PHYSICS BEYOND χPT

A. Pich

IFIC, Univ. Valencia – CSIC

- $F_\pi(t)$: F. Guerrero, A.P., PLB 412 (1997) 382
  F. Guerrero, PRD 57 (1998) 4136
  D. Gómez–Dumm, A.P., J. Portolés, PRD 62 (2000) 054014
- $F_S^{\pi\pi}(t)$: E. Pallante, A.P., hep-ph/0007208
- ε'/ε: E. Pallante, A.P., PRL 84 (2000) 2568;
  E. Pallante, A.P., hep-ph/0007208
  E. Pallante, A.P., I. Scimemi, IFIC/00-31
  G. Ecker etal., PLB 477 (2000) 88
- $F_S^{K\pi}(t)$, m_s: M. Jamin, J.A. Oller, A.P., hep-ph/0006045
  M. Jamin, J.A. Oller, A.P., IFIC/00-32
- $K_L \to \mu^+\mu^-$: D. Gómez–Dumm, A.P., PRL 80 (1998) 4633

EVOLUTION FROM M_W TO M_K

Scale	Fields	Eff. Theory
M_W	W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model
	↓ OPE	
$\lesssim m_c$	$\gamma, g\ ;\ \mu, e, \nu_i$ s, d, u	$\mathcal{L}_{\text{QCD}}^{(n_f=3)}$, $\mathcal{L}_{\text{eff}}^{\Delta S=1,2}$
	↓ $N_C \to \infty$	
M_K	$\gamma\ ;\ \mu, e, \nu_i$ π, K, η	ChPT

χPT

Chiral Symmetry: $[\mathcal{M} \equiv \text{diag}(m_u, m_d, m_s) = 0]$

$$\mathcal{L}_{QCD} \doteq i\,\bar{q}_L \not{D} q_L + i\,\bar{q}_R \not{D} q_R \qquad ; \qquad q^T = (u,\, d,\, s)$$

$$q_{L,R} \to g_{L,R}\, q_{L,R} \qquad ; \qquad g_{L,R} \in SU(3)_{L,R}$$

$$SU(3)_L \otimes SU(3)_R \;\to\; SU(3)_V \;+\; 8 \;\; 0^- \;\text{Goldstones}$$

$$\langle \bar{q}_L^j q_R^i \rangle \;\Rightarrow\; U = \exp\left(i\sqrt{2}\Phi/f\right) \qquad ; \qquad U \to g_R\, U\, g_L^\dagger$$

$$\Phi = \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$

Low–Energy Expansion: $(p^{2n},\, m_q^n)$ $\qquad \mathcal{L} = \sum_n \mathcal{L}_{2n}$

$$\mathcal{L}_2 = \frac{f^2}{4} \langle D_\mu U^\dagger D^\mu U + 2\, B_0 \left(U^\dagger \mathcal{M} + \mathcal{M}^\dagger U\right) \rangle$$

$$= D_\mu \pi^+ D_\mu \pi^- - M_\pi^2 \pi^+ \pi^- + \cdots$$

$$+ \frac{1}{6f^2} \left(\pi^+ \overleftrightarrow{D}_\mu \pi^-\right) \left(\pi^+ \overleftrightarrow{D}^\mu \pi^-\right) + \cdots$$

$$\frac{M_\pi^2}{m_u + m_d} = \frac{M_{K^0}^2}{m_s + m_d} = \frac{M_{K^+}^2}{m_s + m_u} = B_0 = -\frac{\langle \bar{q} q \rangle}{f^2}$$

$O(p^4)$ χPT

i) $\mathcal{L}_4$ at tree level (Gasser–Leutwyler)

$$\begin{aligned}
\mathcal{L}_4 = & \, L_1 \langle D_\mu U^\dagger D^\mu U \rangle^2 + L_2 \langle D_\mu U^\dagger D_\nu U \rangle \langle D^\mu U^\dagger D^\nu U \rangle \\
& + L_3 \langle D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U \rangle + L_4 \langle D_\mu U^\dagger D^\mu U \rangle \langle U^\dagger \chi + \chi^\dagger U \rangle \\
& + L_5 \langle D_\mu U^\dagger D^\mu U \left(U^\dagger \chi + \chi^\dagger U \right) \rangle + L_6 \langle U^\dagger \chi + \chi^\dagger U \rangle^2 \\
& + L_7 \langle U^\dagger \chi - \chi^\dagger U \rangle^2 + L_8 \langle \chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger \chi \rangle \\
& - i \, L_9 \langle F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U \rangle + L_{10} \langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu} \rangle
\end{aligned}$$

$$F_J^{\mu\nu} \equiv \partial^\mu J^\nu - \partial^\nu J^\mu - i[J^\mu, J^\nu] \quad ; \quad J^\mu = l^\mu, r^\mu \quad ; \quad \chi \equiv 2\, B_0\, \mathcal{M}$$

ii) $\mathcal{L}_2$ at one loop (Unitarity)

$$T_4 \sim p^4 \left\{ a \, \log(p^2/\mu^2) + b(\mu) \right\}$$

- Chiral Logarithms unambiguously predicted
- L_i's fixed by QCD dynamics
  Reabsorb one–loop divergences ➡ $L_i^r(\mu)$

iii) Wess–Zumino–Witten term (chiral anomaly)

$$\pi^0 \to \gamma\gamma \quad ; \quad \eta \to \gamma\gamma \quad ; \quad \eta \to \pi\pi\gamma \quad ; \quad \cdots$$

$O(p^4)$ χPT COUPLINGS

i	$L_i^r(M_\rho) \times 10^3$	Source	Γ_i
1	0.4 ± 0.3	K_{e4}, $\pi\pi \to \pi\pi$	3/32
2	1.4 ± 0.3	K_{e4}, $\pi\pi \to \pi\pi$	3/16
3	-3.5 ± 1.1	K_{e4}, $\pi\pi \to \pi\pi$	0
4	-0.3 ± 0.5	Zweig rule	1/8
5	1.4 ± 0.5	$F_K : F_\pi$	3/8
6	-0.2 ± 0.3	Zweig rule	11/144
7	-0.4 ± 0.2	Gell-Mann–Okubo, $L_{5,8}$	0
8	0.9 ± 0.3	$M_{K^0} - M_{K^+}$, L_5, $(m_s - \hat{m}) : (m_d - m_u)$	5/48
9	6.9 ± 0.7	$\langle r^2 \rangle_V^\pi$	1/4
10	-5.5 ± 0.7	$\pi \to e\nu\gamma$	−1/4

$$L_i = L_i^r(\mu) + \Gamma_i \, \frac{\mu^{D-4}}{32\pi^2} \left\{ \frac{2}{D-4} + \gamma_E - \log(4\pi) - 1 \right\}$$

$$L_i^r(\mu_2) = L_i^r(\mu_1) + \frac{\Gamma_i}{(4\pi)^2} \log\left(\frac{\mu_1}{\mu_2}\right)$$

$\Lambda_\chi \sim 4\pi f_\pi \sim 1.2$ GeV ➡ $L_i \sim \frac{f_\pi^2/4}{\Lambda_\chi^2} \sim 2 \times 10^{-3}$

L_i'S FROM RESONANCE EXCHANGE

i	$L_i^r(M_\rho)$	V	A	S	S_1	η_1	Total	Total[b]
1	0.4 ± 0.3	0.6	0	−0.2	0.2	0	0.6	0.9
2	1.4 ± 0.3	1.2	0	0	0	0	1.2	1.8
3	-3.5 ± 1.1	−3.6	0	0.6	0	0	−3.0	−4.9
4	-0.3 ± 0.5	0	0	−0.5	0.5	0	0.0	0.0
5	1.4 ± 0.5	0	0	1.4[a]	0	0	1.4	1.4
6	-0.2 ± 0.3	0	0	−0.3	0.3	0	0.0	0.0
7	-0.4 ± 0.2	0	0	0	0	−0.3	−0.3	−0.3
8	0.9 ± 0.3	0	0	0.9[a]	0	0	0.9	0.9
9	6.9 ± 0.7	6.9[a]	0	0	0	0	6.9	7.3
10	-5.5 ± 0.7	−10.0	4.0	0	0	0	−6.0	−5.5

[a] Input

[b] $L_1^V = L_2^V/2 = -L_3^V/6 = L_9^V/8 = -L_{10}^{V+A}/6 = f_\pi^2/(16 M_V^2)$

PION FORM FACTOR

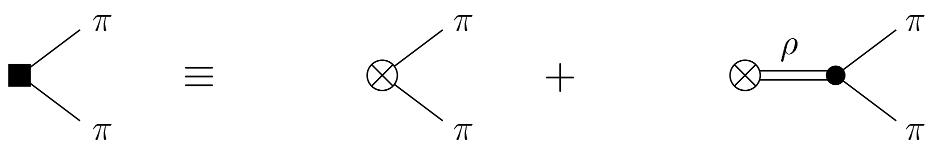
$$\langle \pi^0 \pi^- | \bar{d} \gamma^\mu u | 0 \rangle \equiv \sqrt{2}\, F_\pi(s) \left(p_{\pi^-} - p_{\pi^0} \right)^\mu$$

- $O(p^4)$ χPT:

$$F_\pi(s) = 1 + \frac{2\, L_9^r(\mu)}{f_\pi^2}\, s - \frac{s}{96\pi^2 f_\pi^2}\, A\left(\frac{M_\pi^2}{s}, \frac{M_\pi^2}{\mu^2}\right)$$

$$A(x,y) \equiv \log y + 8x - \frac{5}{3} + \sigma_x^3 \log\left(\frac{\sigma_x + 1}{\sigma_x - 1}\right) \qquad ; \qquad \sigma_x \equiv \sqrt{1 - 4x}$$

- $N_C \to \infty$: (1 Resonance only)



$$F_\pi(s) = 1 + \frac{F_V G_V}{f_\pi^2}\, \frac{s}{M_\rho^2 - s} = \frac{M_\rho^2}{M_\rho^2 - s} \qquad \left(\lim_{s\to\infty} F_\pi(s) = 0\right)$$

- $O(p^4)$ χPT + $N_C \to \infty$:

$$F_\pi(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2}\, A\left(\frac{M_\pi^2}{s}, \frac{M_\pi^2}{\mu^2}\right)$$

- Omnès Summation:

$$F_\pi(s) = \frac{M_\rho^2}{M_\rho^2 - s}\, \exp\left\{ -\frac{s}{96\pi^2 f_\pi^2}\, A\left(\frac{M_\pi^2}{s}, \frac{M_\pi^2}{M_\rho^2}\right)\right\}$$

Omnès Problem

$$\text{Im}\, A_J^I(s+i\epsilon) = \frac{1}{2}\sum_n \langle (\pi\pi)_J^I | T^\dagger | n\rangle \langle n | O | i(q)\rangle$$

$$\text{Im}\, A_J^I = \left(\text{Im}\, A_J^I\right)_{2\pi} = e^{-i\delta_J^I} \sin\delta_J^I\, A_J^I = e^{i\delta_J^I} \sin\delta_J^I\, A_J^{I*}$$

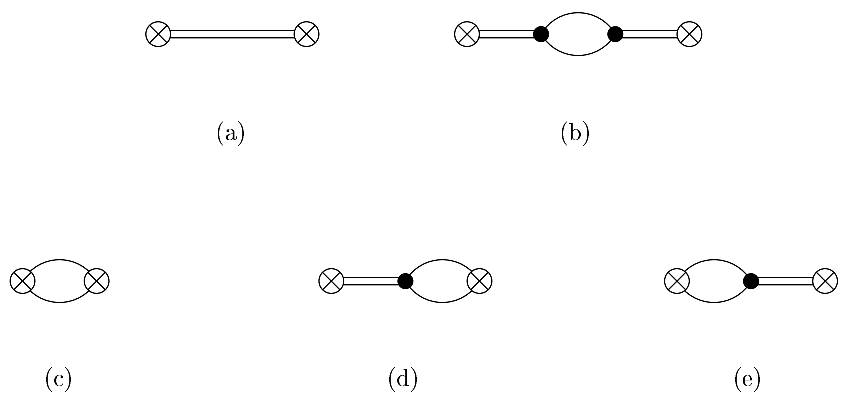
$$= \sin\delta_J^I\, |A_J^I| = \tan\delta_J^I\, \text{Re}\, A_J^I$$

$$A_J^I(s) = \sum_{k=0}^{n-1} \frac{(s-s_0)^k}{k!} \frac{d^k A_J^I}{ds^k}\bigg|_{s=s_0} + \frac{(s-s_0)^n}{\pi}\int_{4M_\pi^2}^\infty \frac{dz}{(z-s_0)^n} \frac{\tan\delta_J^I(z)\,\text{Re}A_J^I(z)}{z-s-i\epsilon}$$

$$A_J^I(s) = \exp\left\{\sum_{k=0}^{n-1} \frac{(s-s_0)^k}{k!} \frac{d^k}{ds^k} \log\left\{A_J^I(s)\right\}\Big|_{s=s_0} + \frac{(s-s_0)^n}{\pi}\int_{4M_\pi^2}^\infty \frac{dz}{(z-s_0)^n} \frac{\delta_J^I(z)}{z-s-i\epsilon}\right\}$$

$$\equiv \Omega_{I,J}(s,s_0)\; A_J^I(s_0)$$

- <u>Rho Width:</u> Dyson Summation



$$F_\pi(s) \approx \frac{M_\rho^2}{M_\rho^2 - s - i\, M_\rho\, \Gamma_\rho(s)}$$

$$\Gamma_\rho(s) = \theta(s - 4M_\pi^2)\ \sigma_\pi^3\ \frac{M_\rho\, s}{96\pi f_\pi^2}$$

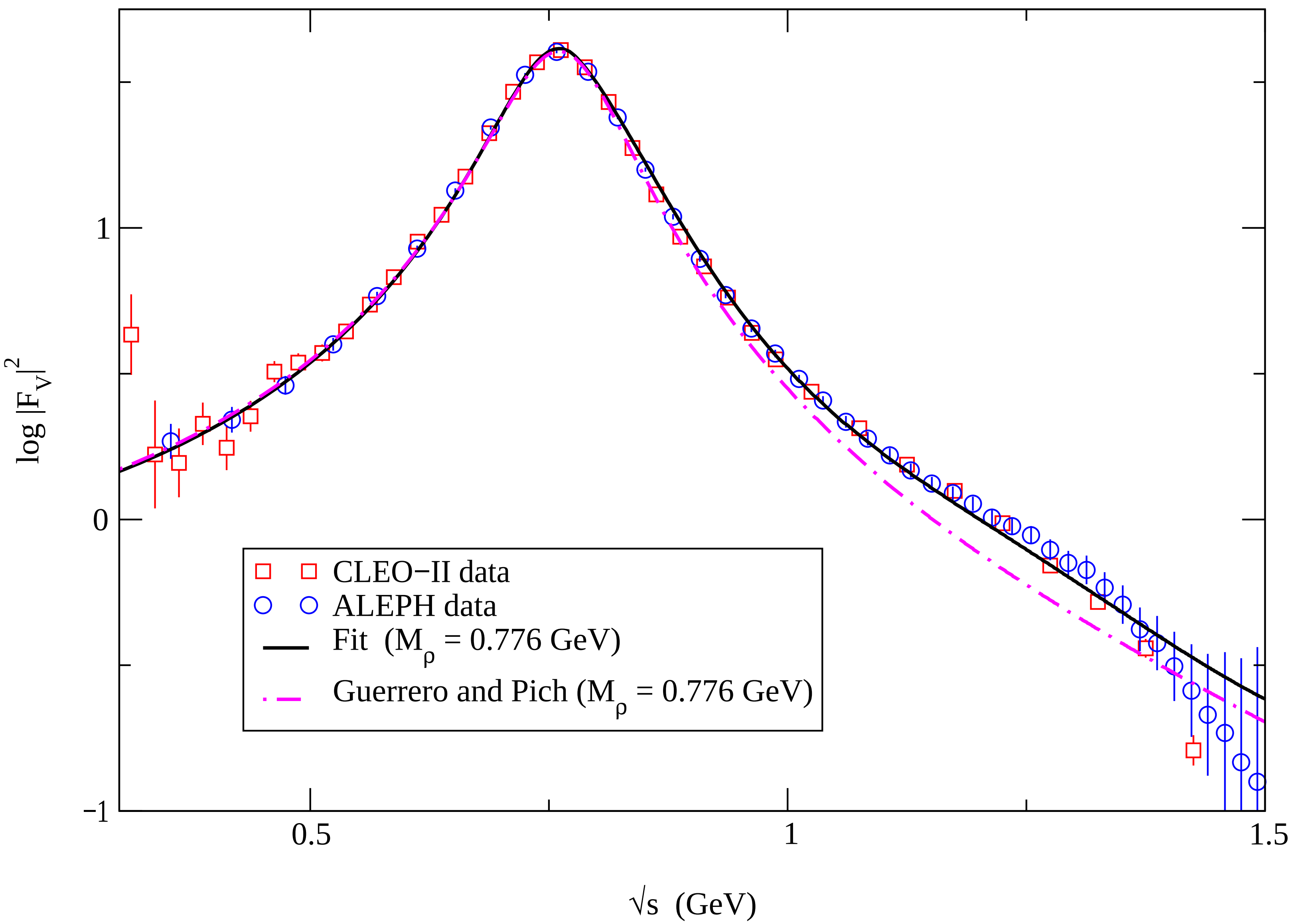
$\Rightarrow$ $\Gamma_\rho(s) = 144$ MeV [exp: (150.7 ± 1.1) MeV]

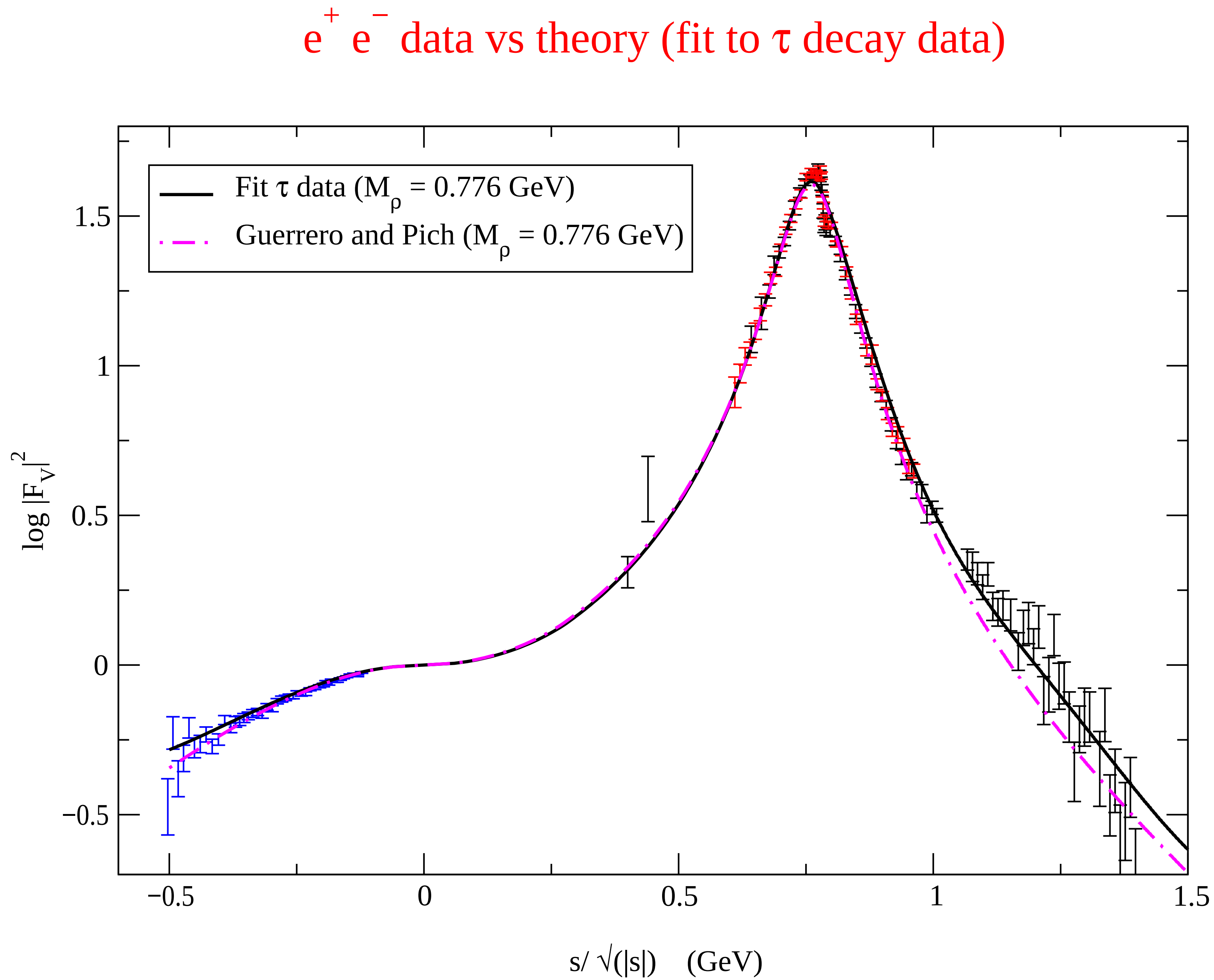
$$\delta_1^1(s) = \arctan\left\{\frac{M_\rho\,\Gamma_\rho(s)}{M_\rho^s - s}\right\} \approx \frac{s\,\sigma_\pi^3}{96\pi f_\pi^2} + \cdots$$

$\Downarrow$

$$F_\pi(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)}\ \exp\left\{-\frac{s}{96\pi^2 f_\pi^2}\, \mathrm{Re}\left[A\left(\frac{M_\pi^2}{s}, \frac{M_\pi^2}{M_\rho^2}\right)\right]\right\}$$

τ decay data vs theory




e+ e− data vs theory (fit to τ decay data)
Fit τ data (Mρ = 0.776 GeV)
Guerrero and Pich (Mρ = 0.776 GeV)
log |FV|2
s/ √(|s|)   (GeV)
1.5
1
0.5
0
−0.5
−0.5
0
0.5
1
1.5

SCALAR FORM FACTOR

$$\langle \pi^i(p')|\bar{u}u + \bar{d}d|\pi^k(p)\rangle \;\equiv\; \delta^{ik}\, F_S^\pi(t)$$

$$F_S^\pi(t) \;=\; F_S^\pi(0)\;\left\{1 + g(t) + O(p^4)\right\}$$

$$g(t) \;=\; \frac{t}{f^2}\left\{\left(1 - \frac{M_\pi^2}{2t}\right)\bar{J}_{\pi\pi}(t) + \frac{1}{4}\,\bar{J}_{KK}(t) + \frac{M_\pi^2}{18t}\,\bar{J}_{\eta\eta}(t) + 4(L_5^r + 2L_4^r)(\mu) + \frac{5}{4(4\pi)^2}\left(\ln\frac{\mu^2}{M_\pi^2} - 1\right) - \frac{1}{4(4\pi)^2}\ln\frac{M_K^2}{M_\pi^2}\right\}$$

$$\bar{J}_{PP}(t) \;=\; \frac{1}{(4\pi)^2}\left\{2 - \sigma_P \ln\left(\frac{\sigma_P+1}{\sigma_P-1}\right)\right\} \qquad ; \qquad \sigma_P \equiv \sqrt{1 - \frac{4M_P^2}{t}}$$

$$F_S^\pi(t) = \Omega_0(t,t_0)\; F_S^\pi(t_0) \approx \Omega_0(t,t_0)\; F_S^\pi(0)\,\{1 + g(t_0)\}$$

t_0	$g(t_0)$	$\Re_0(M_K^2, t_0)$	$\lvert F_S^\pi(M_K^2)/F_S^\pi(0)\rvert$
0	0	1.45	1.45
M_π^2	0.042	1.40	1.46
$2M_\pi^2$	0.091	1.34	1.46
$3M_\pi^2$	0.15	1.26	1.45
$4M_\pi^2$	0.26	1.11	1.40
M_K^2	$0.54 - 0.46\, i$	$\equiv 1$	1.61

$\rightarrow$ $$\lvert F_S^\pi(M_K^2)/F_S^\pi(0)\rvert = 1.55 \pm 0.10$$

$$\eta_{+-} \equiv \frac{T(K_L \to \pi^+\pi^-)}{T(K_S \to \pi^+\pi^-)} \approx \varepsilon_K + \varepsilon'_K$$

$$\eta_{00} \equiv \frac{T(K_L \to \pi^0\pi^0)}{T(K_S \to \pi^0\pi^0)} \approx \varepsilon_K - 2\varepsilon'_K$$

$$\varepsilon_K = (2.280 \pm 0.013) \times 10^{-3} \; e^{i\phi_{\varepsilon_K}}$$

$$\phi_{\varepsilon_K} = (43.47 \pm 0.51)^\circ$$

$$\mathrm{Re}\left(\frac{\varepsilon'_K}{\varepsilon_K}\right) \approx \frac{1}{6}\left\{1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2\right\} = (19.3 \pm 2.4) \times 10^{-4}$$

➡ DIRECT CP VIOLATION

$10^4\,\mathrm{Re}(\varepsilon'_K/\varepsilon_K)$ =	33 ± 11	NA31	(1988)
	32 ± 30	E731	(1988)
	20 ± 7	NA31	(1993)
	7.4 ± 5.9	E731	(1993)
	28.0 ± 4.1	KTeV	(1999)
	14.0 ± 4.3	NA48	(1999–2000)

$K \to 2\pi$ ISOSPIN AMPLITUDES

$|\pi\pi\rangle$: $J = 0$, $\mathcal{CP} = +$, $I = 0, 2$ (Bose)

$$A[K^0 \to \pi^+\pi^-] \equiv A_0\, e^{i\,\delta^0} + \frac{1}{\sqrt{2}} A_2\, e^{i\,\delta^2}$$
$$A[K^0 \to \pi^0\pi^0] \equiv A_0\, e^{i\,\delta^0} - \sqrt{2}\, A_2\, e^{i\,\delta^2}$$
$$A[K^+ \to \pi^+\pi^0] \equiv \frac{3}{2} A_2\, e^{i\,\delta^2}$$

$\Delta I = 1/2$ Rule :

$$\omega \equiv \frac{\mathrm{Re}\,(A_2)}{\mathrm{Re}\,(A_0)} \approx \frac{1}{22}$$

Strong Final State Interactions :

$$\delta^0 - \delta^2 = 45^\circ \pm 6^\circ$$

$$\varepsilon'_K = \frac{-i}{\sqrt{2}}\, e^{i\,(\delta^2 - \delta^0)}\, \omega \left\{ \frac{\mathrm{Im}\,(A_0)}{\mathrm{Re}\,(A_0)} - \frac{\mathrm{Im}\,(A_2)}{\mathrm{Re}\,(A_2)} \right\}$$

SHORT DISTANCES

$$\mathcal{L}_{eff}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) Q_i(\mu)$$

$$\begin{aligned}
Q_1 &= \left(\bar{s}_\alpha u_\beta\right)_{V-A} \left(\bar{u}_\beta d_\alpha\right)_{V-A} \\
Q_2 &= (\bar{s}u)_{V-A} (\bar{u}d)_{V-A} \\
Q_{3,5} &= (\bar{s}d)_{V-A} \sum_q (\bar{q}q)_{V\mp A} \\
Q_{4,6} &= \left(\bar{s}_\alpha d_\beta\right)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V\mp A} \\
Q_{7,9} &= \tfrac{3}{2} (\bar{s}d)_{V-A} \sum_q e_q (\bar{q}q)_{V\pm A} \\
Q_{8,10} &= \tfrac{3}{2} \left(\bar{s}_\alpha d_\beta\right)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V\pm A}
\end{aligned}$$

$q > \mu$: $\quad C_i(\mu) = z_i(\mu) - y_i(\mu) \left(V_{td} V_{ts}^* / V_{ud} V_{us}^*\right)$

$O(\alpha_s^n t^n)$, $O(\alpha_s^{n+1} t^n)$, $[t \equiv \log (M/m)]$ (Munich / Rome)

$q < \mu$: $\quad \langle \pi\pi | Q_i(\mu) | K \rangle$?

Physics does not depend on μ

Buras et al

$$\frac{\varepsilon'_K}{\varepsilon_K} \sim \text{Im}\,(V_{ts}^* V_{td})\,\left[P^{(1/2)} - P^{(3/2)}\right]$$

$$P^{(1/2)} = r \sum_i y_i(\mu)\,\langle Q_i(\mu)\rangle_0\,\left(1 - \Omega_{IB}\right)$$

$$P^{(3/2)} = \frac{r}{\omega} \sum_i y_i(\mu)\,\langle Q_i(\mu)\rangle_2$$

Experiment:

$$r = \frac{G_F\,\omega}{2|\varepsilon|\text{Re}\,(A_0)}$$

$$\omega = \frac{\text{Re}\,(A_2)}{\text{Re}\,(A_0)} \approx \frac{1}{22} \quad ; \quad \text{Re}(A_0) = 3.37 \times 10^{-7}\ \text{GeV}$$

Theory:

$$\langle Q_i(\mu)\rangle \equiv \langle Q_i\rangle_{vs}\, B_i(\mu)$$

$$\frac{\varepsilon'_K}{\varepsilon_K} \sim \left[\frac{110\ \text{MeV}}{m_s(2\ \text{GeV})}\right]^2 \left\{B_6^{(1/2)}\left(1 - \Omega_{IB}\right) - 0.4\, B_8^{(3/2)}\right\}$$

$$B_6^{(1/2)} = 1.0 \pm 0.3 \quad ; \quad B_8^{(3/2)} = 0.8 \pm 0.2$$

$$\Omega_{IB} \approx \Omega_{\eta+\eta'} = 0.25 \pm 0.08 \quad \longrightarrow \quad 0.16$$

$\pi^0 - \eta$ MIXING (Ecker et al)

$\pi^0 \approx \pi_3 + \varepsilon_{\pi^0\eta}\, \eta_8$ $\qquad A_0 \longrightarrow A_2^{IB} \propto \varepsilon_{\pi^0\eta}\, A_0$

$$\mathcal{A}(K^0 \to \pi^0\pi^0) \approx \mathcal{A}(K^0 \to \pi_3\pi_3) + 2\, \varepsilon_{\pi^0\eta}\, \mathcal{A}(K^0 \to \eta_8\pi_3)$$

$$\Omega_{IB} = \frac{\mathrm{Im}\,(A_2^{IB})}{\omega\, \mathrm{Im}\,(A_0)} = \frac{2\sqrt{2}\, \varepsilon_{\pi^0\eta}}{3\sqrt{3}\, \omega}$$

$O(p^2)$ CHPT: $\qquad \widehat{m} \equiv (m_u + m_d)/2$

$$\varepsilon^{(2)}_{\pi^0\eta} = \frac{\sqrt{3}\,(m_d - m_u)}{4\,(m_s - \widehat{m})} \quad \longrightarrow \quad \Omega_{IB} \approx 0.13$$

$O(p^4)$ CHPT: $\qquad \varepsilon_{\pi^0\eta} = \varepsilon^{(2)}_{\pi^0\eta} + \varepsilon^{(4)}_{\pi^0\eta}$

$$\boxed{\Omega_{IB} \approx \Omega_{\eta+\eta'} = 0.16 \pm 0.03}$$

$$\varepsilon^{(4)}_{\pi^0\eta} / \varepsilon^{(2)}_{\pi^0\eta} \propto \{(3\, L_7 + L_8(\mu)) + \chi\ \mathrm{logs}\}$$

η': $\quad \varepsilon^{(4)}_{\pi^0\eta}(L_7)/\varepsilon^{(2)}_{\pi^0\eta} = 1.10 \qquad ; \qquad a_0$: $\quad \varepsilon^{(4)}_{\pi^0\eta}(L_8)/\varepsilon^{(2)}_{\pi^0\eta} = -0.83$

RECENT THEORETICAL ESTIMATES

Group	$B_6^{(1/2)}$	$B_8^{(3/2)}$	$\varepsilon'_K/\varepsilon_K \times 10^4$	
Munich	1.0 ± 0.3	0.8 ± 0.2	$9.2^{+6.8}_{-4.0}$	(G)
			$1.4 \to 32.7$	(S)
Rome	1.0 ± 1.0	0.71 ± 0.13	$8.1^{+10.3}_{-9.5}$	(G)
			$-13 \to 37$	(S)
Trieste	1.3	0.84	22 ± 8	(G)
			$9 \to 48$	(S)
Dortmund	$1.50 \to 1.62$	$B_6^{(1/2)}/1.72$	$6.8 \to 63.9$	(S)
Dubna–DESY	1.0	1.0	$-3.2 \to 3.3$	(S)
Granada–Lund	2.5 ± 0.4	1.35 ± 0.20	34 ± 18	
Montpellier	2.99 ± 0.34	1.70 ± 0.39	25.5 ± 8.4	
Taipei	1.5	1.5	$7 \to 16$	
Valencia	1.55	0.92	17 ± 6	

$O(G_F p^2)$ χPT

$$\mathcal{L}_2^{\Delta S=1} = \Big\{ G_8 \, f^4 \, \langle \lambda L_\mu L^\mu \rangle + G_{27} \, f^4 \, \Big(L_{\mu 23} L^\mu_{11} + \frac{2}{3} L_{\mu 21} L^\mu_{13} \Big) + e^2 f^6 G_{EW} \, \langle \lambda U^\dagger \mathcal{Q} U \rangle \Big\}$$

$$G_R \equiv - \frac{G_F}{\sqrt{2}} \, V_{ud} V_{us}^* \, g_R \quad , \quad \lambda \equiv \lambda^{(6-i7)}/2 \quad , \quad L_\mu = -i U^\dagger D_\mu U$$

$$A_0 = \sqrt{2} f_\pi \left\{ \left(G_8 + \frac{1}{9} G_{27} \right) (M_K^2 - M_\pi^2) - \frac{2}{3} f_\pi^2 e^2 G_{EW} \right\}$$

$$A_2 = \frac{2}{9} f_\pi \left\{ 5 \, G_{27} \, (M_K^2 - M_\pi^2) - 3 f_\pi^2 e^2 G_{EW} \right\}$$

$$\delta^0 = \delta^2 = 0$$

$\left[\Gamma(K \to 2\pi) + \delta^I \right]_{Exp}$: $\quad g_8 \approx 5.1 \; ; \; |g_{27}/g_8| \approx 1/18$

$N_C \to \infty$: $\quad g_8 = \left(\frac{3}{5} C_2 - \frac{2}{5} C_1 + C_4\right) - 16 L_5 \left(\frac{\langle \bar{\Psi} \Psi \rangle (\mu)}{f_\pi^3} \right)^2 C_6(\mu)$

$$g_{27} = \frac{3}{5} (C_2 + C_1) \quad ; \quad e^2 \, g_{EW} = -3 \left(\frac{\langle \bar{\Psi} \Psi \rangle (\mu)}{f_\pi^3} \right)^2 C_8(\mu)$$

- Equivalent to standard calculations of B_i
- μ dependence only captured for $Q_{6,8}$
- Good estimate of Im (g_I) . Large $\frac{1}{N_C}$ corrections to Re (g_I) (Pich – de Rafael '91)

1–Loop χPT

(Kambor et al '90)

- Large enhancement of A_0 ($\sim$ 40–50%)
- $\delta^I \neq 0$ (still $\delta^0 < \delta^0_{exp}$)
- Loop correction dominated by infrared $\log(M_\pi)$ terms from π–π loops (FSI)
- $O(p^4)$ local terms fixed at $N_C \to \infty$
- Not included in Lattice – $1/N_C$ estimates
  ($\delta^I = 0$ at $N_C \to \infty$)
  Overlooked in *Standard* predictions of ε'/ε

Higher Orders ?

FINAL STATE INTERACTIONS

Watson Theorem: $\mathcal{A}_I \equiv A[K \to (\pi\pi)_I] \equiv A_I \, e^{i\delta^I}$

SU(3): $\mathcal{A}_I = (M_K^2 - M_\pi^2) \, a_I$

Omnès: $a_I(M_K^2) = \Omega_I(M_K^2, s_0) \, a_I(s_0)$

$$\Omega_I(s,s_0) \equiv \exp\left\{\frac{(s-s_0)}{\pi}\int \frac{dz}{(z-s_0)}\frac{\delta^I(z)}{(z-s-i\epsilon)}\right\} \equiv e^{i\delta^I(s)} \, \Re_I(s,s_0)$$

- $\delta^I(s) = \delta^I(s)|_{\pi\pi}$ below inelastic threshold
- Arbitrary number of subtractions

$$\Omega_I^{(n)}(s,s_0) = \exp\left\{\mathcal{P}_n(s,s_0) + \frac{(s-s_0)^n}{\pi}\int \frac{dz}{(z-s_0)^n}\frac{\delta^I(z)}{(z-s-i\epsilon)}\right\}$$

- All–order resummation of infrared chiral logs (same $\forall n$)
- Polynomial ambiguity $\Rightarrow$ Short–distance information

$$\Re_0 = 1.55 \pm 0.10 \quad ; \quad \Re_2 = 0.92 \pm 0.03$$

$K \to 2\pi$

$$\mathcal{A}_I \equiv A[K \to (\pi\pi)_I] \equiv A_I \, e^{i\delta^I} = (M_K^2 - M_\pi^2)\, a_I \, e^{i\delta^I}$$

$$a_I(s) = a_I(0) \left\{1 + g_I(s) + O(p^4)\right\}$$

$$g(s) = \frac{s}{f^2} \left\{ \left(1 - \frac{M_\pi^2}{2s}\right) \bar{J}_{\pi\pi}(s) + \frac{1}{4} \bar{J}_{KK}(s) + \frac{M_\pi^2}{18s} \bar{J}_{\eta\eta}(s) + 4(L_5^r + 2L_4^r)(\mu) + \frac{5}{4(4\pi)^2} \left(\ln \frac{\mu^2}{M_\pi^2} - 1 \right) - \frac{1}{4(4\pi)^2} \ln \frac{M_K^2}{M_\pi^2} \right\}$$

$$g_0^{(8)}(s) = \frac{s}{f^2} \left\{ \left(1 - \frac{M_\pi^2}{2s}\right) \bar{J}_{\pi\pi}(s) - \frac{1}{4} \left(1 - \frac{M_K^2}{s}\right) \bar{J}_{KK}(s) + \frac{M_\pi^2}{18s} \bar{J}_{\eta\eta}(s) + C_5^8(\mu) + \frac{3}{4(4\pi)^2} \left(\ln \frac{\mu^2}{M_\pi^2} - 1 \right) + \frac{1}{4(4\pi)^2} \ln \frac{M_K^2}{M_\pi^2} \right\}$$

$$g_0^{(27)}(s) = \frac{s}{f^2} \left\{ \left(1 - \frac{M_\pi^2}{2s}\right) \bar{J}_{\pi\pi}(s) - \frac{3}{2} \left(1 - \frac{M_K^2}{s}\right) \bar{J}_{KK}(s) - \frac{M_\pi^2}{2s} \bar{J}_{\eta\eta}(s) + C_5^{27}(\mu) - \frac{1}{2(4\pi)^2} \left(\ln \frac{\mu^2}{M_\pi^2} - 1 \right) + \frac{3}{2(4\pi)^2} \ln \frac{M_K^2}{M_\pi^2} \right\}$$

$$g_2(s) = \frac{s}{f^2} \left\{ -\frac{1}{2} \left(1 - \frac{2M_\pi^2}{s}\right) \bar{J}_{\pi\pi}(s) + \bar{C}_5^{27}(\mu) - \frac{1}{2(4\pi)^2} \left(\ln \frac{\mu^2}{M_\pi^2} - 1 \right) \right\}$$

INCLUDING FSI

$$\frac{\varepsilon'_K}{\varepsilon_K} \sim \left[\frac{110\,\text{MeV}}{m_s(2\,\text{GeV})}\right]^2 \left\{B_6^{(1/2)}\,(1-\Omega_{IB}) - 0.4\,B_8^{(3/2)}\right\}$$

- $B_6^{(1/2)}\Big|_{N_C\to\infty} \approx B_8^{(3/2)}\Big|_{N_C\to\infty} \approx 1$
- $B_6^{(1/2)} \approx B_6^{(1/2)}\Big|_{N_C\to\infty} \times \Re_0 \approx 1.55$
- $B_8^{(3/2)} \approx B_8^{(3/2)}\Big|_{N_C\to\infty} \times \Re_2 \approx 0.92$
- $\Omega_{IB} \approx 0.16 \times \Re_2/\Re_0 \approx 0.09$

(Wolfe–Maltman: 0.08 ± 0.05)

→ $$\frac{\varepsilon'}{\varepsilon} = (17 \pm 6) \times 10^{-4}$$

SUMMARY

- χPT $\equiv$ Low–Energy Symmetry Constraints
  - Known infrared logarithms
  - Unknown chiral couplings

    ➔ Short Distances

- $1/N_C$: QCD Matching

  Resonance Exchanges

- Omnès: FSI (elastic unitarity)

  Exponentiation of chiral logarithms

- Unitarity: Coupled channels

- Many applications:
  - Estimates of chiral couplings
  - Hadronic Form Factors / Matrix Elements
  - ε'/ε
  - m_s
  - $K_L \to \mu^+\mu^-$