The width difference
of $B_s$-mesons

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1. Heavy Quark Expansion

Voloshin, Shifman; Bigi, Uraltsev, Vainshtein

Optical theorem for total decay rate $\Gamma$:

$$\Gamma \propto \text{Im} \langle B | i \int d^4x \, T \mathcal{H}_{\text{eff}}(x) \mathcal{H}_{\text{eff}}(0) | B \rangle$$

$\mathcal{H}_{\text{eff}}$ is the effective $|\Delta B|=1$ hamiltonian.

HQE = Operator product expansion:

$$\Gamma \propto G_F^2 \sum_j m_b^{8-d_j} c_j (\mu/m_b) \langle B | \mathcal{O}_j(\mu) | B \rangle \mathcal{O} \left( \Lambda_{QCD}^{d_j-3} \right)$$

$c_j$: Wilson coefficients containing physics from scales $\geq \mu = \mathcal{O}(m_b)$

$\mathcal{O}_j$: local operators with dimension $d_j \geq 3$.

Effect: Expansion of $\Gamma$ in $\Lambda_{QCD}/m_b$ and $\alpha_s(m_b)$. 
\[ \Gamma (B \rightarrow all) = \Gamma (b \rightarrow all) + \mathcal{O} \left( \frac{\Lambda_{QCD}^2}{m_b^2} \right) \]

First term: QCD corrected parton model.

First corrections are \( \mathcal{O} \left( \frac{\Lambda_{QCD}^2}{m_b^2} \right) \) and come from Fermi motion of the b-quark and the chromomagnetic interaction with the light degrees of freedom.

Validity of HQE \( \leftrightarrow \) Quark-hadron duality
2. Lifetime differences

Dominant source of lifetime differences between \(B_d\), \(B_s\) and \(B^\pm\) mesons: Participation of the spectator quark in the weak decay. Effect of order \(\mathcal{O}\left(16\pi^2\frac{\Lambda^3_{QCD}}{m_b^3}\right)\)

(Except: \(\tau(B_s) - \tau(B_d)\) stems from \(SU(3)_F\) breaking in \(\mathcal{O}\left(\frac{\Lambda^2_{QCD}}{m_b^2}\right)\) matrix elements.)

\(B_d - B^\pm\) lifetime difference:

Bigi, Shifman, Uraltsev, Vainshtein
Neubert, Sachrajda
Lifetime difference of $B_s$ mesons:

$$B_s \sim \bar{b}s \quad \bar{B}_s \sim b\bar{s}$$

Standard Model: Negligible CP-violation in $B_s$–$ar{B}_s$–mixing:

$$| B_{L,H} \rangle = \frac{1}{\sqrt{2}} \left[ | B_s \rangle \mp | \bar{B}_s \rangle \right]$$

Width difference

$$\Delta \Gamma_{B_s} \equiv \Gamma_L - \Gamma_H$$

$$= -\frac{1}{M_{B_s}} \text{Im} \langle \bar{B}_s | i \int d^4 x \ T \mathcal{H}_{eff}(x) \mathcal{H}_{eff}(0) | B_s \rangle$$

from final states common to $B_S$ and $\bar{B}_S$

Measurement at Tevatron Run-II: Compare average $B_s$ lifetime $\tau(B_s)$ measured in $B_s \rightarrow D_s^- \pi^+$ with $\tau(B_{s,L}) = 1/\Gamma_L$ measured in $B_s \rightarrow \psi\phi$ (CP-even component).
Compare:

\[ \frac{\tau(B^+)}{\tau(B_d)}: \]
insensitive to new physics \(\Rightarrow\) tests HQE

\[ \frac{\tau(B_s)}{\tau(B_d)} \simeq 1 \pm \mathcal{O}(1\%) \] (in Standard Model):
mildly sensitive to new physics in penguin coefficients

Keum, U.N.

\[ \frac{\tau(B_{s,L})}{\tau(B_{s,H})}: \]
New CP-violating physics in \(B_S-\bar{B}_S\)-mixing can suppress \(\Delta \Gamma_{B_S}\) below its SM value.

Grossman
Why calculate lifetime differences to $\mathcal{O}(\alpha_s)$?

- to reduce the sizable $\mu$-dependence
- consistent use of $\Lambda_{\overline{MS}}$
- meaningful use of lattice results for hadronic matrix elements like $\langle \bar{B}_s | \mathcal{O} | B_s \rangle$, $\langle \bar{B}_s | \mathcal{O}_S | B_s \rangle$
- QCD corrections are of order 30%.
- verify infrared safety of the $c_j$’s.
- Test of quark-hadron duality: Need to go beyond leading logarithmic approximation.
3. The width difference of $B_S$–mesons

Result:

$$\text{Im} \langle \bar{B}_s | i \int d^4 x \ T \mathcal{H}_{\text{eff}}(x) \mathcal{H}_{\text{eff}}(0) | B_s \rangle$$

$$= - \frac{G_F^2 m_b^2}{12 \pi} (V_{cb}^* V_{cs})^2 \left[ F(z) \langle \bar{B}_s | Q | B_s \rangle + F_S(z) \langle \bar{B}_s | Q_S | B_s \rangle \right]$$

$F$ and $F_S$ are IR-safe functions of $z = m_c^2 / m_b^2$.

IR-singularities cancel via two mechanisms:

1. Bloch-Nordsieck cancellations among different cuts of the same diagram
2. Factorization of IR-singularities, which end up in

$$\langle \bar{B}_s | \mathcal{O} | B_s \rangle, \langle \bar{B}_s | \mathcal{O}_S | B_s \rangle$$
Nonperturbative QCD in

\[
\langle \bar{B}_s | Q | B_s \rangle = \frac{8}{3} f_{B_s}^2 M_{B_s}^2 B
\]

\[
\langle \bar{B}_s | Q_S | B_s \rangle = -\frac{5}{3} f_{B_s}^2 M_{B_s}^2 \frac{M_{B_s}^2}{(m_b + \bar{m}_s)^2} B_S
\]

Include corrections of order $\Lambda_{QCD}/m_b$:


\[
\left( \frac{\Delta \Gamma}{\Gamma} \right)_{B_s} = \left( \frac{f_{B_s}}{245 \text{ MeV}} \right)^2 [0.008 B + 0.204 B_S - 0.086]
\]

for the $\overline{\text{MS}}$-scheme at $\mu = m_b$.

Quenched lattice QCD:

\[
B(\mu = m_b) = 0.80 \pm 0.15 \quad \text{Hashimoto (Lattice '99)}
\]

\[
B_S(\mu = m_b) = 1.19 \pm 0.20 \quad \text{Yamada et al. (Hiroshima)}
\]

\[
\left( \frac{\Delta \Gamma}{\Gamma} \right)_{B_s} = \left( \frac{f_{B_s}}{245 \text{ MeV}} \right)^2 (0.162 \pm 0.041 \pm ??? \text{ (latt. syst.)})
\]
Summary

1. Need $\mathcal{O}(\alpha_s)$ corrections to test the HQE predictions for the lifetime differences of $B$ mesons.

2. New CP-violation in $B_s$-mixing affects $\Delta \Gamma_{B_s}$.

3. Next-to-leading QCD-corrections to $\Delta \Gamma_{B_s}$ are infrared safe and reduce $\Delta \Gamma_{B_s}$ by 30%. 
   $\Delta \Gamma_{B_s} / \Gamma_{B_s} = (16 \pm 7)\%$. 