the large-Dynkin-index) limit.

## Summary

- There are hints which, although certainly ambiguous, suggest that theories with positive $\beta$ function in the IR can have a UV-stable fixed point.

- We have been unable to rule out this scenario.

- If a UV-stable fixed point exists, it can provide a possible solution to the SUSY flavor problem, because gaugino masses have power-law growth.

- Constraints from the scaling dimension of gauge-invariant operator near the fixed point may be non-trivial, and need more investigation.