S-matrix approach to the Z resonance

Tord Riemann, DESY

1 The problem of mass and width

2 From amplitudes to cross-sections and asymmetries

3 Some experimental results

4 Renormalization and gauge-invariance

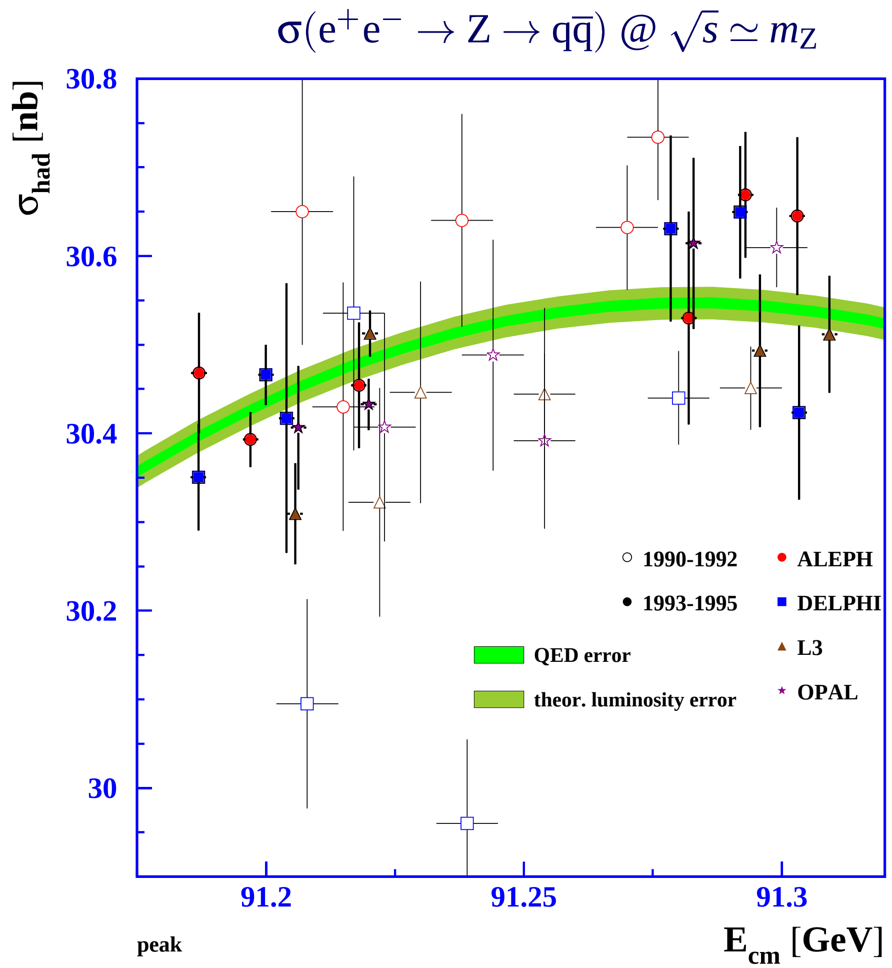
5 Width and life-time

6 Summary

1.

The problem of mass and width

real measurements: 4×27 cross sections



vertical error bars are statistical only
horizontal error bars from beam energy uncertainty fully correlated
theoretical errors (green band) fully correlated
1990-1992 data have larger luminosity errors

$M_Z = 91.186$ GeV

$M_Z = 91.186$ GeV

6

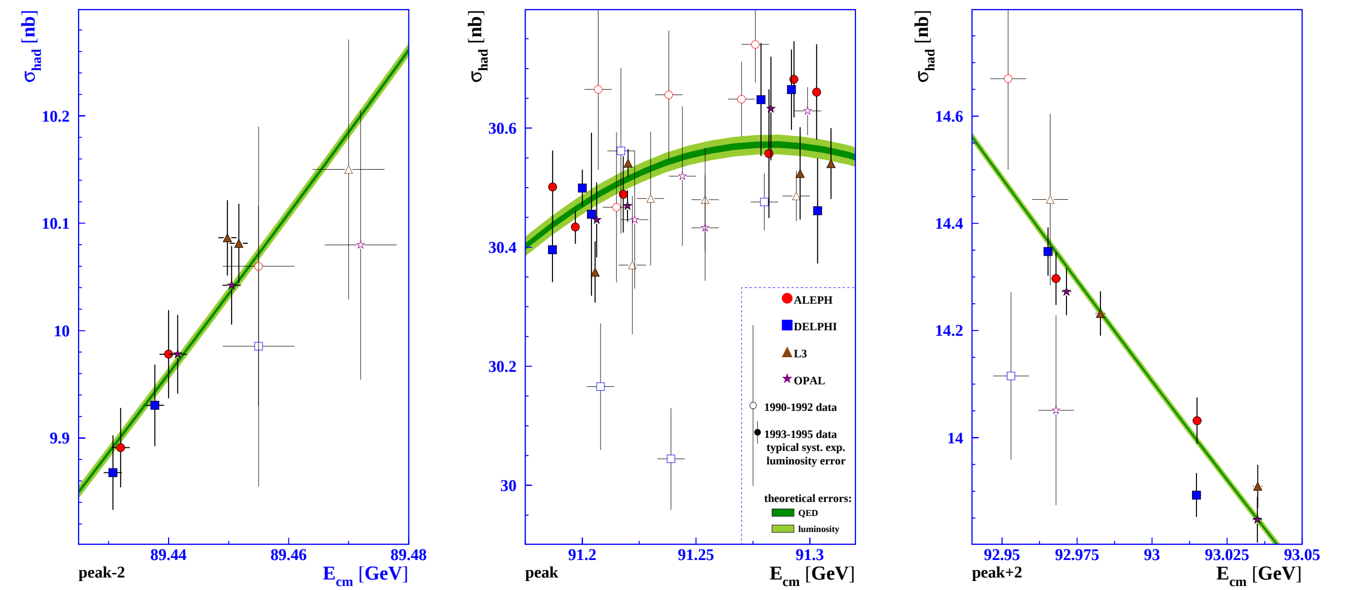


Figure 2: *Measurements by the four experiments of the hadronic cross-sections around the three principal energies. The vertical error bars show the statistical errors only. The open symbols represent the early measurements with typically much larger systematic errors than the later ones, shown as full symbols. Typical experimental systematic errors on the determination of the luminosity are also indicated; these are almost fully correlated within each experiment, but uncorrelated among the experiments. The horizontal error bars show the uncertainties in* LEP *centre-of-mass energy, where the errors for the period 1993–1995 are smaller than the symbol size in some cases. The bands represent the result of the model-independent fit to all data, including the two most important common theoretical errors from the unfolding of photon radiation and from the calculations of the small-angle Bhabha cross-section.*

Energy scans around Z peak 1990–1995

total cross sections ...

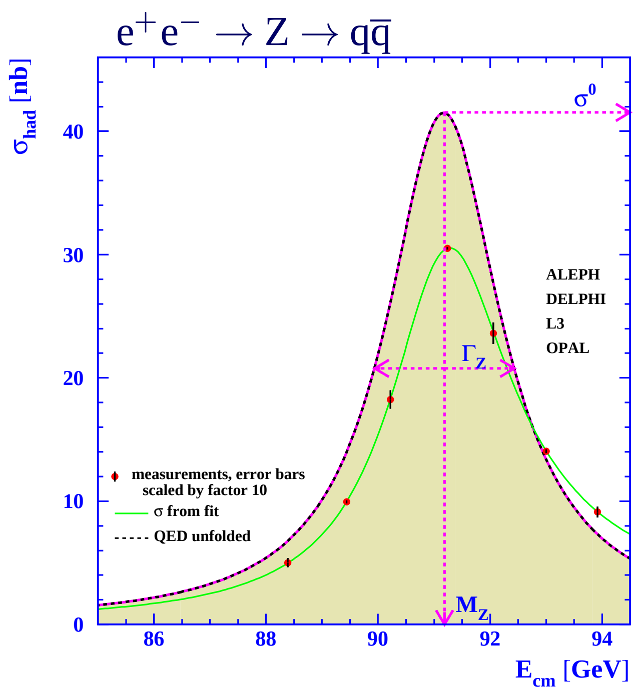
$e^+e^- \to$ hadrons

$e^+e^- \to e^+e^-$

$e^+e^- \to \mu^+\mu^-$

$e^+e^- \to \tau^+\tau^-$

... as a function of E_{cm}



∼30 per channel around 7 "energy points",

$$\sigma(E_i) = \frac{N^{\rm cand}_{\rm f\bar{f}}(E_i) - N^{\rm bkg}_{\rm f\bar{f}}(E_i)}{\varepsilon_{\rm ac}(E_i)} \frac{1}{\int L\ (E_i)},$$

parameterised in terms of 6 "pseudo-observables":

- m_Z
- Γ_Z
- $\sigma^o_{\rm had} = \frac{12\pi}{m_Z^2} \frac{\Gamma_{\rm ee}\Gamma_{\rm had}}{\Gamma_Z^2}$
- $R_e = \Gamma_{\rm had} / \Gamma_{\rm ee}$
- $R_\mu = \Gamma_{\rm had} / \Gamma_{\mu\mu}$
- $R_\tau = \Gamma_{\rm had} / \Gamma_{\tau\tau}$ $\quad\quad \Gamma_{\rm ff} \propto \left({g_v^f}^2 + {g_a^f}^2 \right)$ for f=e, μ, τ

Günter Quast, CERN & Mainz — Loops & Legs 2000

The Standard 'model-independent' approach at LEP

The γZ interference is assumed to be known

Definition of Z mass and width

Relativistic Z propagator with s dependent width function:

$$\Gamma_Z(s) \approx \frac{s}{M_Z^2}\Gamma_Z$$

The linear approximation is good for $Z \to f\bar{f}$ decay channels far away from the production thresholds.

'Conventional' LEP propagator with s-dependent width:

$$\chi(s) = \frac{G_\mu}{s - M_Z^2 + i\frac{s}{M_Z^2}M_Z\Gamma_Z}$$

Relativistic Breit-Wigner propagator, with $\bar{G}_\mu = G_\mu/(1 + i\frac{\Gamma}{M})$:

$$\chi(s) = \frac{\bar{G}_\mu}{s - \bar{m}_Z^2 + i\bar{m}_Z\bar{\Gamma}_Z}$$

Non-relativistic Breit-Wigner propagator:

$$\chi(s) = \frac{\bar{G}_\mu}{s - (M_R - \frac{i}{2}\Gamma_R)^2}$$

The resulting Z mass values differ:

Bardin Leike Riemann Sachwitz 1988

$$\bar{m}_Z = M_R - \frac{1}{8}\frac{\Gamma_R^2}{M_R} = M_Z - \frac{1}{2}\frac{\Gamma_Z^2}{M_Z}$$

or numerically:

$$\bar{m}_Z = M_Z - 34 \text{ MeV}, \qquad \mathrm{M_R} = \mathrm{M_Z} - 26 \text{ MeV}$$

How to look at the Z line shape?

Question: What is $\sigma^0_{tot}(s')$ in terms of M_Z and Γ_Z?

Maybe a pure Breit-Wigner function ...

$$\sigma^{(Z)}_{BW}(s) \sim \frac{M_Z^2 \cdot R}{\left|s - M_Z^2 + iM_Z\Gamma_Z\right|^2}.$$

The usual description is:

$$\sigma(s) \quad = \quad \int \frac{ds'}{s}\ \sigma^0(s')\ \rho(s'/s)\ +\ \int \frac{ds'}{s}\ \sigma^0_{ifi}(s,s')\ \rho^{ifi}$$

The $\rho(s'/s)$ and $\rho^{ifi}(s'/s)$ (ifi=ini-fin) have to be calculated.

Simplest: Bonneau-Martin formula:

$$\rho^{ini}_{tot}(s'/s) = soft + vertex + \frac{\alpha}{\pi} Q_e^2 \left(\ln \frac{s}{m_e^2} - 1 \right) \frac{1 + (s'/s)^2}{1 - s'/s}$$

See also: Bardin et al., `ZFITTER`, hep-ph/9908433

a shift of the peak position arises:

$$\begin{aligned} \sqrt{s_{\max}} - M_Z = \delta_{QED} \quad &\approx \quad \frac{\pi}{8}\beta\left(1 + \delta^{soft+virtual}\right)\Gamma_Z \\ &\approx \quad 90\,\mathrm{MeV}. \end{aligned}$$

- Which and how many free parameters have to be introduced?
- Should one measure at many energies or only at the peak itself?

From: Bardin, Leike, Riemann, Sachwitz, PLB 206 (1988) 539

Table 1. Results of a MINUIT fit to the Z-boson line shape as described in sect. 3.

	Born		Born + QED-corrections	
	$\Gamma_Z(s)$	Γ_Z	$\Gamma_Z(s)$	Γ_Z
M_Z [GeV]	93.000±0.013	92.966±0.013	93.000±0.016	92.966±0.016
Γ_Z [GeV]	2.498±0.009	2.498±0.009	2.498±0.011	2.498±0.011

Figure caption

Fig. 1: Total cross-sections $\sigma_B, \sigma_{RC}^{QED}$ for the reaction $e^+e^- \to \mu^+\mu^-(\gamma)$ in Born approximation (left scale) and including $O(\alpha)$ QED-corrections (right scale). Peaks of σ with energy-dependent width $\Gamma_Z(s)$ are shifted by 34 MeV to the left.

11

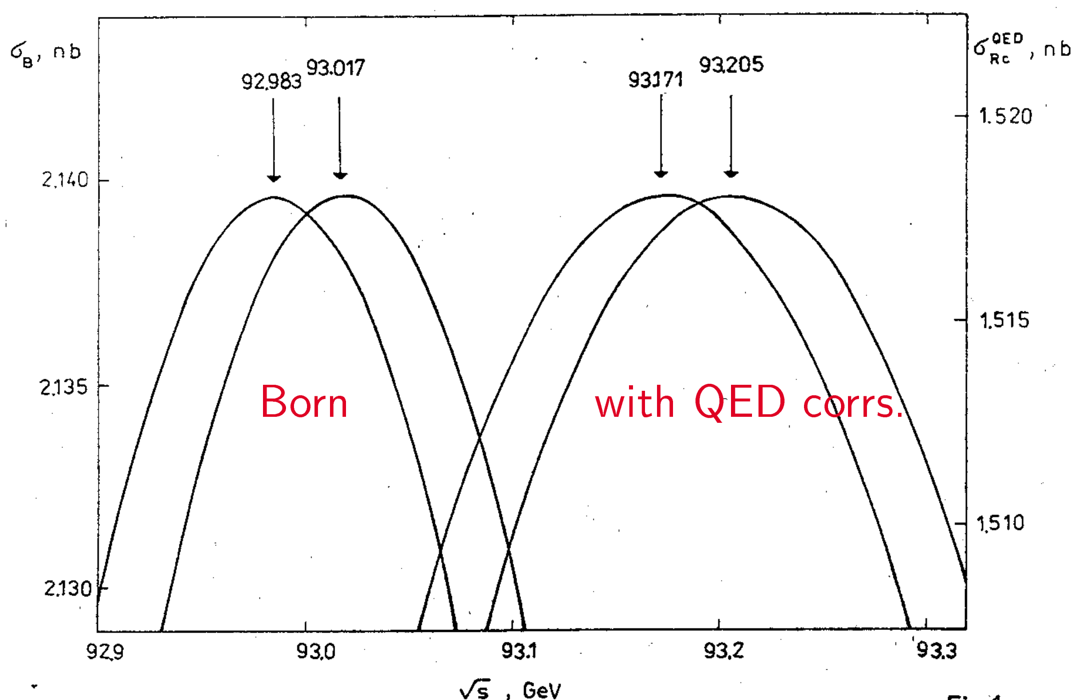


Fig.1

$M_Z = 93$ GeV, $\Gamma_Z = 2.5$ GeV

2.

From amplitudes to cross-sections and asymmetries

From an amplitude to the total cross-section

Using the S-matrix definition of the Z resonance the (complete) amplitude is:

Stuart 1991

Leike, Riemann, Rose 1991

Bohm, Harshman 2000

$$\mathcal{M} = \frac{R_Z}{s - s_0} + \frac{R_\gamma}{s} + B(s)$$

for a Z and a γ and some non-resonant background.

When listening to a talk on S-matrix aspects of renormalization of the SM by Robin Stuart at CERN in 1991, I had the idea to apply the S-matrix ansatz for $\sigma^0_{tot}(s')$ immediately to the data.

Then we tried it out...
Leike, Riemann, Rose 1991

The following ansatz is a good choice without explicit reference to the Standard Model:

$$\sigma_0(s) = \frac{4}{3}\pi\alpha^2 \left[\frac{r^\gamma}{s} + \frac{s \cdot R + (s - M_Z^2) \cdot J}{\left| s - M_Z^2 + i M_Z \Gamma_Z(s) \right|^2} + B(s) \right].$$

$$\Gamma_Z(s) = \frac{s}{M_Z^2}\Gamma_Z \quad \text{or} \quad \Gamma_\mathrm{Z}(\mathrm{s}) = \Gamma_\mathrm{Z}$$

The line shape is then described by five parameters:

- $r^\gamma \sim \alpha^2_{em}(M_Z^2)$ – may be assumed to be known
- $M_Z, \;\; \Gamma_Z$
- . R – measure of the Z peak height
- . J – measure of the γZ interference
- $B(s)$ – some slowly varying background.

Thus, essentially: M_Z, Γ_Z, R, J are the unknowns.

Analysing the Z resonance

Compare the simple Breit-Wigner

$$\sigma_0^{(Z)}(s) \sim \frac{M_Z^2 \cdot R}{\left|s - M_Z^2 + iM_Z\Gamma_Z\right|^2}$$

and our preferred ansatz

$$\sigma_0(s) = \frac{4}{3}\pi\alpha^2 \left[\frac{r^\gamma}{s} + \frac{s \cdot R + (s - M_Z^2) \cdot J}{\left|s - M_Z^2 + is\Gamma_Z/M_Z\right|^2}\right]$$

From the replacements $M_Z^2 \cdot R \to M_Z^2 \cdot s, \quad M_Z\Gamma_Z \to s/M_Z \cdot \Gamma_Z,$
and from the γZ interference J
shifts arise:

$$\begin{aligned}\sqrt{s_{\max}} - M_Z &= \delta_{QED} \oplus \frac{1}{4}\frac{\Gamma_Z^2}{M_Z}\left(1 + \frac{J}{R}\right) \ominus \frac{1}{2}\frac{\Gamma_Z^2}{M_Z} \\ &\sim \left[90 \; + \; 17 \times \left(1 + \frac{J}{R}\right) - \; 34 \;\right] \text{ MeV}\end{aligned}$$

The γZ interference J and M_Z are strongly anti-correlated.

Standard Model prediction:

$$\frac{J}{R} \otimes 17 \text{ MeV} = \frac{0.22}{2.969} \otimes 17 \text{ MeV} = 1.26 \text{ MeV}$$

Cross-sections

Consider four independent helicity amplitudes in the case of massless fermions f:

$$\mathcal{M}^{fi}(s) \quad = \quad \frac{R_\gamma^f}{s} + \frac{R_Z^{fi}}{s - s_Z} + B(s)$$

The position of the Z pole in the complex s plane is given by s_Z:

$$s_Z = \overline{m}_Z^2 - i\overline{m}_Z\overline{\Gamma}_Z.$$

There are four residua R_Z^{fi} per channel:

$$\begin{aligned}
R_Z^{f0} &= R_Z(e_L^- e_R^+ \longrightarrow f_L^- f_R^+), \\
R_Z^{f1} &= R_Z(e_L^- e_R^+ \longrightarrow f_R^- f_L^+), \\
R_Z^{f2} &= R_Z(e_R^- e_L^+ \longrightarrow f_R^- f_L^+), \\
R_Z^{f3} &= R_Z(e_R^- e_L^+ \longrightarrow f_L^- f_R^+).
\end{aligned}$$

They yield four helicity cross-sections $\sigma_i \sim |\mathcal{M}^{fi}(s)|^2$: which add up incoherently to the following measurable cross-sections:

$$\begin{array}{ccccc}
 & & \sigma_T^0(s) & = & +\,\sigma_0 + \sigma_1 + \sigma_2 + \sigma_3, \\
\sigma^0_{\textit{lr-pol}}(s) & = & \sigma^0_{FB}(s) & = & +\,\sigma_0 - \sigma_1 + \sigma_2 - \sigma_3, \\
\sigma^0_{\textit{FB-lr}}(s) & = & \sigma^0_{pol}(s) & = & -\,\sigma_0 + \sigma_1 + \sigma_2 - \sigma_3, \\
\sigma^0_{lr}(s) & = & \sigma^0_{\textit{FB-pol}}(s) & = & -\,\sigma_0 - \sigma_1 + \sigma_2 + \sigma_3.
\end{array}$$

All these cross-sections may be parameterized by the following master formula:

$$\sigma_A^0(s) \quad = \quad \frac{4}{3}\pi\alpha^2 \left[\frac{r_A^{\gamma f}}{s} + \frac{s r_A^f + (s - \overline{m}_Z^2) j_A^f}{(s - \overline{m}_Z^2)^2 + \overline{m}_Z^2 \overline{\Gamma}_Z^2} + B(s) \right],$$

where the definitions of the r and j depend on the label $A =$ *T*, *FB*, ..., e.g.

$$r_{FB}^f = \sum_{i=0}^{3} (-1)^i \; |R_Z^{fi}|^2$$

Asymmetries

Without QED corrections, asymmetries are defined by:

$$\mathcal{A}_A^0(s) = \frac{\sigma_A^0(s)}{\sigma_T^0(s)}, \quad A \neq T.$$

They take an extremely simple form around the Z resonance:
Riemann 1992

$$\mathcal{A}_A^0(s) \quad = \quad {A_0}^A + {A_1}^A \left(\frac{s}{\overline{m}_Z^2} - 1 \right) + A_2^A \left(\frac{s}{\overline{m}_Z^2} - 1 \right)^2 + \ldots$$

$$\mathcal{A}_A^0(s) \quad \approx \quad {A_0}^A + {A_1}^A \left(\frac{s}{\overline{m}_Z^2} - 1 \right)$$

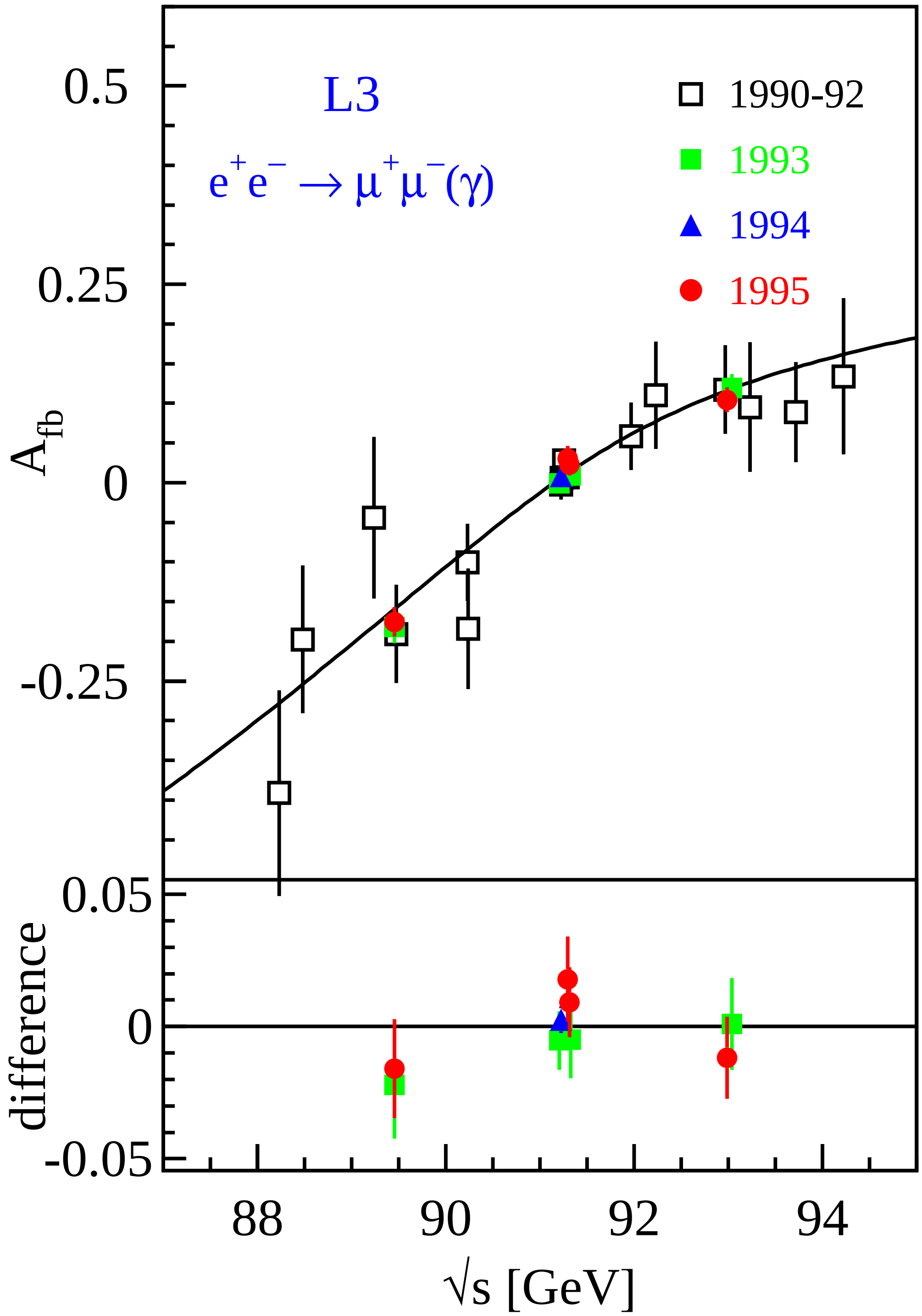
At LEP 1, the higher order terms may be neglected since $(s/\overline{m}_Z^2 - 1)^2 < 2 \times 10^{-4}$.

The coefficients have a quite simple form:

$${A_0}^A \quad = \quad \frac{r_A^f}{r_T^f},$$

and

$${A_1}^A \quad = \quad \left[\frac{j_A^f}{r_A^f} - \frac{j_T^f}{r_T^f} \right] {A_0}^A.$$



How to describe e.g. the Forward-Backward Asymmetry?

With QED corrections, not much changes: The coefficients get s dependent (and on cuts):

$$A_0^{FB} \to \bar{A}_0^{FB} \approx const \times A_0^{FB}$$

and

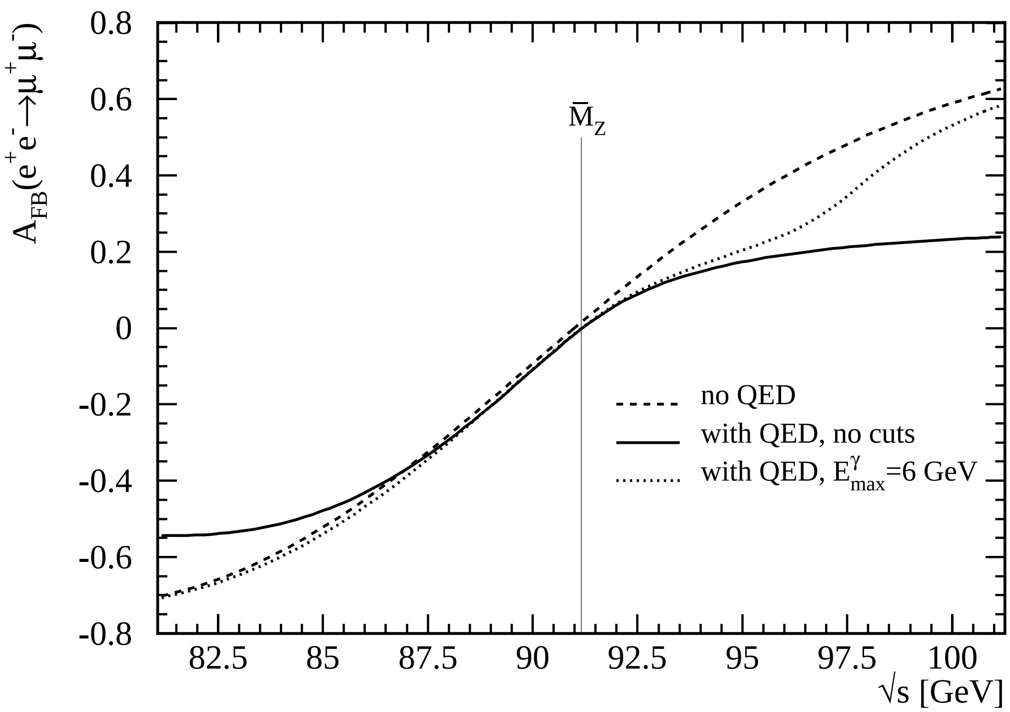
$$A_1^{FB} \to \bar{A}_1^{FB} \approx C(s) \times A_1^{FB}$$

where $C(s)$ is a tricky function of s reflecting the radiative tail properties of the Z exchange part.

Remember: $A_1^{FB} \sim j/r$, where:

j due to γZ

r due to Z exchange (with tail) interference (no tail)



Cut-dependence of $\mathcal{A}_{FB}(e^+e^- \to \mu^+\mu^-)$ near the Z peak

From: `SMATASY`, S. Kirsch, T. Riemann 1995

Some theoretical papers

• Gounaris, Sakurai, 1968

Finite width corrections to the vector meson dominance prediction for $\rho \to e^+e^-$

Early discussion of several width approaches, mass shift from energy dependent width

• Passarino, Veltman, 1979

One loop corrections for $e^+e^- \to \mu^+\mu^-$ in the Weinberg model

First complete electroweak calculation, not with resonance treatment

• Wetzel, 1983

Electroweak radiative corrections for $e^+e^- \to \mu^+\mu^-$ at LEP energies

Resonance treatment; energy-dependent width with higher orders

• Consoli, Sirlin, 1986

The role of the one loop electroweak effects in $e^+e^- \to \mu^+\mu^-$

Discussion of Z mass and complex pole location, but then as of higher order neglected

• Bardin, Leike, Riemann, Sachwitz, 1988

Energy dependent width effects in e^+e^- annihilation near the Z boson pole

Observe the mass shift between constant and S-dependent width of $\frac{1}{2}\Gamma^2/M_Z$=34 GeV

Some theoretical papers

- Willenbrock, Valencia, 1991

On the definition of the Z boson mass

....

- Sirlin,1991

Theoretical considerations concerning the Z^0 mass

...

- Stuart, 1991

Gauge invariance, analyticity and physical observables at the Z^0 resonance

....

- Leike, Riemann, Rose, 1991

S-matrix approach to the Z line shape

....

- Riemann, 1992

Cross-section asymmetries around the Z peak

....

- Bohm, Harshman, 2000

On the mass and width of the Z boson and other relativistic quasistable particles

....

- Freitas, Heinemeyer, Hollik, Walter, Weiglein, 2000

Calculation of fermionic two loop contributions to μ- decay and for the $M_W - M_Z$ interdependence

....

- A summary of status: Riemann, Goslar 1996

The Z boson resonance parameters

. . .

3.

Experimental results

Some experimental papers

• Review: PDG, Review of Particle Physics, EPJC 15 (2000), p.257
and refs. therein: L3, PLB (1997) and OPAL, PLB (1997)

• see also:
`http://lepewwg.web.cern.ch/LEPEWWG/lineshape/`
`http://lepewwg.web.cern.ch/LEPEWWG/smatrix/`
`http://lepewwg.web.cern.ch/LEPEWWG/lep2/`

• First LEP collab. paper: L3 Collab., PLB (1993)
An S-matrix analysis of the Z resonance

• TOPAZ Collab., 1995
Measurement of the total hadronic cross-section and determination of ΓZ interference in e^+e^- annihilation
combining KEK data with OPAL data

• OPAL Collab. , 1997
Production of fermion pair events in e^+e^- collisions at 161 GeV

• L3 Collab., 1997
Measurement of hadron and lepton pair production at 161 – 172 GeV at LEP

Some recent experimental papers with full statistics

• L3 Collab., EPJC (2000)

Measurements of cross-sections and forward backward asymmetries at the Z resonance and determination of electroweak parameters

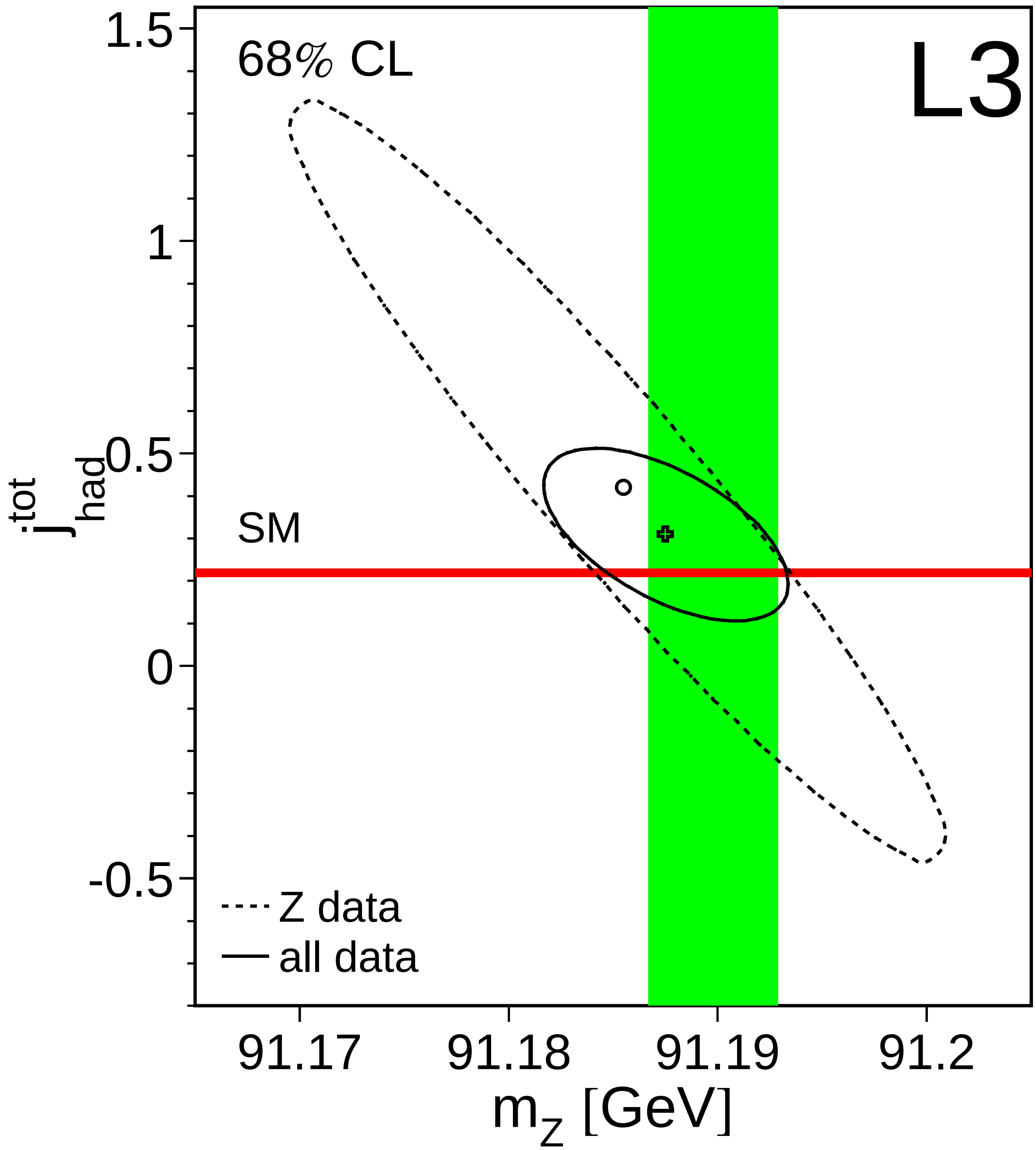
Based on only LEP 1 data ...see Tables 32,33,34 there

• *L3 Collab., subm. to PRL (2000)*

Determination of γZ interference in e^+e^- annihilation at LEP

Especially on γZ interference, m_z correlation with j_{had}^{tot}

Based on LEP 1 and LEP 2 data ...see improvements in Tables 1,2,3 $\rightarrow$ *see Tables and Figure*

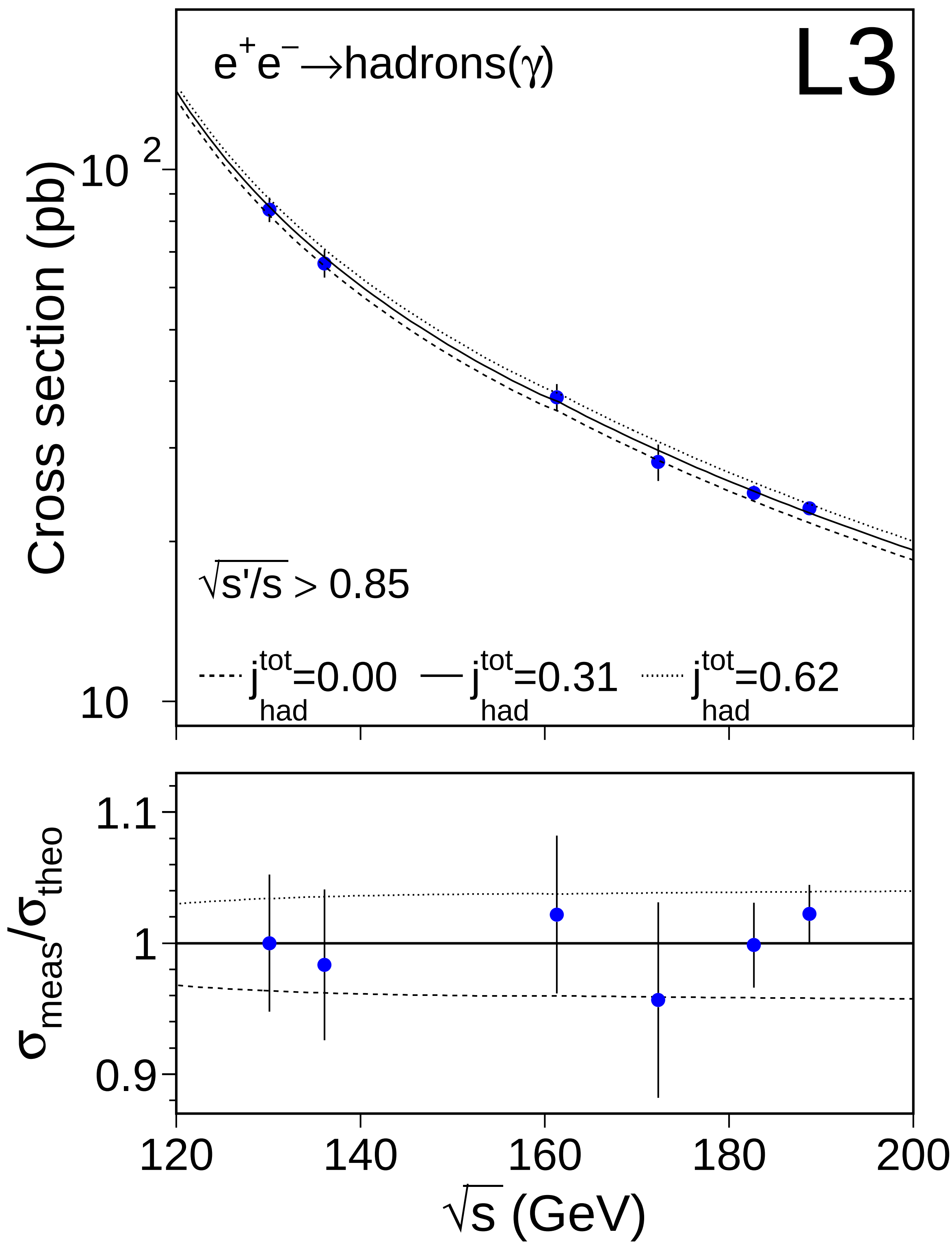


Dashed line: Z resonance data only – circle: central fit

Solid line: LEP 2 data added – cross: central fit

Horizontal band: SM prediction for j_{had}^{tot}

Vertical band: m_Z fit with SM prediction for γZ interference



Hadronic Cross-Section at LEP 2

Parameter	Treatment of Charged Leptons Non–Universality	Treatment of Charged Leptons Universality	Theory uncertainty	Standard Model
m_Z [MeV]	91188.3 ± 3.9	91187.5 ± 3.9	0.6	—
Γ_Z [MeV]	2502.8 ± 4.1	2502.5 ± 4.1	0.1	$2492.7\,^{+3.8}_{-5.2}$
r_{had}^{tot}	2.9856 ± 0.0092	2.9848 ± 0.0092	0.0003	$2.9584\,^{+0.0088}_{-0.0119}$
r_e^{tot}	0.14317 ± 0.00075	—	0.00002	
r_μ^{tot}	0.14287 ± 0.00079	—	0.00002	
r_τ^{tot}	0.14375 ± 0.00102	—	0.00002	
r_ℓ^{tot}	—	0.14318 ± 0.00059	0.00002	$0.14242\,^{+0.00035}_{-0.00049}$
j_{had}^{tot}	0.30 ± 0.13	0.31 ± 0.13	0.04	0.21 ± 0.01
j_e^{tot}	-0.030 ± 0.045	—	0.002	
j_μ^{tot}	-0.001 ± 0.027	—	0.002	
j_τ^{tot}	0.061 ± 0.031	—	0.002	
j_ℓ^{tot}	—	0.017 ± 0.019	0.002	0.0041 ± 0.0003
r_e^{fb}	0.00177 ± 0.00111	—	0.000002	
r_μ^{fb}	0.00333 ± 0.00064	—	0.000002	
r_τ^{fb}	0.00448 ± 0.00092	—	0.000002	
r_ℓ^{fb}	—	0.00332 ± 0.00047	0.000002	0.00255 ± 0.00023
j_e^{fb}	0.700 ± 0.075	—	0.001	
j_μ^{fb}	0.807 ± 0.034	—	0.001	
j_τ^{fb}	0.732 ± 0.044	—	0.001	
j_ℓ^{fb}	—	0.770 ± 0.026	0.001	0.799 ± 0.001
χ^2 /d.o.f.	30.4/28	33.0/36		—

Table 1: Results of the fits in the S–Matrix framework without and with the assumption of lepton universality. The theory uncertainties on the S–Matrix parameters are determined from the 0.5% uncertainty on the ZFITTER predictions for cross sections. The Standard Model expectations are calculated using the parameters listed in Equation 1.

Fit is based on LEP 1 and LEP 2 data.

Shown are $m_Z = \bar{m}_Z + 34.1$ MeV and $\Gamma_Z = \bar{\Gamma}_Z + 0.9$ MeV.

	m_Z	Γ_Z	r^{tot}_{had}	r^{tot}_{ℓ}	j^{tot}_{had}	j^{tot}_{ℓ}	r^{fb}_{ℓ}	j^{fb}_{ℓ}
m_Z	1.00	0.05	0.06	–0.02	–0.57	–0.24	0.05	–0.06
Γ_Z		1.00	0.92	0.69	0.01	0.01	0.02	0.05
r^{tot}_{had}			1.00	0.71	0.01	0.00	0.03	0.05
r^{tot}_{ℓ}				1.00	0.04	0.08	0.05	0.08
j^{tot}_{had}					1.00	0.21	–0.03	0.06
j^{tot}_{ℓ}						1.00	0.04	0.25
r^{fb}_{ℓ}							1.00	0.11
j^{fb}_{ℓ}								1.00

Table 3: Correlation coefficients of the S–Matrix parameters listed in Table 1 assuming lepton universality.

Largest correlations are between m_Z and j^{tot}_{had}, j^{tot}_{l}
and between Γ_Z and r^{tot}_{had}, r^{tot}_{l}.

The central values agree nicely with those of the Standard Model fit.

4.

Renormalization

and

gauge-invariance

Renormalization and Gauge-invariance

See many papers, e.g. Consoli, Sirlin 1986

... Sirlin 1991, Willenbrock 1991, Stuart 1991

... Freitas, Heinemeyer, Hollik, Walter, Weiglein 2000

$$D(s) = \frac{1}{s - M_0^2 - \Pi(s)}$$

On-shell renormalization condition:

$$M_0^2 = M_Z^2 - \Re e\ \Pi(M_Z^2)$$

leads to nearly non-influenced imaginary part (width) of Π, i.e. s-dependent Γ_Z:

$$D(s) = \frac{1}{s - M_Z^2 - [\Pi(s) - \Re e\ \Pi(M_Z^2)]}$$

Complex pole renormalization condition:

$$M_0^2 = \bar{s} - \Pi(\bar{s})$$

with $\quad \bar{s} = \bar{m}_Z^2 - i\bar{m}_Z\bar{\Gamma}_Z \quad or \quad \bar{s} = (M_R - \frac{i}{2}\Gamma_R)^2$

leads to nearly constant width function, i.e. s-independent Γ_Z:

$$D(s) = \frac{1}{s - \bar{m}_Z^2 - [\Pi(s) - \Pi(\bar{s})] + i\bar{m}_Z\bar{\Gamma}_Z}$$

$\longrightarrow$ The difference $M_Z^2 - \bar{m}_Z^2 = \bar{\Gamma}_Z^2 + O(\alpha^3)$ is order $O(\alpha^2)$ and gauge-dependent.

5.

Width and life-time

Width and Life-time of the Resonance

A. Bohm and N. Harshman and collab., 1997–2000
Study of the ambiguity of mass and width definitions of relativistic resoances from a mathematical point of view.

They use relativistic Gamov vectors and rigged Hilbert spaces and study:

- Resonance width Γ defined by a Breit-Wigner line shape

- Resonance life time τ defined by the exponential decay law

Demand:

$$\Gamma = \frac{1}{\tau}$$

and select this way:

$$D(s) = \frac{1}{s - (M_R - \frac{i}{2}\Gamma_R)^2}$$

6.

Summary

Summary

- We have compared two approaches to the Z boson line shape:
  – the model-independent Standard LEP approach
  – the S-matrix approach

  They differ in the determination of M_Z and in the treatment of the resonance shape

- The Z line shape may be described by 4 independent parameters (per channel):
  $M_Z, \quad \Gamma_Z, \quad R_T, \quad J_T$
  – if QED is assumed to be a known phenomenon

- The γZ interference is an independent quantity, which enlarges the error for M_Z.

- Asymmetries depend on two parameters (per channel):
  $R_{asy}, \quad J_{asy}$
  The asymmetries' variations with s near the peak are due to the γZ interference

- Several mass definitions are used
  Only one of them, M_R, is gauge invariant and leads to a nice relation to the life time
  $M_R = M_Z - 26$ MeV
  $D^{-1} = s - (M_R - \frac{i}{2}\Gamma_R)^2$

- We strongly recommend the four LEP collaborations to perform a combined line shape fit in the S-matrix approach