Electroweak Measurements on the Z Resonance
results from
ALEPH, DELPHI, L3, OPAL and SLD

Outline:

- LEP Lineshape and Lepton Asymmetry Measurements
- LEP tau polarization
- SLD Right-Left Asymmetries
- LEP and SLD heavy quark results
LEP Lineshape and Lepton Asymmetry Measurements

Lowest order differential cross section (ignoring masses and initial and final state QCD and QED corrections):

\[
\frac{2s}{\pi \alpha^2 N_c} \frac{d\sigma}{d \cos \theta} = \frac{1}{d \cos \theta} \left[ q_f^2 \left(1 + \cos^2 \theta \right) \right.
\]

\[-8 \text{Re}\{\chi(s) q_f (g_{Ve} g_{Nf} \left(1 + \cos^2 \theta \right) + 2 g_{Ae} g_{Af} \cos \theta)\} \right]
\]

\[16|\chi(s)|^2 [(g_{Ve}^2 + g_{Ae}^2)(g_{Vf}^2 + g_{Af}^2) \left(1 + \cos^2 \theta \right) + 8 g_{Ve} g_{Ae} g_{Vf} g_{Af} \cos \theta] \]

Where:

- \( \chi(s) = \frac{G_F m_Z^2}{8\pi \alpha \sqrt{2}} \frac{s}{s - m_Z^2 + i s \Gamma_Z / m_Z} \)

- \( g_{Vf} \) and \( g_{Af} \) are replaced with \textit{effective} couplings in the IBA which we will use from now on.

- \( N_c \) is the number of “colors” for the final state fermions

For standard LEP fits interference term and imaginary parts of \( g_{Vf} \) and \( g_{Af} \) are fixed to the Standard Model prediction.

(See talk on s-matrix approach)
LEP experiments measure cross sections

![Graph showing measurements for LEP experiments]

Experimental results reported after unfolding for huge ($\sim 25\%$) initial-state radiative corrections. Electrons are corrected for t-channel effects.

(These corrections are known to third-order, see: Berends, Neerven, Burgers / Montagna, Nicrosini, and Piccinini / Skrzypek / Jadach, Pietrzyk, and Skrzypek).
and forward-backward asymmetries

combination of 200 - 400 data points/experiment too difficult ⇒ use pseudo-observables which are closely related to the measured quantities
LEP Model (in)Dependent Parameter Set
(also called pseudo-observables)

<table>
<thead>
<tr>
<th></th>
<th>Total Error</th>
<th>Theory Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_Z )</td>
<td>2.1 MeV</td>
<td>0.3 MeV</td>
</tr>
<tr>
<td></td>
<td>((0.2 \times 10^{-4}))</td>
<td>((0.03 \times 10^{-4}))</td>
</tr>
<tr>
<td>( \Gamma_Z )</td>
<td>2.3 MeV</td>
<td>0.2 MeV</td>
</tr>
<tr>
<td></td>
<td>((9.2 \times 10^{-4}))</td>
<td>((0.8 \times 10^{-4}))</td>
</tr>
<tr>
<td>( \sigma^0_{\text{had}} )</td>
<td>( \frac{12\pi \Gamma_Z e^+e^- - \Gamma_{\text{had}}}{m_Z^2} )</td>
<td>0.037 nb (0.022 \text{ nb})</td>
</tr>
<tr>
<td></td>
<td>( \Gamma_Z )</td>
<td>((8.9 \times 10^{-4}))</td>
</tr>
<tr>
<td>( R_\ell )</td>
<td>0.025</td>
<td>0.004*</td>
</tr>
<tr>
<td></td>
<td>((12 \times 10^{-4}))</td>
<td>((1.9 \times 10^{-4}))</td>
</tr>
<tr>
<td>( A^\text{pole}_{\text{FB}} )</td>
<td>( \frac{3}{4} A_e A_f )</td>
<td>0.00095 (0.0001^*)</td>
</tr>
<tr>
<td></td>
<td>((5.6%))</td>
<td>((0.6%))</td>
</tr>
</tbody>
</table>

\[
\Gamma_{\ell \ell} = \frac{G_F N_c m_N^2}{6\pi \sqrt{2}} \left( R^\ell_{\text{V}} g_{\ell \ell}^2 + R^\ell_{\text{A}} g^2_{\ell \ell} \right) + \Delta_{\text{QCD}} \\
A_f = 2 \frac{g^2_{\ell \ell}}{g^2_{\ell \ell} + g_{\ell \ell}}
\]

\( R^\ell_{\text{V}} \) and \( R^\ell_{\text{A}} \) give corrections for final-state QED and QCD effects as well as quark masses, \( \Delta_{\text{QCD}} \) for non-factorizable QCD effects.

* Theory error for electrons is larger 0.024 (\( R_\ell \)) and 0.0014 (\( A^\text{pole}_{\text{FB}} \))

In general our results greatly exceeded expectations:

<table>
<thead>
<tr>
<th></th>
<th>Expectation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z ) Decays/experiment</td>
<td>(5 - 10 \times 10^6)</td>
<td>(5 \times 10^6)</td>
</tr>
<tr>
<td>Systematic on energy</td>
<td>10 MeV</td>
<td>(&lt; 2.0 \text{ MeV})</td>
</tr>
<tr>
<td>Systematic on luminosity</td>
<td>(\sim 1%)</td>
<td>(\sim 0.05% \text{ (Exp)}) (0.054% \text{ (Th)})</td>
</tr>
</tbody>
</table>
In last two years unfolding procedure and theoretical errors on the pseudo-observables examined in great detail:

- Detailed comparisons of MIZA, TOPAZ0, ZFITTER

- Important theory changes
  - 1999 turning on 3rd order corrections shifts:
    - $\sigma^0_{\text{had}}$ by 0.023nb ($\sim 6 \times 10^{-4}$)
    - $m_Z$ and $\Gamma_Z$ by $\sim 0.5$ MeV
  - 2000 Inclusion of radiative final-state pair correction shifts $m_Z$ by $\sim 0.5$ MeV and $\Gamma_Z$ by $\sim 0.8$ MeV
• Experimental change: use optimal weight of 1993 and 1995 data for observables, eg use separate $m_Z$ masses for different energy calibrations:

\[
\begin{align*}
1990-1992 & \quad 91.1904 \pm 0.0065 \\
1993-1994 & \quad 91.1882 \pm 0.0033 \\
1995 & \quad 91.1866 \pm 0.0024 \\
\text{average} & \quad 91.1874 \pm 0.0021
\end{align*}
\]

Status

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH</td>
<td>Final</td>
</tr>
<tr>
<td>DELPHI</td>
<td>Final</td>
</tr>
<tr>
<td>L3</td>
<td>Final</td>
</tr>
<tr>
<td>OPAL</td>
<td>“public reading”</td>
</tr>
</tbody>
</table>
The total width $\Gamma_Z$ and $A_{\text{FB}}^{\text{pole}}$ both favor a light Higgs, but this interpretation depends on $\Delta \alpha_{\text{had}}$. 
\( \sigma_{\text{had}}^0 \) and \( R_\ell \) are important for \( \Gamma_{\text{inv}} \) and \( \alpha_s \) because they depend on the ratio of widths.
Ratio of invisible to leptonic widths:

\[
\frac{\Gamma_{\text{inv}}}{\Gamma_{\ell\ell}} = \frac{Br(Z \to \text{inv})}{Br(Z \to \ell^+\ell^-)} = 5.942 \pm 0.016
\]

\(\text{(SM} \frac{\Gamma_{\text{inv}}}{\Gamma_{\ell\ell}} = 5.9736 \pm 0.0048)\)

or

\[N_\nu = 2.984 \pm 0.008\]

or

\[\Delta \Gamma_{\text{inv}} < 2.0\text{MeV} \ (95\% \text{C.L.})\]

This result benefits from the improved Bhabha theory error (Jadach, Placzek, Richter-Was, Ward, Z. Was/ Montagna, Moretti, Nicrosini, Pallavicini, Piccinini) and from the improved initial-state radiative corrections.

In terms LEP observables:

\[
\frac{\Gamma_{\text{inv}}}{\Gamma_{\ell\ell}} = \left( \frac{12\pi R_\ell}{m_Z^2 \sigma_{\text{had}}^0} \right)^{\frac{1}{2}} - R_\ell - 3
\]
Determination of $\alpha_s$ from $R_l \propto 1 + \frac{\alpha_s}{\pi} \ldots$

(take $m_h = 100\text{GeV}$ for the following)

$$\alpha_s(m_Z) = 0.1228 \pm 0.0038 \pm 0.0033(m_h=900\text{GeV}) - 0.0000(m_h=100\text{GeV})$$

and surprisingly from $\sigma_\ell^0 \propto \left(\frac{\Gamma_{\ell\ell}}{\Gamma_Z}\right)^2 \propto 1 - 1.4 \frac{\alpha_s}{\pi} \ldots$

$$\alpha_s(m_Z) = 0.1183 \pm 0.0030 \pm 0.0026(m_h=900\text{GeV}) - 0.0000(m_h=100\text{GeV})$$

Grand LEPEWG fit:

$$\alpha_s(m_Z) = 0.1183 \pm 0.0027$$

(Note: Theory error is under discussion.)

PDG 2000 average $\alpha_s(m_Z) = 0.1181 \pm 0.002$

NNLO average (no LEP) $\alpha_s(m_Z) = 0.1178 \pm 0.0034$

Tau Polarization

\[ P_T \equiv \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} \]

where \( \sigma_+(-) \) is for positive(negative) helicity

At Born level

\[ P_T(\cos \theta_{T^-}) = \frac{\langle P_T \rangle \left( 1 + \cos^2 \theta_{T^-} \right) + \frac{8}{3} A_{pol}^{FB} \cos \theta_{T^-}}{\left( 1 + \cos^2 \theta_{T^-} \right) + \frac{8}{3} A_{pol}^{FB} \cos \theta_{T^-}} \]

with \( \langle P_T \rangle = -A_T \) and \( A_{pol}^{FB} = -\frac{3}{4} A_e \)

New result from OPAL:
Combined results favor a low Higgs mass:

\[ \chi^2/\text{dof} = 0.9/3 \]

\[ \chi^2/\text{dof} = 3.1/3 \]

ALEPH preliminary
DELPHI Final
L3 Final
OPAL new result, paper in preparation
Flag ship measurement:

\[ A_{LR} \simeq \frac{1}{|P_e|} \frac{\sigma_l - \sigma_r}{\sigma_l + \sigma_r} = A_e \]

\( \sigma_r \) and \( \sigma_l \) are cross section with left and right polarization

- In contrast to LEP radiative and interference corrections are very small.

Measurement statistics limited (\( \sim 1.5\% \)), main systematics:

- Polarization (0.5%)
- Energy dependence (0.4%, based on energy scan)

\textit{Final} SLD result on \( A_e \) (including input from electrons) is

\[ A_e = 0.1516 \pm 0.0021 \]
From polarized forward-backward asymmetry

\[ A_f^{\text{FB}} = \frac{4}{3|P_e|} \frac{(\sigma_l(F) - \sigma_l(B)) - (\sigma_r(F) - \sigma_r(B))}{(\sigma_l(F) + \sigma_l(B)) + (\sigma_r(F) + \sigma_r(B))} \]

SLD obtains:

\[ A_e = 0.1455 \pm 0.0060 \]

\[ A_\mu = 0.142 \pm 0.015 \]

\[ A_T = 0.136 \pm 0.015 \]

The polarized forward-backward asymmetry can also be used for \(b\bar{b}\) and \(c\bar{c}\) final states ...
**A_b Measurements (Summer-2000)**

- SLD JetC: 0.882 ± 0.020 ± 0.029
- SLD Lepton: 0.922 ± 0.029 ± 0.024
- SLD K\(^{-}\) tag: 0.960 ± 0.040 ± 0.069
- SLD Vtx-Q: 0.926 ± 0.019 ± 0.027
- SLD Average: 0.914 ± 0.024
- ALEPH Lept: 0.886 ± 0.036 ± 0.023
- DELPHI Lept: 0.916 ± 0.051 ± 0.023
- L3 Lept: 0.868 ± 0.055 ± 0.030
- OPAL Lept: 0.850 ± 0.038 ± 0.021
- ALEPH JetC: 0.968 ± 0.034 ± 0.030
- DELPHI JetC: 0.890 ± 0.042 ± 0.020
- L3 JetC: 0.801 ± 0.105 ± 0.051
- OPAL JetC: 0.894 ± 0.048 ± 0.036
- LEP Average: 0.880 ± 0.020

**SLD Measurements (Summer-00)**

- SLD JetC: 0.894 ± 0.048 ± 0.036
- OPAL Jet: 0.580 ± 0.060 ± 0.045
- L3 JetC: 0.890 ± 0.042 ± 0.020
- OPAL JetC: 0.894 ± 0.048 ± 0.036

**ТЕП Measurements: A \(_b\) = 4 \(A^{b}\) / 3 \(A_L\)**

Using \(A_L=0.1500\pm0.0016\) (Combine SLD \(A_L\) and LEP \(A_L\))

**A_c Measurements (Summer-00)**

- SLD K & vtx-Q: 0.603 ± 0.028 ± 0.023
- SLD Lepton: 0.567 ± 0.051 ± 0.064
- SLD D\(^{-}\) D\(^{+}\): 0.690 ± 0.042 ± 0.021
- SLD soft \(\pi\): 0.685 ± 0.052 ± 0.038
- SLD Average: 0.635 ± 0.027
- ALEPH Lept: 0.58 ± 0.05 ± 0.03
- DELPHI Lept: 0.68 ± 0.08 ± 0.06
- L3 Lept: 0.82 ± 0.29 ± 0.19
- OPAL Lept: 0.58 ± 0.06 ± 0.04
- ALEPH D\(^{-}\): 0.63 ± 0.08 ± 0.02
- DELPHI D\(^{-}\): 0.64 ± 0.08 ± 0.04
- OPAL D\(^{-}\): 0.64 ± 0.10 ± 0.05
- LEP Average: 0.612 ± 0.032

**SLD Measurements: A \(_c\) = 4 \(A^{c}\) / 3 \(A_L\)**

Using \(A_L=0.1500\pm0.0016\) (Combine SLD \(A_L\) and LEP \(A_L\))

**LEP/SLD average**

\(A_b = 0.898 \pm 0.015\)

\(A_c = 0.624 \pm 0.020\)

**Standard Model**

\(0.935\) \((-2.5 \sigma)\)

\(0.668\) \((-2.2 \sigma)\)
Updates to LEP results heavy flavor results:

- New b and c lepton based asymmetries from DELPHI
- New results on b-branching ratios from DELPHI (DELPHI 2000-114) and ALEPH (ALEPH 2000-69)

The asymmetry results are mostly final, but updates expected for the high $p_t$ leptons and improved jet-charge from ALEPH and DELPHI.


<table>
<thead>
<tr>
<th>Source</th>
<th>$A_{FB}^b$ ($\times10^{-3}$)</th>
<th>$A_{FB}^c$ ($\times10^{-3}$)</th>
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</thead>
<tbody>
<tr>
<td>statistics</td>
<td>1.8</td>
<td>3.2</td>
</tr>
<tr>
<td>internal sys.</td>
<td>0.8</td>
<td>1.5</td>
</tr>
<tr>
<td>light quarks</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>QCD</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>$\text{Br}(D^+ \to K^- \pi^+ \pi^+)$</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>$\text{Br}(D_s \to \phi \pi^+)$</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>gluon splitting</td>
<td>0.1</td>
<td>0</td>
</tr>
</tbody>
</table>

Energy Dependence (see also LEP 2 results):
$R_b$ and $R_c$ no longer important for grand electroweak fit:
b-quark and c-quark couplings:

\[ \text{Preliminary} \]

\[ g_{Ac} \quad 0.48 \quad 0.49 \quad 0.5 \quad 0.51 \quad 0.52 \quad 0.53 \]

\[ g_{Vc} \quad 0.15 \quad 0.16 \quad 0.17 \quad 0.18 \quad 0.19 \quad 0.2 \]

\[ 68.3 \quad 95.5 \quad 99.5 \% \text{ CL} \]

\[ g_{Ab} \quad -0.54 \quad -0.53 \quad -0.52 \quad -0.51 \quad -0.5 \quad -0.49 \]

\[ -0.36 \quad -0.34 \quad -0.32 \quad -0.3 \quad -0.28 \quad -0.26 \]

\[ \text{SM} \]
\[ A_{FB}^{b\bar{b}} = \frac{3}{4} A_e A_b \quad \text{and} \quad A_{FB}^{c\bar{c}} = \frac{3}{4} A_e A_c \]

both favor a heavy Higgs.
In terms of $\sin^2 \theta_{\text{lep}}^{\text{eff}} \equiv \frac{1}{4} \left( 1 - \frac{g_V}{g_A} \right)$

SLD-LEP Weak Mixing Angle Results

- **World Avg**: $\sin^2 \theta_W = 0.23147 \pm 0.00017$ ($\chi^2$/NDF=12.9/7)
- **Leptons Only**: $\sin^2 \theta_W = 0.23113 \pm 0.00020$ ($\chi^2$/NDF=2.6/4)
- **Hadrons Only**: $\sin^2 \theta_W = 0.23231 \pm 0.00031$ ($\chi^2$/NDF=0.2/2)

Difference between leptons and hadrons is -3.2 $\sigma$, but overall $\chi^2$ is ok (C.L = 7.5%).
Interpretation in terms of Higgs mass depends on crucially on running of $\alpha$

$$\alpha(m_Z^2) = \frac{\alpha(0)}{1 - \Delta\alpha_\ell(m_Z^2) - \Delta\alpha^5_{\text{had}}(m_Z^2) - \Delta\alpha_{\text{top}}(m_Z^2)}$$

New value of $\Delta\alpha^5_{\text{had}}$ from BES (Very Preliminary)
Comparison with other results:

Red error bars indicate determination with minimal theoretical input.

new publications added up to Oct.1998
Contours in $\Delta \alpha^5_{\text{had}}, m_{\text{Higgs}}$ plane are shown below (includes data presented here, $m_W$ and $m_t$):

Bands show Eidelmann and Jegerlehner 95

$$\Delta \alpha^5_{\text{had}} = 0.02804 \pm 0.00065$$

and very preliminary Pietrzyk, ICHEP 2000

$$\Delta \alpha_{\text{had}} = 0.02755 \pm 0.00046$$

Contours are $1\sigma$ (46% C.L.), $2\sigma$ (91%) and $3\sigma$ (99.5% C.L.)
Higgs limit (including \(m_t\), \(m_W\), \(\nu N\) results to be presented, see talks by Scott Willenbrock, Kevin McFarland, and Stephan Wynhoff):

\[
\Delta \alpha_{\text{had}}^{(5)} = 0.02804 \pm 0.00065 \quad m_{\text{Higgs}} = 62^{+53}_{-30} \text{ GeV}
\]

\[
\Delta \alpha_{\text{had}}^{(5)} = 0.02755 \pm 0.00046 \quad m_{\text{Higgs}} = 90^{+63}_{-39} \text{ GeV}
\]

LEPEWWG 95\% C.L. upper limit

\[m_{\text{Higgs}} < 170\text{GeV}\]

Very preliminary Pietryzk update gives 95\% C.L.

\[m_{\text{Higgs}} < 210\text{GeV}\]
Conclusion

- Z-resonance measurements are nearly complete, most results are final

- Most of the precision observables favor a light Higgs, but $A_{FB}^b$ and $A_{FB}^c$ favor a heavy Higgs.

- Interpretation of result crucially depends on an accurate value of $\Delta\alpha_{\text{had}}$

Thanks to ALEPH, DELPHI, L3, OPAL, SLD, LEP-EWWG, and for fits from T. Abe, M. Grünewald, G. Quast, K. Mönig and B. Pietrzyk.
$S$-$T$ plot:

- $U=0$ Constraint
  - $S = 0.02 \pm 0.10$
  - $T = 0.03 \pm 0.10$
  - $M_H < 199$ GeV (95%)

$R^-$ measured

- $m_t^{\text{ref}} = 175$ GeV
- $m_H^{\text{ref}} = 100$ GeV

$\sin^2 \theta_W$ (all)
### Osaka 2000 Measurements

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Value</th>
<th>Pull</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_Z$ [GeV]</td>
<td>$91.1875 \pm 0.0021$</td>
<td>.05</td>
</tr>
<tr>
<td>$\Gamma_Z$ [GeV]</td>
<td>$2.4952 \pm 0.0023$</td>
<td>-.42</td>
</tr>
<tr>
<td>$\sigma^0_{\text{hadr}}$ [nb]</td>
<td>$41.540 \pm 0.037$</td>
<td>1.62</td>
</tr>
<tr>
<td>$R_l$</td>
<td>$20.767 \pm 0.025$</td>
<td>1.07</td>
</tr>
<tr>
<td>$A_{\text{fb}}^{0,l}$</td>
<td>$0.01714 \pm 0.00095$</td>
<td>.75</td>
</tr>
<tr>
<td>$A_e$</td>
<td>$0.1498 \pm 0.0048$</td>
<td>.38</td>
</tr>
<tr>
<td>$A_\tau$</td>
<td>$0.1439 \pm 0.0042$</td>
<td>-.97</td>
</tr>
<tr>
<td>$\sin^2\theta_{\text{eff}}$</td>
<td>$0.2321 \pm 0.0010$</td>
<td>.70</td>
</tr>
<tr>
<td>$m_W$ [GeV]</td>
<td>$80.427 \pm 0.046$</td>
<td>.55</td>
</tr>
<tr>
<td>$R_b$</td>
<td>$0.21653 \pm 0.00069$</td>
<td>1.09</td>
</tr>
<tr>
<td>$R_c$</td>
<td>$0.1709 \pm 0.0034$</td>
<td>-.40</td>
</tr>
<tr>
<td>$A_{\text{fb}}^{0,b}$</td>
<td>$0.0990 \pm 0.0020$</td>
<td>-2.38</td>
</tr>
<tr>
<td>$A_{\text{fb}}^{0,c}$</td>
<td>$0.0689 \pm 0.0035$</td>
<td>-1.51</td>
</tr>
<tr>
<td>$A_b$</td>
<td>$0.922 \pm 0.023$</td>
<td>-.55</td>
</tr>
<tr>
<td>$A_c$</td>
<td>$0.631 \pm 0.026$</td>
<td>-1.43</td>
</tr>
<tr>
<td>$\sin^2\theta_{\text{eff}}$</td>
<td>$0.23098 \pm 0.00026$</td>
<td>-1.61</td>
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<tr>
<td>$\sin^2\theta_W$</td>
<td>$0.2255 \pm 0.0021$</td>
<td>1.20</td>
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<tr>
<td>$m_W$ [GeV]</td>
<td>$80.452 \pm 0.062$</td>
<td>.81</td>
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<tr>
<td>$m_t$ [GeV]</td>
<td>$174.3 \pm 5.1$</td>
<td>-.01</td>
</tr>
<tr>
<td>$\Delta\alpha^{(5)}_{\text{had}}(m_Z)$</td>
<td>$0.02804 \pm 0.00065$</td>
<td>-.29</td>
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</tbody>
</table>