Coherent Exclusive Exponentiation
of 2f Processes
in e^+e^- Annihilation

Z. Wąs
Institute of Nuclear Physics, Kraków, Poland
and
CERN-TH, Geneva

Main Topics:

- **Introduction.**
- **Physics motivation.**
- **Basic steps to CEEX.**
- **Summary, outlook.**

Papers on CEEX by S. Jadach, B.F.L. Ward and Z. Wąs:

Phys. Lett. B449 (1999) 97, (CERN-TH/98-253), CERN-TH/98-235, subm. to EPJ,

DESY-99-106 (CERN-TH/99-235), 88p., Comput. Phys. Commun. in press

UTHEP-99-09-01 (CERN-TH/2000-087), in preparation.

Slides are on http://home.cern.ch/wasm

Theory versus experiment

I hope that we can agree, that at least in simplified way:

The purpose of any theory is to represent in as compact as possible and unambiguous way as large as possible domain of experimental data (past and future).

The purpose of any measurement is to confirm existing theory in new conditions, explore the limits of its validity or, eventually find where it is necessary to develop the new theory.

Each side stimulate another:

New data stimulate creation of new theories.

New theories stimulate new experiments.

At technical level:

The better, more precise are experiments, the better more precise theoretical predictions must be to draw conclusions on the differences or agreements between data and experiment.

Of course there is a lot of work needed to study internal structure of theory and equally much for experiments.

Phenomenology of SM is important

I just can not resist to mention

the three dark sides to fight,

which take most of our time:

Physical uncertainty

- Order of perturbation expansion, - mass terms
- narrow resonance approximations
- Fits to data for vacuum polarization, - etc etc..

Technical uncertainty

- Correctness of the integration/generation algorithms
- Technical approximations in M.E. and/or phase space
- Computer bugs.

Common sense uncertainty

- Mismatch in input parameters
- Accidental misuse of codes
- All other unexpected little things, obvious but invisible.

Design and implementation of algorithms for tests is difficult part of the work, but it is necessary if one wants to exploit fully data obtained from huge effort of experimental communities.

The $\sim$ 0.1 % precision level means:

Present, and near future experiments of HEP do not require the precision to cross the threshold of about 0.01 % $\sim \alpha^2_{QED}$.

That is why:

Complete electroweak calculations beyond one loop are not necessary, if all possible enhancement factors of higher orders, such as: $\ln \frac{s}{m_e^2}$, $\ln \frac{M_Z}{\Gamma_Z}$, $\frac{M_t}{M_Z}$, α_s, ... are taken into account.

This required work of many:

QCD calculations, low energy hadr. contributions to vac. pol.

Electroweak calculations

Pair effects

These essential topics will/were covered in other talks.

QED effects can be separated.

That is good news, because that is what makes relations with the measurement conditions most complicated.

Pair effects can not be separated from the cut off-treatment even at 0.5 % precision level, but do not need to be included in the same segment of the calculation.

Solution advantages:

The experimental side:

Monte Carlo programs based on exponentiation are:

- Sufficient for the 0.061 % -0.4 % precision and LEP 2 observables already now.
- Precision can be maintained in case of complicated experimental cuts and detector resolution. That is, can be assured at the early stage of data handling.

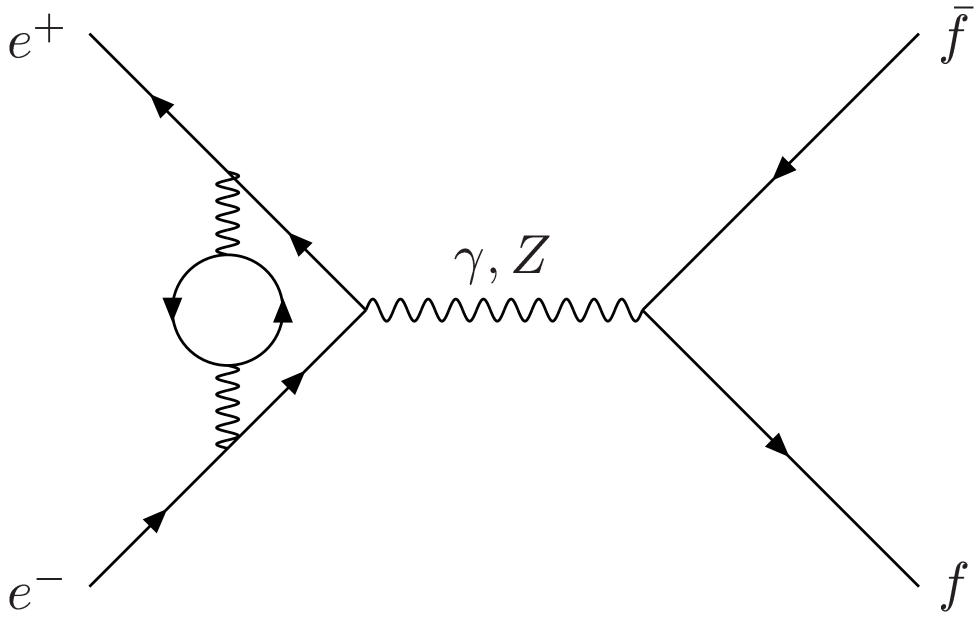
Physical side

- YFS machinery is available and proven since many years. From the beginning, it gives full control over potentially largest terms to all orders of perturbation expansion. From the beginning it gives full coverage of phase-space, including configurations with any number of additional photons.
- Perturbation correction can be added order-by-order in a systematic way.

Technical side

- YFS machinery is available and proven since many years.
- At zeroth order matrix element has a unique (scaling) symmetry, which was convenient in constructing algorithm.
- Pair effects can be treated as independent processes.

A typical example of virtual pair correction



Virtual Pair effect can be added to F_1 electric form-factor using old and well known formula:

$$F_1^{pair}(s) = \sum_f \left\{ -\frac{1}{36} L_f^3 + \frac{19}{72} L_f^2 + \left(\frac{1}{18}\pi^2 - \frac{265}{216}\right) L_f + C_F \right\}, \quad (1)$$

$$C_F = \begin{cases} \frac{383}{108} - \frac{11}{6}\frac{\pi^2}{6}, & m_f = m_F, \\ -\frac{1}{3}\zeta(3) + \frac{3355}{1296} - \frac{19}{18}\frac{\pi^2}{6}, & m_f \gg m_F. \end{cases} \quad (2)$$

$$L_f = \log\frac{s}{m_f^2}, \quad (3)$$

See for instance G. Burgers, *Phys. Lett.*, **B164**, (1985), 167

m_f denotes mass of the fermion in the loop and m_F mass of the incoming fermion (typically an electron).

EEX = Exclusive EXponentiation

Based on Yennie-Frautschi-Suura (1961)

YFS2, YFS3, KORALZ, $e^+e^- \to f\bar{f} + n\gamma$

YFS3WW, $e^+e^- \to W^+W^- + n\gamma$

YFS3ZZ, $e^+e^- \to ZZ + n\gamma$

BHLUMI, BHWIDE, $e^+e^- \to e^+e^- + n\gamma$

CEEX = Coherent Exclusive EXponentiation

New one, by Jadach-Ward-Wąs (1998), an extension of Yennie-Frautschi-Suura

$\mathcal{KK}$MC, $e^+e^- \to f\bar{f} + n\gamma,\ f = \mu, \tau, d, u, s, c, b$

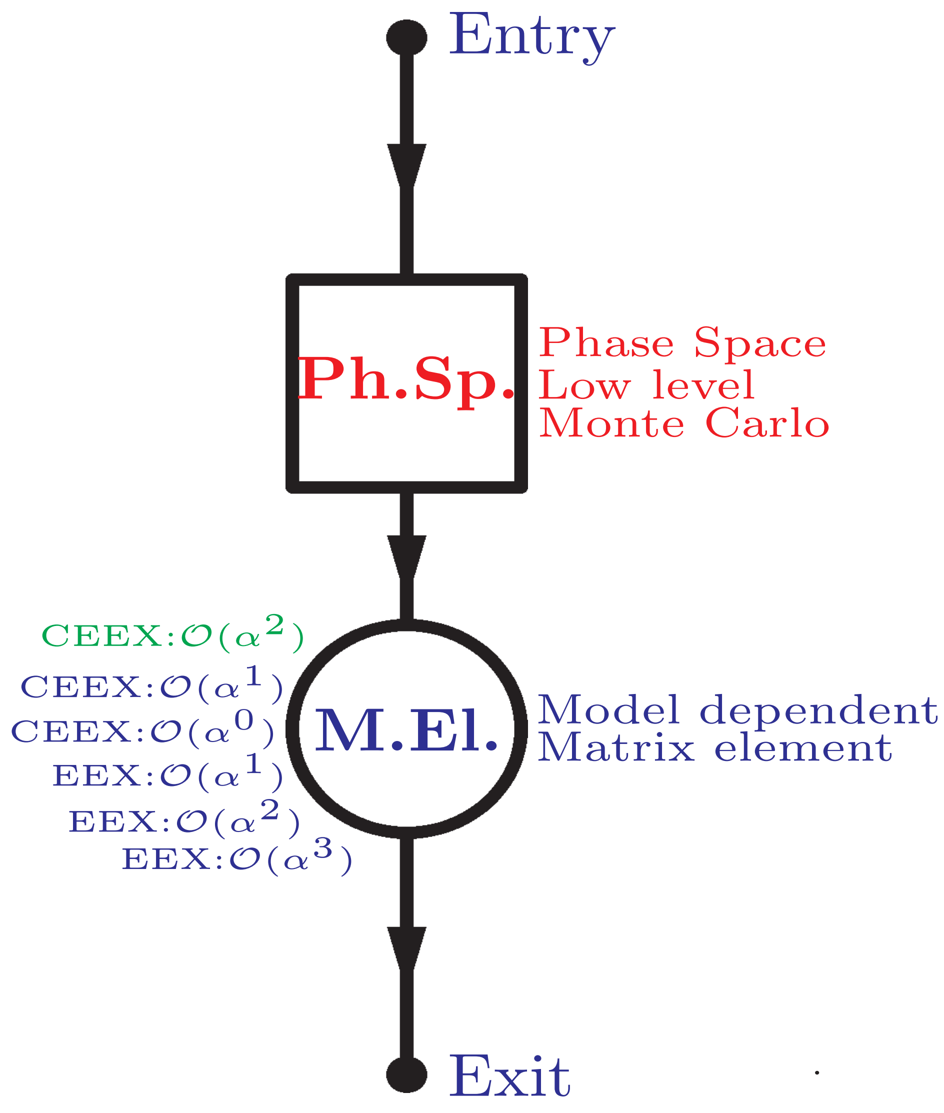
IEX = Inclusive EXponentiation

Semi-analytical exponentiation, performed at the level of earlier integrated cross-sections:

$\mathcal{KK}$sem, CALASY, Zfitter, Gentle...

TERMINOLOGY

General structure of $\mathcal{KK}$ MC (flowchart)



- **Universal Phase-space Monte Carlo simulator**
- **Library of several types of QED matrix elements**
- **Library of SM/QCD formf-factors**
- **Tau decays or hadronization come after**

Why COHERENT?

Because: Friendly to Quantum coherence;

Coherence among Feynman diagrams: Narrow resonances,

$\gamma \oplus Z$ exchanges, $t \oplus s$ channels, ISR⊕FSR real emissions, etc.

Complete $\left|\sum_{diagr.}^{n} \mathcal{M}_i\right|^2$ rather then often incomplete $\sum_{i,j}^{n^2} \mathcal{M}_i \mathcal{M}_j^*$

Why EXCLUSIVE?

Because: Phase-Space $\times \sum_{spin} \left|\mathcal{M}\right|^2$

Contrary to Parton-Showers, Leading-Logs, Structure-Functions

which are all inherently inclusive.

Why EXPONENTIATION?

Infrared (IR) contributions summed to $\mathcal{O}(\alpha^\infty)$

Going **beyond** Yennie-Frautschi-Suura (EEX):

Real IR singularities isolated/reorganized at the **spin amplitude**

level, while IR cancellations occur for probabilities (integrated

x-sections). Paradox???

Differences with older EEX/YFS? See next slide

EEX= Exclusive EXponentiation = variant of YFS expon.

Phase space–near factorization

$$
\begin{aligned}
&dLips_{n+1}(P) = \\
&\left[2M\,dk_\gamma \frac{2k_\gamma}{M}\, d\cos\theta\, d\phi \frac{1}{8(2\pi)^3}\right] \times dLips_n(p \to k_1 \ldots k_n) \\
= \quad &\left[k_\gamma\, dk_\gamma\, d\cos\theta\, d\phi \frac{1}{2(2\pi)^3}\right] \times dLips_n(p \to k_1 \ldots k_n), \qquad (4)
\end{aligned}
$$

If we have l photons accompanying n other particles, then the factor in brackets become iterated. Statistical factor $\frac{1}{l!}$ has to be introduced:

$$
dLips_{n+l}(P) = \frac{1}{l!} \prod_{i=1}^{l} \left[k_{\gamma_i}\, dk_{\gamma_i}\, d\cos\theta_i\, d\phi_i \frac{1}{2(2\pi)^3}\right] \times dLips_n(p \to k_1 \ldots k_n). \qquad (5)
$$

If we do not ask questions about phase-space limit this look like perfect term in expansion of exponent[a].

As the leading parts of the Matrix elements squared, have the same factorization property, the exponents of the YFS expansion form.

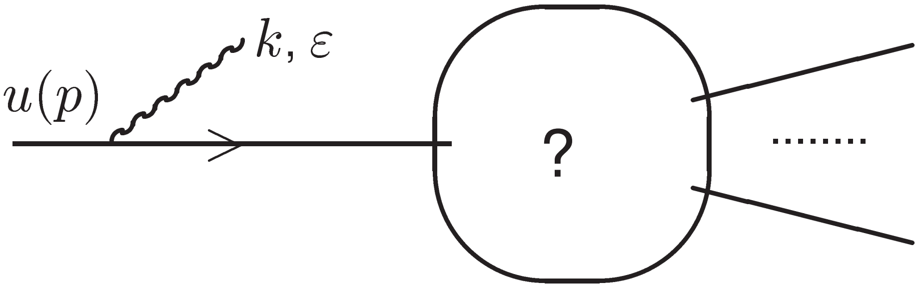
Now CEEX matrix element

In CEEX we want to rewrite this factorization at the Matrix Element level, to enjoy all coveniences of the spin amplitude methods: shorter formulas, interferences 'for free', spin amplitudes techniques, full density matrices 'very cheaply' etc.

[a] In reality it can be realized relatively easy in M.C. because of the scale symmetry present in the expression.

CEEX in practice

M.E. factorization



Feynman diagram of photon emission from initial state electron. Dots represent all other fields entering amplitude.

$$\mathcal{M} = \dots \frac{\not p - \not k + m}{-2pk} e \not\varepsilon u(p,s). \tag{6}$$

$$\begin{aligned}
\mathcal{M}^{\mathcal{IR}} &= \frac{-e}{2pk} \dots (\not p + m) \not\varepsilon u(p,s) \\
&= \frac{-e}{2pk} \dots \Big(2p\varepsilon + \not\varepsilon(-\not p + m)\Big) u(p,s) \\
&= \frac{-e}{2pk} \dots (2p\varepsilon) u(p,s) \\
&= e \frac{-\varepsilon p}{pk} \dots u(p,s) \\
&= -e \frac{\varepsilon p}{pk} \mathcal{M}_B,
\end{aligned} \tag{7}$$

Where $\mathcal{M}_B$ is (up to some transfers in propagators) Born-level spin amplitude.

M.E. factorization

We can write finally:

$$\mathcal{M} = -e\frac{\varepsilon p}{pk}\mathcal{M}_B + \mathcal{O}(k), \tag{8}$$

or,

$$\mathcal{M} = -e\frac{\varepsilon p}{pk}\mathcal{M}_B + \ \dots\ e\frac{\not{k}}{2pk}\ \not{\varepsilon}u(p,s). \tag{9}$$

Important for CEEX was to be able to calculate consistently the first term as analytic expression and second term from the spin amplitude techniques.

We have chosen conventions of Kleiss-Stirling but we need full control of the complex phases between contributions.

As a welcomed by-product, this enabled calculation of density matrices for all fermions without any approximations.

In the above, essentials of arguments are present, but all is far from rigorous.

We use K-S spin amplitude techniques for the calculation of IR-finite parts of spin amplitudes only.

EEX $\leftrightarrow$ CEEX

$$e^-(p_1,\lambda_1) + e^+(p_2,\lambda_2) \to f(q_1,\lambda'_1) + \bar{f}(q_2,\lambda'_2) + \gamma(k_1,\sigma_1) + \ldots + \gamma(k_n,\sigma_n)$$

EEX/YFS very schematically

$$\sigma = \sum_{n=0}^{\infty} \int_{m_\gamma} d\Phi_{n+2}\, e^{Y(m_\gamma)} D_n(q_1, q_2, k_1, \ldots, k_n)$$

$\mathcal{O}(\alpha^1)$ Example:

$$D_0 = \bar{\beta}_0$$

$$D_1(k_1) = \bar{\beta}_0 \tilde{S}(k_1) + \bar{\beta}_1(k_1)$$

$$D_2(k_1,k_2) = \bar{\beta}_0 \tilde{S}(k_1)\tilde{S}(k_2) + \bar{\beta}_1(k_1)\tilde{S}(k_2) + \bar{\beta}_1(k_2)\tilde{S}(k_1)$$

Real soft factors:

$$\tilde{S}(k) = \sum_\sigma |\mathfrak{s}_\sigma(k)|^2 = |\mathfrak{s}_+|^2(k) + |\mathfrak{s}_-(k)|^2 = -\frac{\alpha}{\pi}\left(\frac{q_1}{kq_1} - \frac{q_2}{kq_2}\right)^2$$

IR-finite building blocks:

$\bar{\beta}_0 = \sum_\lambda |\mathcal{M}_\lambda|^2$, λ = fermion helicities, σ = photon helicity

$$\bar{\beta}_1(k) = \sum_{\lambda\sigma} |\mathcal{M}^{1-\mathrm{PHOT}}_{\lambda\sigma}|^2 - \sum_\sigma |\mathfrak{s}_\sigma(k)|^2 \sum_\lambda |\mathcal{M}^{\mathrm{Born}}_\lambda|^2$$

Everything in terms of $\sum_{spin} |\ldots|^2$!!!!

CEEX schematically

$$\sigma = \sum_{n=0}^{\infty} \int_{m_\gamma} d\Phi_{n+2} \sum_{\lambda,\sigma_1,\dots,\sigma_n} \left| e^{B(m_\gamma)} \mathcal{M}^{\lambda}_{n,\sigma_1,\dots,\sigma_n}(k_1,\dots,k_n) \right|^2$$

$\mathcal{O}(\alpha^1)$ Example:

$\mathcal{M}^{\lambda}_{0} = \hat{\beta}^{\lambda}_{0}$, λ=fermion helicities,

$$\mathcal{M}^{\lambda}_{1,\sigma_1}(k_1) = \hat{\beta}^{\lambda}_{0}\mathfrak{s}_{\sigma_1}(k_1) + \hat{\beta}^{\lambda}_{1,\sigma_1}(k_1)$$

$$\mathcal{M}^{\lambda}_{2,\sigma_1,\sigma_2}(k_1,k_2) = \hat{\beta}^{\lambda}_{0}\mathfrak{s}_{\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2) + \hat{\beta}^{\lambda}_{1,\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2) + \hat{\beta}^{\lambda}_{1,\sigma_2}(k_2)\mathfrak{s}_{\sigma_1}(k_1)$$

IR-finite building blocks:

$$\hat{\beta}^{\lambda}_{0} = \left(e^{-B}\mathcal{M}^{\text{Born+Virt.}}_{\lambda}\right)\Big|_{\mathcal{O}(\alpha^1)},$$

$$\hat{\beta}^{\lambda}_{1,\sigma}(k) = \mathcal{M}^{\lambda}_{1,\sigma}(k) - \hat{\beta}^{\lambda}_{0}\mathfrak{s}_{\sigma}(k)$$

Everything in terms of $\mathcal{M}$-spin-amplitudes !!!!

CEEX practical considerations

Practical problems

Deficiencies of YFS **Exclusive Exponentiation** (EEX) technique, as implemented in KORALZ/YFS3, BHLUMI, BHWIDE, KORALW:

- Lack of the ISR/FSR or Up/Down interferences
  (except BHWIDE, but difficult to upgrade...)
- Simplified treatment of spin, neglected transverse polarization
  (except KORALB, but no exponentiation...)
- Approximate matrix element for 2 and 3 hard large p_T photons.
  (Multiphotons are important for searches.)

The CEEX solution

New Coherent Exclusive Exponentiation CEEX solves many (all?):

- ISR$\otimes$FSR interferences are included in a natural way.
  Spin amplitudes for ISR and FSR summed squared numerically.
  (The BHWIDE approach gets cumbersome beyond first order)
- Exact treatment of fermion spin polarizations (transv.&longitudinal).
  Numerical Wigner rotations of spin density matrices available.
  (Necessary for interfacing with decay M.C. simulating fermion decays).
- Exact matrix element for 2 and 3 and more photons using
  Kleiss-Stirling (KS) spinor technique.

MC Implementation of CEEX

Comparison of $\mathcal{KK}$ MC with its predecessors

Feature	KORALB	KORALZ	$\mathcal{KK}$ 4.13	$\mathcal{KK}$ 2000+?
QED type	$\mathcal{O}(\alpha)$	EEX	CEEX, EEX	CEEX, EEX
CEEX(ISR+FSR)	none	none	$\{\alpha, \alpha L; \alpha^2 L^2, \alpha^2 L^1\}$	$\{\ldots \alpha^2 L^1; \alpha^3 L^3\}$
EEX(ISR*FSR)	none	$\{\alpha, \alpha L, \alpha^2 L^2\}$	$\{\alpha, \alpha L, \alpha^2 L^2, \alpha^3 L^3\}$	$\{\ldots \alpha^2 L^2, \alpha^3 L^3\}$
ISR-FSR int.	$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha)$	$\{\alpha, \alpha L\}_{\rm CEEX}$	$\{\alpha, \alpha L\}_{\rm CEEX}$
Exact bremss.	1 γ	1, 2coll. γ	1, 2, 3coll. γ	up to 3 γ
Electroweak	No Z-res.	DIZET 6.x	DIZET 6.x	New version?
Beam polar.	long+trans.	longit.	long+trans.	long+trans.
τ polar.	long+trans.	longit.	long+trans.	long+trans.
Hadronization	—	JETSET	JETSET	PYTHIA
τ decay	TAUOLA	TAUOLA	TAUOLA	TAUOLA
Inclusive mode	—	No	Yes	Yes
Beamstrahlung	—	No	Yes	Yes
Beam spread	—	No	Yes	Yes
$\nu\nu$ channel	—	Yes	No	Yes
ee channel	—	No	No	Yes
tt channel	—	No	No	yes?
WW channel	—	No	No	yes?

Full Scale CEEX

Full Scale CEEX $\mathcal{O}(\alpha^r)$, r=1,2, master formula

Polarized total x-section:

$$\sigma^{(r)} = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\tau_n(p_a+p_b; p_c, p_d, k_1, \dots, k_n)\; e^{2\alpha \Re B_4} \sum_{\sigma_i,\lambda,\bar{\lambda}} \sum_{i,j,l,m=0}^{3}$$

$$\hat{\varepsilon}_a^i \hat{\varepsilon}_b^j \sigma^i_{\lambda_a \bar{\lambda}_a} \sigma^j_{\lambda_b \bar{\lambda}_b} \mathfrak{M}_n^{(r)} \begin{pmatrix} p\; k_1\; k_2 & \dots & k_n \\ \lambda\, \sigma_1\, \sigma_2 & \dots & \sigma_n \end{pmatrix} \left[\mathfrak{M}_n^{(r)} \begin{pmatrix} p\; k_1\; k_2 & \dots & k_n \\ \bar{\lambda}\, \sigma_1\, \sigma_2 & \dots & \sigma_n \end{pmatrix} \right]^{\star} \sigma^l_{\bar{\lambda}_c \lambda_c} \sigma^m_{\bar{\lambda}_d \lambda_d} \hat{h}_c^l \hat{h}_c^m$$

CEEX amplitudes:

$$\mathfrak{M}_n^{(1)} \begin{pmatrix} p\; k_1 & \dots & k_n \\ \lambda\, \sigma_1 & \dots & \sigma_n \end{pmatrix} = \sum_{\wp \in \mathcal{P}} \prod_{i=1}^{n} \mathfrak{s}_{[i]}^{\{\wp_i\}} \left\{ \beta_0^{(1)} \left({p \atop \lambda}; X_\wp \right) + \sum_{j=1}^{n} \frac{\beta^{(1)}_{1\{\wp_j\}} \left({p\; k_j \atop \lambda\, \sigma_j}; X_\wp \right)}{\mathfrak{s}_{[j]}^{\{\wp_j\}}} \right\}$$

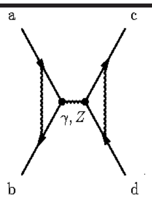
$$\mathfrak{M}_n^{(2)} \begin{pmatrix} p\; k_1 & \dots & k_n \\ \lambda\, \sigma_1 & \dots & \sigma_n \end{pmatrix} = \sum_{\wp \in \mathcal{P}} \prod_{i=1}^{n} \mathfrak{s}_{[i]}^{\{\wp_i\}}$$

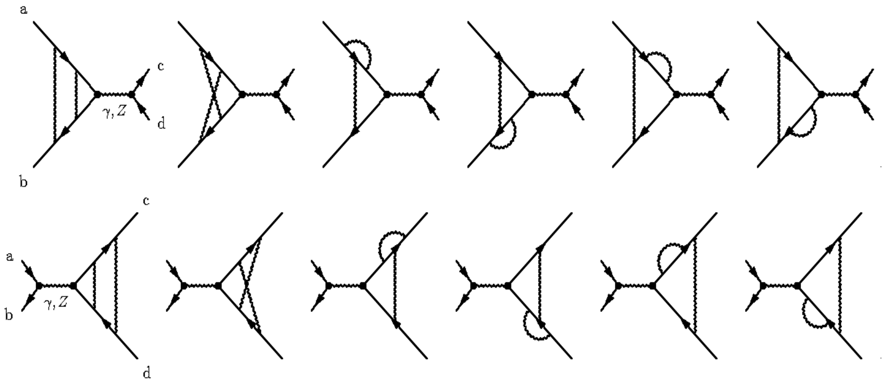
$$\times \left\{ \beta_0^{(2)} \left({p \atop \lambda}; X_\wp \right) + \sum_{j=1}^{n} \frac{\beta^{(1)}_{2\{\wp_j\}} \left({p\; k_j \atop \lambda\, \sigma_j}; X_\wp \right)}{\mathfrak{s}_{[j]}^{\{\wp_j\}}} + \sum_{1 \le j < l \le n} \frac{\beta^{(2)}_{2\{\wp_j, \wp_l\}} \left({p\; k_j\; k_l \atop \lambda\, \sigma_j\, \sigma_l}; X_\wp \right)}{\mathfrak{s}_{[j]}^{\{\wp_j\}} \mathfrak{s}_{[l]}^{\{\wp_l\}}} \right\}$$

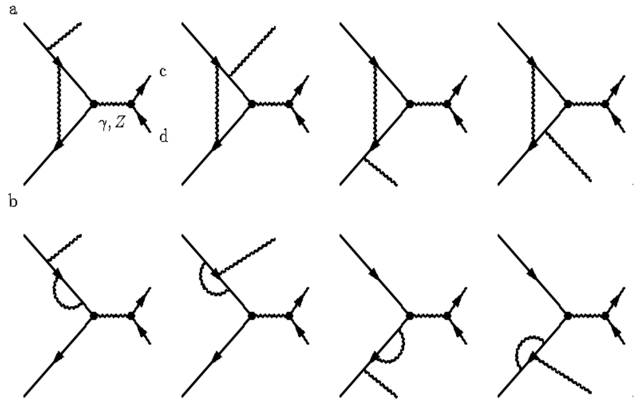
For details see next slides, and for true details our papers.

Full Scale CEEX

$\mathcal{O}(\alpha^2)_{\rm CEEX}$ **Beta's**



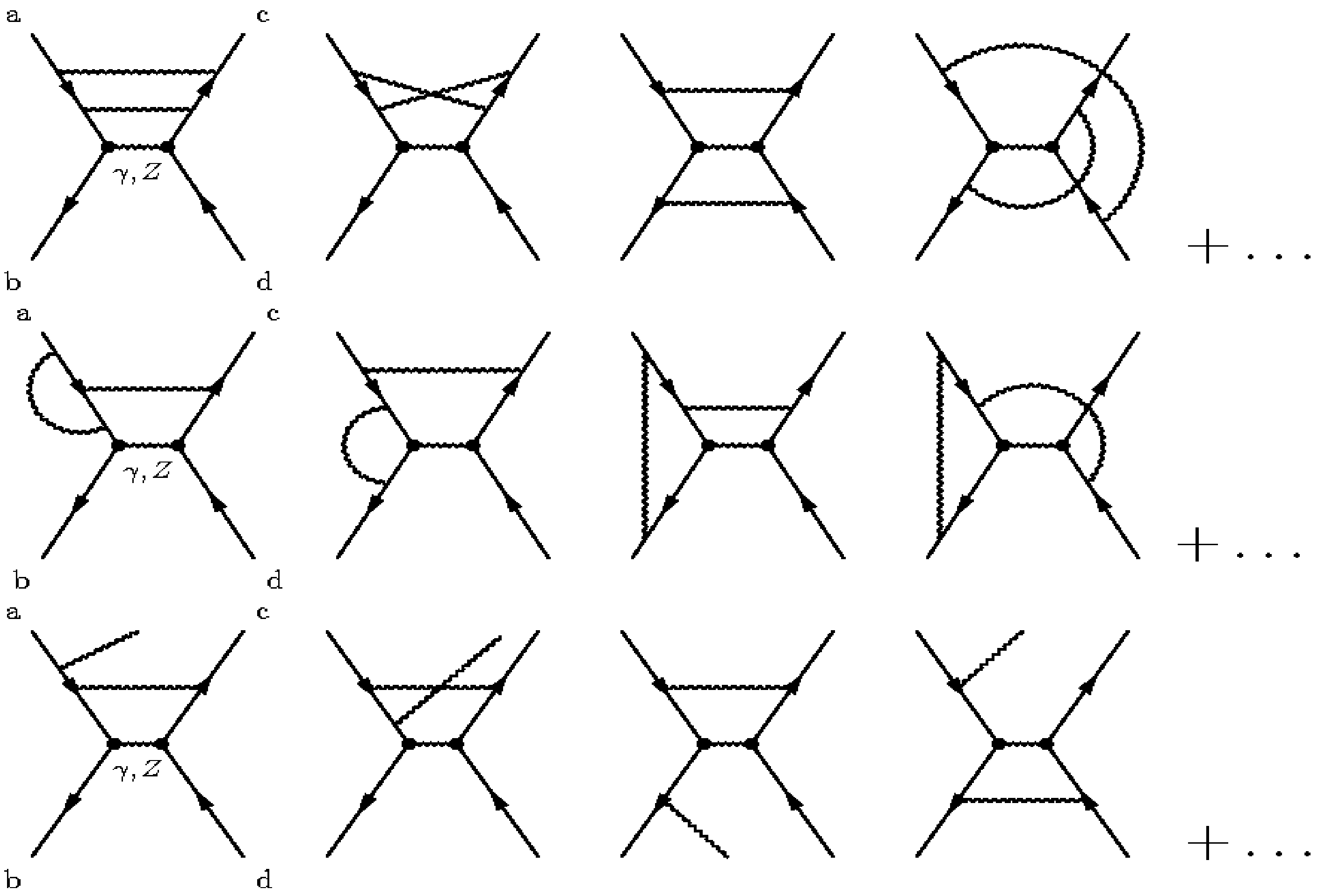




$$
\begin{aligned}
&\beta_0^{(l)}\left({}^{p}_{\lambda};P\right) = \mathfrak{M}_0^{(l)}\left({}^{p}_{\lambda};P\right),\ l=0,1,2,\\
&\beta^{(1+l)}_{1\{I\}}\left({}^{p\,k_1}_{\lambda\,\sigma_1};P-k_1\right) = \mathfrak{M}^{(1+l)}_{1\{I\}}\left({}^{p\,k_1}_{\lambda\,\sigma_1};P-k_1\right) - \beta_0^{(l)}\left({}^{p}_{\lambda};P-k_1\right)\mathfrak{s}^{\{I\}}_{\sigma_1}(k_1),\ l=0,1,\\
&\beta^{(1+l)}_{1\{F\}}\left({}^{p\,k_1}_{\lambda\,\sigma_1};P\right) = \mathfrak{M}^{(1+l)}_{1\{F\}}\left({}^{p\,k_1}_{\lambda\,\sigma_1};P\right) - \beta_0^{(l)}\left({}^{p}_{\lambda};P\right)\mathfrak{s}^{\{F\}}_{\sigma_1}(k_1),\ l=0,1,\\
&\beta^{(2)}_{2\{\omega_1,\omega_2\}}\left({}^{p\,k_1\,k_2}_{\lambda\,\sigma_1\,\sigma_2};X_\omega\right) = \mathfrak{M}^{(2)}_{2\{\omega_1,\omega_2\}}\left({}^{p\,k_1\,k_2}_{\lambda\,\sigma_1\,\sigma_2};X_\omega\right)\\
&\quad -\beta^{(1)}_{1\{\omega_1\}}\left({}^{p\,k_1}_{\lambda\,\sigma_1};X_\omega\right)\mathfrak{s}^{\{\omega_2\}}_{\sigma_2}(k_2) - \beta^{(1)}_{1\{\omega_2\}}\left({}^{p\,k_2}_{\lambda\,\sigma_2};X_\omega\right)\mathfrak{s}^{\{\omega_1\}}_{\sigma_1}(k_1)\\
&\quad -\beta_0^{(0)}\left({}^{p}_{\lambda};X_\omega\right)\mathfrak{s}^{\{\omega_1\}}_{\sigma_1}(k_1)\mathfrak{s}^{\{\omega_2\}}_{\sigma_2}(k_2),
\end{aligned}
$$

Full Scale CEEX

$\mathcal{O}(\alpha^2)_{\rm CEEX}$ Shopping list

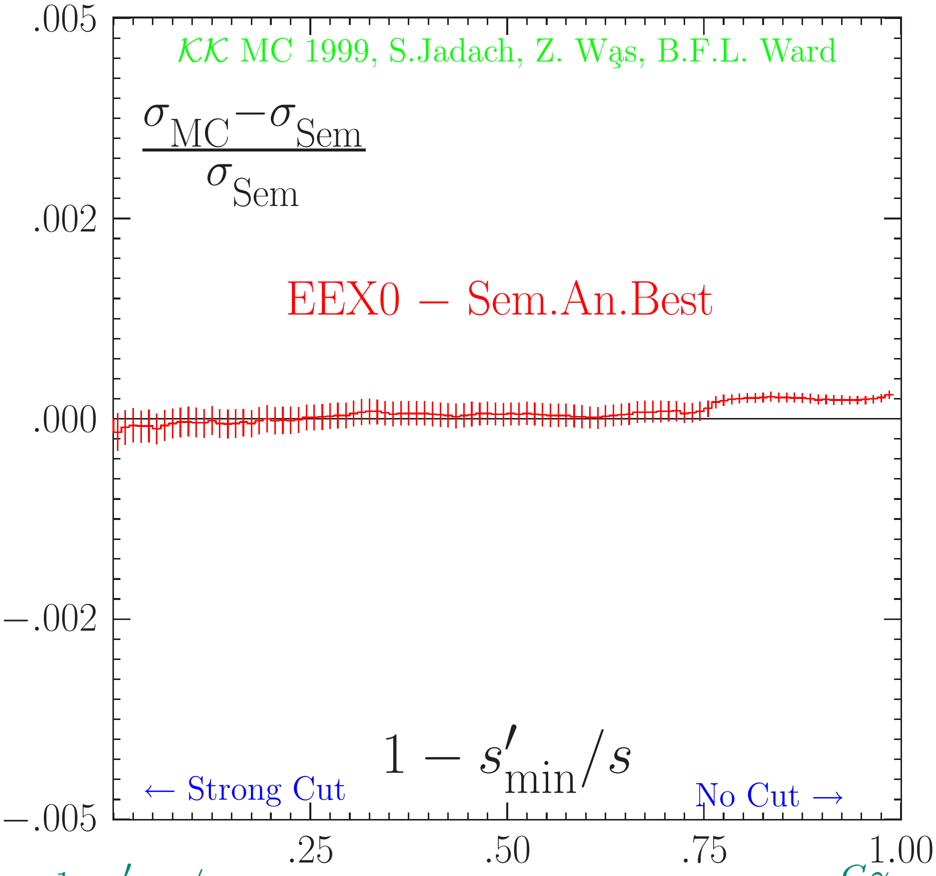


Never ever combine with real emission! (Keep photon mass and publish.)

Never ever square and spin sum! In the M.C. real emission is added anyway.

Baseline test of the Low level MC

$|$ **Matrix Element** $|^2$ **= Born** $\times$ **Product of soft factors**



$$\sigma_{\rm Sem.} = \int_0^{1-s'_{\rm min}/s} dv\ \sigma^f_{\rm Born}(s(1-u)(1-v))\ \frac{e^{-C\gamma_e}}{\Gamma(1+\gamma_e)}$$

$$e^{\frac{1}{4}\gamma_e+\frac{\alpha}{\pi}\left(\frac{1}{2}+\frac{\pi^2}{3}\right)}\ \gamma_e v^{\gamma_e-1}\left[1-\frac{\gamma_e\ln(1-v)}{4}-\frac{\alpha}{\pi}\frac{\ln^2(1-v)}{2}+0\ \gamma_e^2\right]$$

$$\frac{e^{-C\gamma_f}}{\Gamma(1+\gamma_f)}e^{\frac{1}{4}\gamma_f-\frac{1}{2}\gamma_f\ln(1-u)+\frac{\alpha}{\pi}\left(\frac{1}{2}+\frac{\pi^2}{3}\right)}\gamma_f u^{\gamma_f-1}\left[1-\frac{\gamma_f\ln(1-u)}{4}\right]$$

Techn.Error=$|$MC−Sem.Anl.$|$ $\leq 2\cdot 10^{-4}$

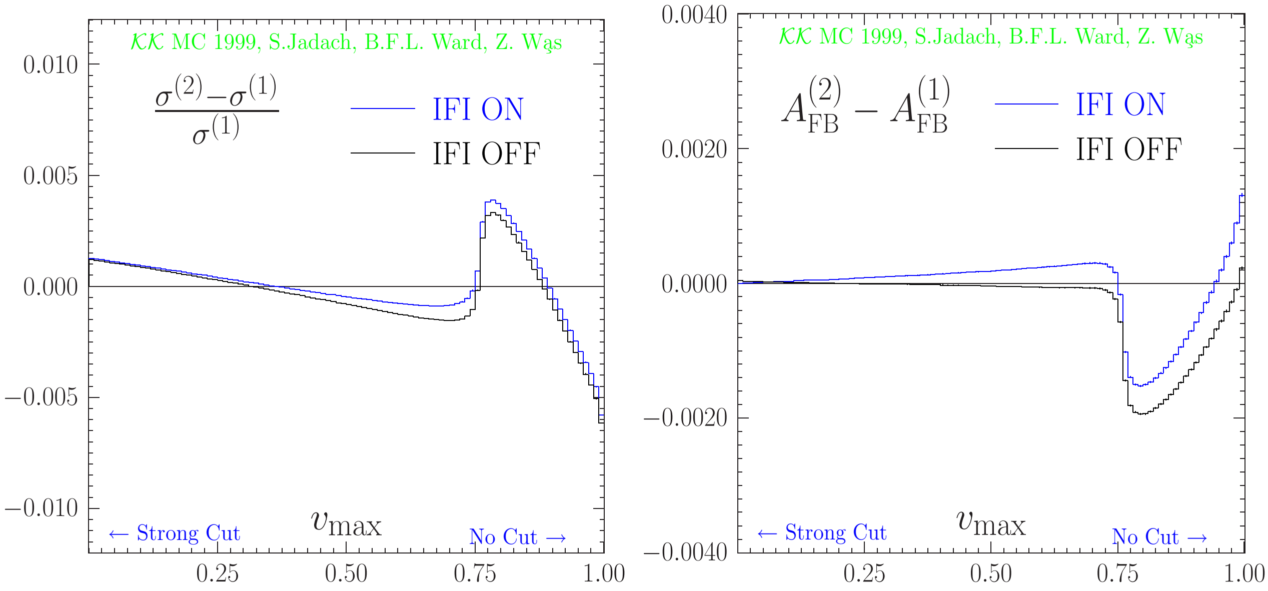
PP = neglected higher orders and subleadings

Our basic estimate: PP = $\frac{1}{2}\left\{\mathcal{O}(\alpha^2)_{\rm CEEX} - \mathcal{O}(\alpha^1)_{\rm CEEX}\right\}$

$\mathcal{KK}$ **MC results for** $e^+e^- \to \mu^+\mu^-$ **at 189GeV**

Photon Energy Cut: $v = 1 - s'/s < v_{\max}$; $s' = m^2_{f\bar{f}}$

Angular Cut: $|\cos\theta| < 1$



The above result imply for Physical Precision:
$\frac{\delta\sigma}{\sigma} \le 0.2\%$, $\delta A_{FB} \le 0.1\%$,
even including radiative-return on-shell Z.

This is confirmed by more tests, see later.

Absolute prediction for σ and A_{FB}

Process: $e^- e^+ \to f\bar{f}$, $f = \mu^-$, **at 189GeV.**

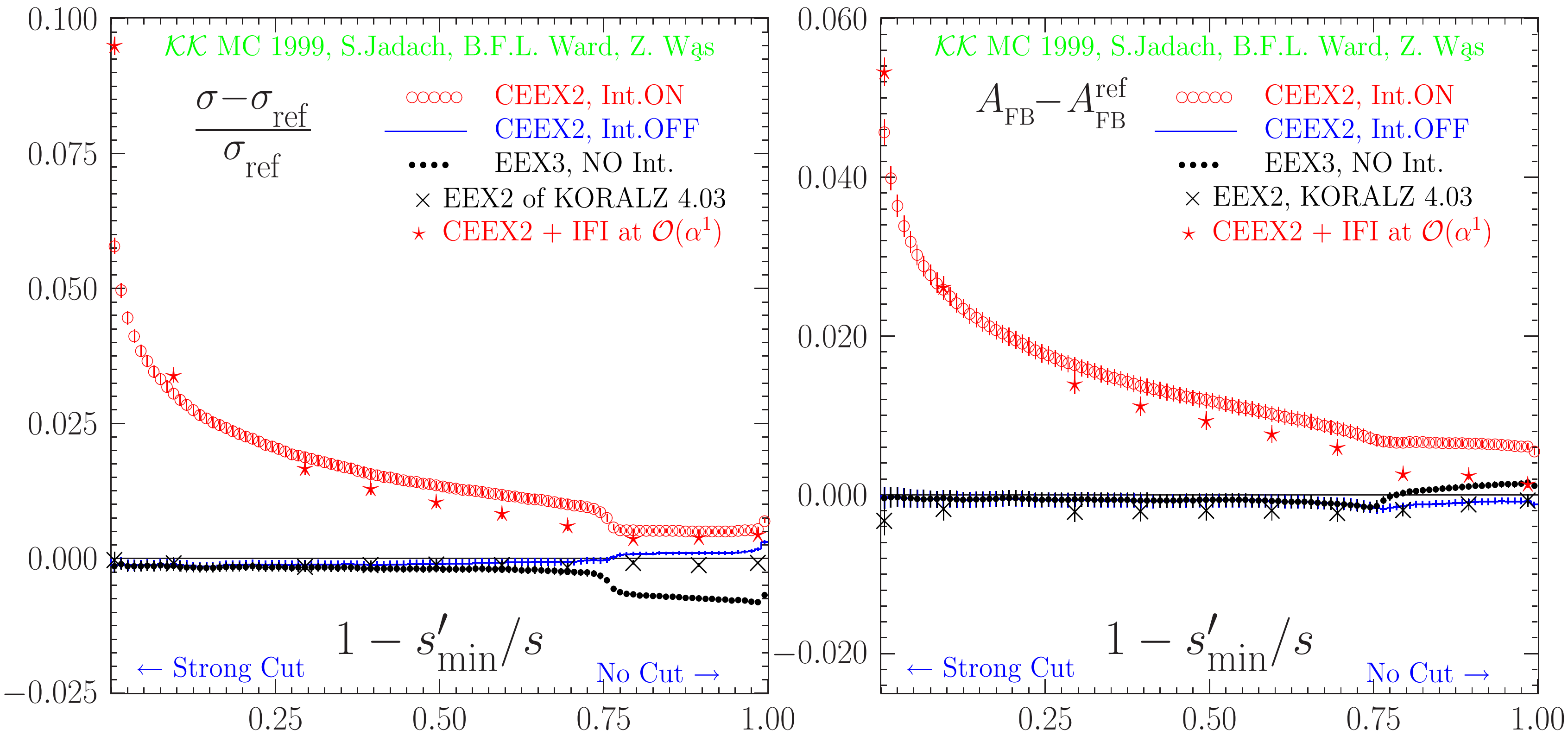
Energy cut: $v < v_{\max}$, **where** $v = 1 - s'/s$, $s' = M^2_{f\bar{f}}$.

Scattering angle for $A_{\rm FB}$ **is** $\theta = \theta^\bullet$. **No cut in** θ. **E-W corr. in** $\mathcal{KK}$ **according to DIZET 6.x.** $\mathcal{O}(\alpha^3)_{\rm LL}$ **EEX3 matrix element in** $\mathcal{KK}$ **MC (No ISR$\otimes$FSR interf.)**

$\mathcal{KK}$**sem is semianalytical part of** $\mathcal{KK}$. (Angle $\theta^\bullet$ is from Phys. Rev. D41, 1425 (1990).)

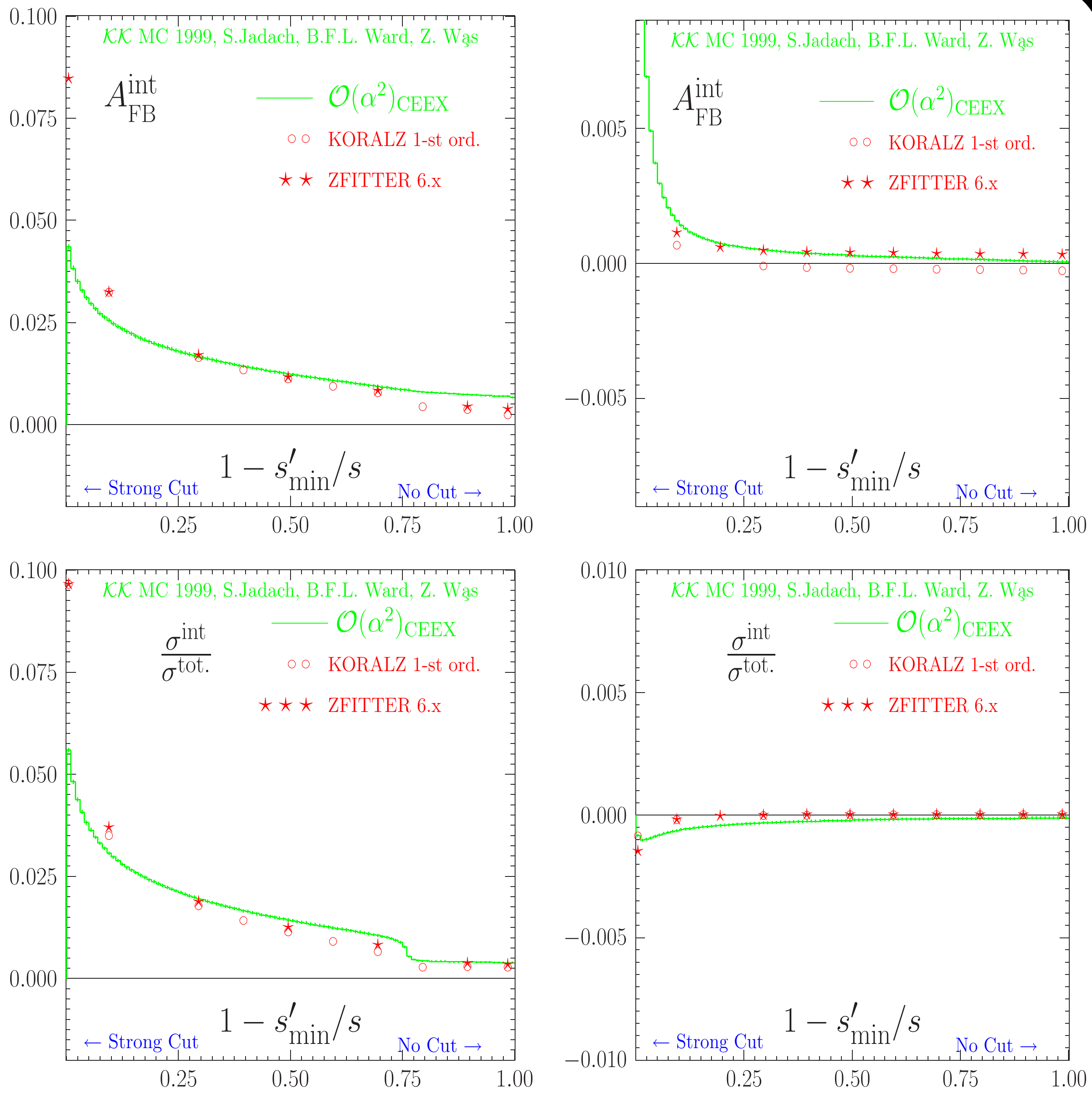
$v_{\max}$	$\mathcal{KK}$sem Refer.	$\mathcal{O}(\alpha^3)_{\rm EEX3}$	$\mathcal{O}(\alpha^2)_{\rm CEEX}$ intOFF	$\mathcal{O}(\alpha^2)_{\rm CEEX}$	KORALZ	KORALZ Interf.
	$\sigma(v_{\max})$ [pb], $\mathcal{KK}$ M.C. and KORALZ 1-st order					
.01	$1.6712 \pm .0000$	$1.6687 \pm .0020$	$1.6690 \pm .0020$	$1.7679 \pm .0024$	$.9639 \pm .0009$	$.1610 \pm .0009$
.10	$2.5198 \pm .0000$	$2.5164 \pm .0023$	$2.5170 \pm .0023$	$2.5967 \pm .0027$	$2.1919 \pm .0010$	$.0880 \pm .0010$
.30	$3.0616 \pm .0000$	$3.0565 \pm .0024$	$3.0581 \pm .0024$	$3.1190 \pm .0029$	$2.7690 \pm .0010$	$.0545 \pm .0010$
.50	$3.3747 \pm .0000$	$3.3682 \pm .0025$	$3.3713 \pm .0025$	$3.4203 \pm .0029$	$3.0565 \pm .0010$	$.0385 \pm .0010$
.70	$3.7225 \pm .0000$	$3.7131 \pm .0025$	$3.7200 \pm .0025$	$3.7596 \pm .0030$	$3.3649 \pm .0010$	$.0246 \pm .0010$
.90	$7.1434 \pm .0000$	$7.0904 \pm .0024$	$7.1496 \pm .0024$	$7.1789 \pm .0030$	$6.3558 \pm .0010$	$.0210 \pm .0010$
.99	$7.6145 \pm .0000$	$7.5511 \pm .0024$	$7.6254 \pm .0024$	$7.6542 \pm .0029$	$6.7004 \pm .0010$	$.0213 \pm .0010$
	$A_{\rm FB}(v_{\max})$, $\mathcal{KK}$ M.C. and KORALZ 1-st order					
.01	$.5654 \pm .0000$	$.5650 \pm .0014$	$.5650 \pm .0014$	$.6111 \pm .0016$	$.5765 \pm .0013$	$.1201 \pm .0013$
.10	$.5664 \pm .0000$	$.5660 \pm .0011$	$.5660 \pm .0011$	$.5922 \pm .0012$	$.5784 \pm .0006$	$.0324 \pm .0006$
.30	$.5692 \pm .0000$	$.5687 \pm .0009$	$.5686 \pm .0009$	$.5856 \pm .0011$	$.5818 \pm .0005$	$.0164 \pm .0005$
.50	$.5744 \pm .0000$	$.5738 \pm .0009$	$.5737 \pm .0009$	$.5863 \pm .0010$	$.5868 \pm .0005$	$.0112 \pm .0005$
.70	$.5864 \pm .0000$	$.5852 \pm .0008$	$.5852 \pm .0008$	$.5947 \pm .0009$	$.5972 \pm .0004$	$.0078 \pm .0004$
.90	$.3105 \pm .0000$	$.3115 \pm .0004$	$.3096 \pm .0004$	$.3170 \pm .0005$	$.3260 \pm .0002$	$.0037 \pm .0002$
.99	$.2851 \pm .0000$	$.2867 \pm .0004$	$.2843 \pm .0004$	$.2912 \pm .0004$	$.3039 \pm .0002$	$.0024 \pm .0002$

Absolute prediction for σ and A_{FB}



Absolute prediction for total cross section an charge asymmetry. They are for $\mu^+\mu^-$ final state at $\sqrt{s} = 189$GeV. Results are plotted as a function of the cut-off on the total photon energy $v_{\max} = 1 - s'_{\min}/s$. The "reference" σ and $A_{\rm FB}$ in first column are from $\mathcal{KK}$sem semi-analytical program.

ISR$\otimes$FSR interference



The s'-cut dependence of $\delta\sigma$ and δA_{FB} due to ISR$\otimes$FSR interference. No θ-cut. **For the ISR$\otimes$FSR KORALZ is the best reference and starting point**

Pure $\mathcal{O}(\alpha^1)$ (no exponentiation) ISR$\otimes$FSR in KORALZ was verified very precisely $\sim 0.01\%$ with special analytical calculation, S.Jadach Z.Wąs, Phys. Rev. D41, 1425 (1990).

$\mathcal{KK}$MC versus $\mathcal{KK}$sem, WT-ed events

Inclusive run of $\mathcal{KK}$MC, $q = d, u, s, c, b$.
Energy: $\sqrt{s}$= 189GeV. FSR is off.
The only cut $v = 1 - s'/s < v_{\max}$.

$v_{\max}$	$\mathcal{KK}$sem Refer.	$\mathcal{O}(\alpha^3)_{\rm EEX3}$	$\mathcal{O}(\alpha^2)_{\rm CEEX}$ intOFF
	$\sigma(v_{\max})$ [pb]		
0.01	14.9309 ± 0.0000	14.9276 ± 0.0078	14.9298 ± 0.0078
0.10	19.5171 ± 0.0000	19.5093 ± 0.0086	19.5135 ± 0.0086
0.20	21.2573 ± 0.0000	21.2487 ± 0.0088	21.2542 ± 0.0088
0.30	22.4477 ± 0.0000	22.4370 ± 0.0089	22.4436 ± 0.0089
0.40	23.4602 ± 0.0000	23.4454 ± 0.0090	23.4539 ± 0.0090
0.50	24.4711 ± 0.0000	24.4515 ± 0.0091	24.4639 ± 0.0091
0.60	25.7152 ± 0.0000	25.6890 ± 0.0092	25.7082 ± 0.0092
0.70	28.2545 ± 0.0000	28.2038 ± 0.0092	28.2482 ± 0.0093
0.80	92.7078 ± 0.0000	91.8508 ± 0.0088	92.8080 ± 0.0098
0.90	95.3717 ± 0.0000	94.4731 ± 0.0087	95.4779 ± 0.0097
0.99	97.0035 ± 0.0000	96.0726 ± 0.0086	97.1242 ± 0.0096
	$A_{\rm FB}(v_{\max})$		
0.01	0.6358 ± 0.0000	0.6357 ± 0.0006	0.6357 ± 0.0006
0.10	0.6358 ± 0.0000	0.6358 ± 0.0005	0.6358 ± 0.0005
0.20	0.6357 ± 0.0000	0.6357 ± 0.0005	0.6357 ± 0.0005
0.30	0.6356 ± 0.0000	0.6355 ± 0.0005	0.6355 ± 0.0005
0.40	0.6352 ± 0.0000	0.6351 ± 0.0005	0.6351 ± 0.0005
0.50	0.6343 ± 0.0000	0.6341 ± 0.0004	0.6341 ± 0.0004
0.60	0.6316 ± 0.0000	0.6313 ± 0.0004	0.6313 ± 0.0004
0.70	0.6169 ± 0.0000	0.6163 ± 0.0004	0.6162 ± 0.0004
0.80	0.2600 ± 0.0000	0.2599 ± 0.0001	0.2586 ± 0.0001
0.90	0.2432 ± 0.0000	0.2434 ± 0.0001	0.2420 ± 0.0001
0.99	0.2354 ± 0.0000	0.2359 ± 0.0001	0.2344 ± 0.0001

Satisfactory agreement $< 0.2\%$ for Z-excl. cuts.
The same for Z-inclusive cuts.
Comparisons with Zfitter just started.

$\mathcal{KK}$MC versus KORALZ 4.02 and 4.04

WT-ed events. $\sqrt{s}$= 189GeV. FSR is off.
$\mathcal{KK}$MC: inclusive run $q = d, u, s, c, b$.
The only cut $v = 1 - s'/s < v_{\max}$.

No.	(a) $\mathcal{O}(\alpha^2)_{\rm CEEX}$ intOFF	(b) KORALZ 4.02	(c) KORALZ 4.04	(b-a)/a	(c-a)/a
	$\sigma(v_{\max})$ [pb], $v_{\max} = 0.1$				
u	3.1761 ± 0.0034	3.1640 ± 0.0198	3.1831 ± 0.0099	-0.0038 ± 0.0063	0.0022 ± 0.0033
d	4.9917 ± 0.0045	5.0101 ± 0.0247	4.9860 ± 0.0123	0.0037 ± 0.0050	-0.0011 ± 0.0026
s	3.1753 ± 0.0034	3.1639 ± 0.0198	3.1831 ± 0.0099	-0.0036 ± 0.0063	0.0025 ± 0.0033
c	4.9918 ± 0.0045	5.0098 ± 0.0247	4.9860 ± 0.0123	0.0036 ± 0.0050	-0.0012 ± 0.0026
b	3.1786 ± 0.0034	3.1036 ± 0.0194	3.1274 ± 0.0097	-0.0236 ± 0.0062	-0.0161 ± 0.0032
all	19.5135 ± 0.0087	19.4515 ± 0.0488	19.4656 ± 0.0243	-0.0032 ± 0.0025	-0.0025 ± 0.0013
	$\sigma(v_{\max})$ [pb], $v_{\max} = 0.2$				
u	3.4653 ± 0.0035	3.4523 ± 0.0207	3.4697 ± 0.0103	-0.0037 ± 0.0061	0.0013 ± 0.0031
d	5.4305 ± 0.0047	5.4465 ± 0.0258	5.4225 ± 0.0128	0.0030 ± 0.0048	-0.0015 ± 0.0025
s	3.4644 ± 0.0035	3.4522 ± 0.0207	3.4697 ± 0.0103	-0.0035 ± 0.0061	0.0015 ± 0.0031
c	5.4301 ± 0.0047	5.4463 ± 0.0258	5.4225 ± 0.0128	0.0030 ± 0.0048	-0.0014 ± 0.0025
b	3.4640 ± 0.0035	3.3880 ± 0.0203	3.4083 ± 0.0101	-0.0219 ± 0.0059	-0.0161 ± 0.0031
all	21.2542 ± 0.0090	21.1854 ± 0.0510	21.1927 ± 0.0254	-0.0032 ± 0.0024	-0.0029 ± 0.0013
	$\sigma(v_{\max})$ [pb], $v_{\max} = 0.3$				
u	3.6654 ± 0.0036	3.6587 ± 0.0213	3.6738 ± 0.0106	-0.0018 ± 0.0059	0.0023 ± 0.0031
d	5.7258 ± 0.0047	5.7361 ± 0.0265	5.7124 ± 0.0132	0.0018 ± 0.0047	-0.0023 ± 0.0024
s	3.6645 ± 0.0036	3.6587 ± 0.0213	3.6738 ± 0.0106	-0.0016 ± 0.0059	0.0025 ± 0.0031
c	5.7238 ± 0.0047	5.7359 ± 0.0265	5.7124 ± 0.0132	0.0021 ± 0.0047	-0.0020 ± 0.0024
b	3.6641 ± 0.0036	3.5894 ± 0.0209	3.6086 ± 0.0104	-0.0204 ± 0.0058	-0.0152 ± 0.0030
all	22.4436 ± 0.0091	22.3787 ± 0.0524	22.3809 ± 0.0261	-0.0029 ± 0.0024	-0.0028 ± 0.0012
	$\sigma(v_{\max})$ [pb], $v_{\max} = 0.9$				
u	19.5444 ± 0.0054	19.5391 ± 0.0499	19.3552 ± 0.0247	-0.0003 ± 0.0026	-0.0097 ± 0.0013
d	18.6747 ± 0.0059	18.6464 ± 0.0487	18.5010 ± 0.0242	-0.0015 ± 0.0026	-0.0093 ± 0.0013
s	19.5439 ± 0.0054	19.5394 ± 0.0499	19.3552 ± 0.0247	-0.0002 ± 0.0026	-0.0097 ± 0.0013
c	18.6673 ± 0.0059	18.6464 ± 0.0487	18.5010 ± 0.0242	-0.0011 ± 0.0026	-0.0089 ± 0.0013
b	19.0477 ± 0.0053	19.1046 ± 0.0488	18.9289 ± 0.0242	0.0030 ± 0.0026	-0.0062 ± 0.0013
all	95.4779 ± 0.0125	95.4759 ± 0.1101	94.6413 ± 0.0546	0.0000 ± 0.0012	-0.0088 ± 0.0006
	$\sigma(v_{\max})$ [pb], $v_{\max} = 0.99$				
u	19.7046 ± 0.0054	19.6951 ± 0.0095	19.5137 ± 0.0048	-0.0005 ± 0.0006	-0.0097 ± 0.0004
d	19.2592 ± 0.0059	19.2389 ± 0.0120	19.0889 ± 0.0060	-0.0011 ± 0.0007	-0.0088 ± 0.0004
s	19.7043 ± 0.0054	19.6954 ± 0.0095	19.5137 ± 0.0048	-0.0004 ± 0.0006	-0.0097 ± 0.0004
c	19.2518 ± 0.0059	19.2390 ± 0.0120	19.0889 ± 0.0060	-0.0007 ± 0.0007	-0.0085 ± 0.0004
b	19.2042 ± 0.0054	19.2577 ± 0.0092	19.0852 ± 0.0046	0.0028 ± 0.0006	-0.0062 ± 0.0004
all	97.1242 ± 0.0126	97.1261 ± 0.0235	96.2903 ± 0.0118	0.0000 ± 0.0003	-0.0086 ± 0.0002

$\mathcal{KK}$MC agrees with KORALZ 4.02 and 4.04 generally better than 1%, except b quark.

$\mathcal{KK}$MC versus ZFITTER

Low precision comparisons from the LEP2 MC workshop for quarks. For the definition of observables see hep-ph/0007180.

obs.	program	ene	value and error estimate			δ ratio
IAleph1	ZFITTER v6.30	189.0	$9.8387\cdot10^{1}$	$+5.22\cdot10^{-3}$	$-4.80\cdot10^{-2}$	
	ZF6.30 no pair	189.0	$9.6865\cdot10^{1}$	$+5.22\cdot10^{-3}$	$-5.11\cdot10^{-3}$	-1.55
	ZFITTER v6.30	206.0	$7.9133\cdot10^{1}$	$+6.58\cdot10^{-3}$	$-4.25\cdot10^{-2}$	
	ZF6.30 no pair	206.0	$7.7853\cdot10^{1}$	$+6.56\cdot10^{-3}$	$-2.83\cdot10^{-3}$	-1.62
IAleph2	ZFITTER v6.30	189.0	$1.9370\cdot10^{1}$	$+1.02\cdot10^{-2}$	$-1.66\cdot10^{-3}$	
	ZF6.30 no pair	189.0	$1.9400\cdot10^{1}$	$+8.28\cdot10^{-3}$	$-1.66\cdot10^{-3}$	0.16
	ZFITTER v6.30	206.0	$1.5456\cdot10^{1}$	$+8.52\cdot10^{-3}$	$-8.30\cdot10^{-4}$	
	ZF6.30 no pair	206.0	$1.5481\cdot10^{1}$	$+6.98\cdot10^{-3}$	$-8.31\cdot10^{-4}$	0.16
IDelphi1	ZFITTER v6.30	189.0	$9.6865\cdot10^{1}$	$+5.22\cdot10^{-3}$	$-5.11\cdot10^{-3}$	
	ZF6.30 no pair	189.0	$9.6865\cdot10^{1}$	$+5.22\cdot10^{-3}$	$-5.11\cdot10^{-3}$	0.00
	ZFITTER v6.30	206.0	$7.7853\cdot10^{1}$	$+6.56\cdot10^{-3}$	$-2.83\cdot10^{-3}$	
	ZF6.30 no pair	206.0	$7.7853\cdot10^{1}$	$+6.56\cdot10^{-3}$	$-2.83\cdot10^{-3}$	0.00
IDelphi2	ZFITTER v6.30	189.0	$2.2001\cdot10^{1}$	$+1.43\cdot10^{-3}$	$-1.88\cdot10^{-3}$	
	ZF6.30 no pair	189.0	$2.2001\cdot10^{1}$	$+1.43\cdot10^{-3}$	$-1.88\cdot10^{-3}$	0.00
	ZFITTER v6.30	206.0	$1.7540\cdot10^{1}$	$+1.39\cdot10^{-3}$	$-9.43\cdot10^{-4}$	
	ZF6.30 no pair	206.0	$1.7540\cdot10^{1}$	$+1.39\cdot10^{-3}$	$-9.43\cdot10^{-4}$	0.00
ILT1	ZFITTER v6.30	189.0	$9.8429\cdot10^{1}$	$+5.20\cdot10^{-3}$	$-4.80\cdot10^{-2}$	
	ZF6.30 no pair	189.0	$9.6906\cdot10^{1}$	$+5.20\cdot10^{-3}$	$-5.11\cdot10^{-3}$	-1.55
	KKMC 4.13	189.0	$9.6911\cdot10^{1}$	$\pm2.54\cdot10^{-2}$		-1.54
	ZFITTER v6.30	206.0	$7.9167\cdot10^{1}$	$+6.58\cdot10^{-3}$	$-4.25\cdot10^{-2}$	
	ZF6.30 no pair	206.0	$7.7886\cdot10^{1}$	$+6.56\cdot10^{-3}$	$-2.83\cdot10^{-3}$	-1.62
	KKMC 4.13	206.0	$7.7907\cdot10^{1}$	$\pm1.99\cdot10^{-2}$		-1.59
ILT2	ZFITTER v6.30	189.0	$9.6751\cdot10^{1}$	$+6.74\cdot10^{-3}$	$-4.39\cdot10^{-2}$	
	ZF6.30 no pair	189.0	$9.5273\cdot10^{1}$	$+6.74\cdot10^{-3}$	$-5.11\cdot10^{-3}$	-1.53
	KKMC 4.13	189.0	$9.5259\cdot10^{1}$	$\pm2.54\cdot10^{-2}$		-1.54
	ZFITTER v6.30	206.0	$7.7847\cdot10^{1}$	$+8.01\cdot10^{-3}$	$-3.92\cdot10^{-2}$	
	ZF6.30 no pair	206.0	$7.6602\cdot10^{1}$	$+7.98\cdot10^{-3}$	$-2.83\cdot10^{-3}$	-1.60
	KKMC 4.13	206.0	$7.6609\cdot10^{1}$	$\pm1.98\cdot10^{-2}$		-1.59
ILT3	ZFITTER v6.30	189.0	$2.2080\cdot10^{1}$	$+3.34\cdot10^{-3}$	$-1.88\cdot10^{-3}$	
	ZF6.30 no pair	189.0	$2.2100\cdot10^{1}$	$+1.43\cdot10^{-3}$	$-1.88\cdot10^{-3}$	0.09
	KKMC 4.13	189.0	$2.2096\cdot10^{1}$	$\pm1.82\cdot10^{-2}$		0.08
	ZFITTER v6.30	206.0	$1.7603\cdot10^{1}$	$+2.95\cdot10^{-3}$	$-9.43\cdot10^{-4}$	
	ZF6.30 no pair	206.0	$1.7620\cdot10^{1}$	$+1.39\cdot10^{-3}$	$-9.44\cdot10^{-4}$	0.10
	KKMC 4.13	206.0	$1.7616\cdot10^{1}$	$\pm1.43\cdot10^{-2}$		0.07
IOpal1	ZFITTER v6.30	189.0	$9.8429\cdot10^{1}$	$+5.20\cdot10^{-3}$	$-4.80\cdot10^{-2}$	
	ZF6.30 no pair	189.0	$9.6906\cdot10^{1}$	$+5.20\cdot10^{-3}$	$-5.11\cdot10^{-3}$	-1.55
	KKMC 4.13	189.0	$9.6911\cdot10^{1}$	$\pm2.54\cdot10^{-2}$		-1.54
	ZFITTER v6.30	206.0	$7.9167\cdot10^{1}$	$+6.58\cdot10^{-3}$	$-4.25\cdot10^{-2}$	
	ZF6.30 no pair	206.0	$7.7886\cdot10^{1}$	$+6.56\cdot10^{-3}$	$-2.83\cdot10^{-3}$	-1.62
	KKMC 4.13	206.0	$7.7907\cdot10^{1}$	$\pm1.99\cdot10^{-2}$		-1.59
IOpal2	ZFITTER v6.30	189.0	$2.2080\cdot10^{1}$	$+3.34\cdot10^{-3}$	$-1.88\cdot10^{-3}$	
	ZF6.30 no pair	189.0	$2.2100\cdot10^{1}$	$+1.43\cdot10^{-3}$	$-1.88\cdot10^{-3}$	0.09
	KKMC 4.13	189.0	$2.2096\cdot10^{1}$	$\pm1.82\cdot10^{-2}$		0.08
	ZFITTER v6.30	206.0	$1.7603\cdot10^{1}$	$+2.95\cdot10^{-3}$	$-9.43\cdot10^{-4}$	
	ZF6.30 no pair	206.0	$1.7620\cdot10^{1}$	$+1.39\cdot10^{-3}$	$-9.44\cdot10^{-4}$	0.10
	KKMC 4.13	206.0	$1.7616\cdot10^{1}$	$\pm1.43\cdot10^{-2}$		0.07

$\mathcal{KK}$MC agrees with ZFITTER in these few cases, better than 0.1%, if pair effects are switched off. That good agreement is however misleading.

Summary on numerical results

- **Technical Precision** $\frac{\delta\sigma}{\sigma} \leq 2 \cdot 10^{-4}$
- **Physical Precision:**
  $\frac{\delta\sigma}{\sigma} \leq 0.2\%$ **and** $\delta A_{FB} \leq 0.1\%$
- **ISR$\otimes$FSR interf. under control at LEP2 energies**

CEEX - Summary and Outlook

CEEX offers clear upgrade path for the precision calculations of the h.o. radiative corrections in the SM/QED, with the Monte Carlo implementation. It goes beyond (extends) the classical Yennie-Frautschi-Suura scheme. $\mathcal{KK}$ MC based on CEEX offers clear upgrade path for the MC event generators to Linear Collider era. CEEX is firmly based on spin amplitudes. The main profits are:

- Interferences ISR$\otimes$FSR included and under firm control
- All kind of coherence effects, including narrow resonances.
- Complete treatment of spin (also transverse) for initial and final states.
- Exact M.E. for 2 high hard photons (3 photons pending).
- Future extension to all-angle second order Bhabha is possible.
- $\nu\bar{\nu}\gamma$ channel is at work and advanced.
- Some terms proportinal to final state mass still missing, trivial things need to be fixed for applications at CMS energies below 40 GeV.

CEEX - Summary and Outlook

First official version 4.13 of $\mathcal{KK}$ MC available. It works for fermion pair production at LEP, LC's, τ and b factories, μ-colliders.

Check

http://home.cern.ch/jadach

http://home.cern.ch/wasm

for slides, programs.