Theoretical Challenges
for a Precision Measurement of the W Mass at hadron colliders

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Outline

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II. The $\mathcal{O}(\alpha)$ corrections to resonant $W$ production
III. The numerical impact on $W$ boson observables:
   • precise measurement of $M_W$ at hadron colliders!

IV. The $\mathcal{O}(\alpha)$ corrections to $Z$ boson production:
   • impact on $M(l^+l^-)$ and $A_{FB}$ and
   • the prospects for a precision measurement of $\sin^2\theta_{eff}^{lept}$
at hadron colliders!
Literature


I. Introduction and Motivation

Precision measurements of SM gauge boson properties
- mass, width, couplings -

at the Tevatron and the LHC require an equally precise theoretical understanding of the underlying production processes.

One of the most important measurements is a precise measurement of the W boson mass.

Prospects for a precise measurement of $M_W$ at hadron colliders *: (assuming PDF and theoretical uncertainties can be drastically reduced)

at present: $p\bar{p}$: 62 MeV, LEP2: 46 MeV
and combined: 37 MeV
expected at LEP2: 35-40 MeV

<table>
<thead>
<tr>
<th>$\delta M_W$ [MeV] (stat. and syst.)</th>
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<tr>
<td>LHC (10 $fb^{-1}$)</td>
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<td>Tevatron RUN II (1 $fb^{-1}$)</td>
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<td>Tevatron (10 $fb^{-1}$)</td>
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$W/Z$ ratios: less sensitive to PDF and QCD uncertainties, electroweak corrections important, multiple photon radiation effects? (U.Baur and T.Stelzer, hep-ph/9910206)

For the envisioned high precision it is crucial that the predictions for W and Z boson observables are well under control:

- **W boson observables:**
  - $M_W$ measurement: from $M_T(W)$, $p_T(l)$ or ratios,
  - $\Gamma_W$ measurement: from W/Z cross section ratio, BR ratio or fit to high $M_T(W)$, and $\sigma_W$ as a luminosity monitor

- **Z boson observables:**
  - $M_W$ measurement: $M_T(Z)$, $p_T(l)$ and $\sigma_Z$ in ratios and $M_Z$ for detector calibration,
  - measurement of $\sin^2 \theta_{eff}^{\text{lept}}$: from $A_{FB}$, and
  - $M(ll)$ distribution at high $M(ll)$: scales of new physics, e.g. $Z'$, extra dimensions!

Electroweak radiative corrections to the W and Z boson production processes need to be included.

Challenging precise theoretical predictions with precise measurements yields

- **consistency checks:**
  compare direct with indirect measurements of input parameters, e.g., the W boson mass
  from the Tevatron and LEPII measurements and from a global $\chi^2$ fit including all electroweak precision data (LEPEWWG, Summer 99/00)

$$M_W = 80.434 \pm 0.037 \text{GeV} \quad M_W = 80.387 \pm 0.026 \text{GeV}$$
information about (not yet) accessible sectors of the MSM, e.g. the Higgs-sector through the virtual presence of Higgs-bosons in loop diagrams (A.Gurtu, ICHEP2000)

\[ M_H < 210 \text{ GeV} (95\% \text{CL}) \ ; \ M_H = 62^{+53}_{-30} \text{ GeV} \]

\[ M_W - M_Z \text{ correlation: } M_W^2 (1 - \frac{M_W^2}{M_Z^2}) = \frac{\pi \alpha(0)}{\sqrt{2} G_F (1 - \Delta r(M_W, m_t, M_H, \ldots))} \]

LEPEWWG Summer 2000 (preliminary)

\[
\begin{align*}
\delta m_{t\bar{t}} &= 2 \text{ GeV} \quad \text{and} \quad \delta M_W = 20 \text{ MeV} \quad \Rightarrow \quad \frac{\delta M_H}{M_H} \approx 30\% \\
\text{and even better from global fits to all EWK data!}
\end{align*}
\]
MSSM predictions:

II. EWK $O(\alpha)$ corrections to resonant $W$ production

The complete $O(\alpha^3)$ parton level cross section of resonant $W$ production via the Drell-Yan mechanism

\[ q_i \bar{q}_j \rightarrow W \rightarrow f \bar{f}'(\gamma) \]

can be written as (W.Hollik and D.W, hep-ph/9606398)

\[
d\hat{\sigma}^{(0+1)} = d\hat{\sigma}^{(0)} \left[ 1 + 2Re(\tilde{F}_{\text{weak}}^{\text{initial}} + \tilde{F}_{\text{weak}}^{\text{final}})(M_W^2) \right] \\
+ \sum_{a=\text{initial, final, } \text{interf.}} [d\hat{\sigma}^{(0)} F_{QED}^a(\hat{s}, \hat{t}) + d\hat{\sigma}_{2\rightarrow 3}^a]
\]

- $\tilde{F}_{\text{weak}}^{\text{initial, final}}(\hat{s} = M_W^2)$:
  gauge invariant modified weak form factors. They comprise the pure weak 1-loop contribution and the IR finite parts of the virtual photon corrections. $W/Z$ box diagrams and terms $\propto s - M_W^2$ are neglected as non-resonant contributions.

- $F_{QED}^{\text{initial, final, interf.}}(\hat{s}, \hat{t})$:
  IR finite, gauge invariant QED-like form factors describing initial and final-state radiation and their interference. They comprise the IR singular parts of the virtual photon contribution and the soft and collinear limits of real photon radiation.

- $d\hat{\sigma}_{2\rightarrow 3}^{\text{initial, final, interf.}}$:
  real hard photon radiation away from soft and collinear singularities.
The Feynman diagrams contributing to $W$ production at $\mathcal{O}(\alpha^3)$

(shaded loop: non-photonic contributions (i.e. $f, H, Z, W$ in loop))

Born-diagram:

$$q_i(p_i) \rightarrow W^+(q) \rightarrow \nu_i(p_f)$$

$$\bar{q}_i(p_{\bar{q}}) \rightarrow t^+(p_{\bar{t}})$$

pure weak contribution:

virtual $\gamma$ contribution:

real $\gamma$ contribution:
Treatment of mass singularities
(same for Z boson production)

$F_{\text{QED}}^{\text{initial}, \text{final}}$ and $d\hat{\sigma}_{2\to3}^{\text{initial}, \text{final}}$ contain large mass singular logarithms: the photon is emitted collinear with a charged fermion and the resulting singularity is regularized by retaining finite fermion masses.

We extract the collinear singularities from $d\hat{\sigma}_{2\to3}^{\text{initial}, \text{final}}$ by defining a collinear region with $\cos \theta > 1 - \delta_\theta$ and perform the cancellation of the mass singularities analytically:

- **Final-state radiation (FSR):**
  in sufficiently inclusive observables the mass singularities completely cancel.

- **Initial-state radiation (ISR):**
  mass singularities always survive but can be absorbed by universal collinear counterterms to the parton distribution functions (in complete analogy to QCD).
    
    - introduces dependence on QED factorization scheme (in analogy to QCD, a $DIS$ and $\overline{MS}$ scheme has been introduced)
    
    - currently no PDFs available which include QED corrections but the effects of QED on the PDFs are expected to be small (H. Spiesberger, LHC report).
III. The numerical impact on $W$ boson observables

The electroweak $\mathcal{O}(\alpha)$ corrections are implemented in the Monte Carlo program WGRAD (U. Baur, S. Keller, D. W., hep-ph/9807417).

We studied the impact of the electroweak radiative corrections in the $W$ resonance region on the $W$ transverse mass distribution, the $W$ to $Z$ transverse mass ratio, the charge asymmetry of leptons in $W \rightarrow l\nu$, the $W$ production cross section and the $W$ to $Z$ cross section ratio.

The transverse mass $M_T$ distribution of the final state $l\nu$ pair is used to extract $M_W$ at the Tevatron

$$M_T = \sqrt{2p_T(l)p_T(\nu)(1 - \cos \Phi^{l\nu})}$$

The $M_T(l^+\nu)$ distributions at the Tevatron with WGRAD:

![Graph showing $M_T(l^+\nu)$ distributions at the Tevatron with WGRAD.](image)
The NLO/LO ratio (only FSR is included) at the Tevatron with WGRAD taking into account lepton identification requirements:

\[ \text{a) } p\bar{p} \rightarrow e^- \nu(\gamma) \]
\[ \sqrt{s} = 1.8 \text{ TeV} \]

solid: no e id. req. included
dash: with e id. req. included

\[ \text{b) } p\bar{p} \rightarrow \mu^+ \nu(\gamma) \]
\[ \sqrt{s} = 1.8 \text{ TeV} \]

solid: no \(\mu\) id. req. included
dash: with \(\mu\) id. req. included

\[ \Rightarrow \text{The effects of FSR are largely reduced in the electron case when lepton id. req. are taken into account.} \]

To simulate the detector responds we apply

- Gaussian smearing of the lepton momenta

- lepton id. req.: charged leptons and photons with small opening angles cannot be discriminated:
  electron and photon momenta are combined when \( \Delta R_{e\gamma} < R_c \)
  muons: events are rejected

- separation cuts:
  \[ p_T(l) > 25 \text{ GeV} \], \[ |\eta(l)| < 1.2 \], \[ \not{p}_T > 25 \text{ GeV} \]
The comparison of the full calculation with WGRAD with the approximation a la Berends et al. (Z. Phys. C 27, 365 (1985)):

\[ \sigma \rightarrow 1^+ \nu(\gamma) \quad \text{FSR only} \]
\[ \sqrt{s} = 1.8 \text{ TeV} \]

no lepton id. req.  

with lepton id. req.  

\[ \Rightarrow \text{approximation} \rightarrow \text{full calculation}: \text{additional shift in} \ M_W \text{ of } \mathcal{O}(10) \text{ MeV} ! \]
The impact of the electroweak corrections on the $p_T(e)$ distribution at the LHC with WGRAD:
(from the LHC workshop report, hep-ph/0003275)

![Graph showing the $p_T(e)$ distribution with and without electron identification.]

- solid: no electron ident.  
- dash: with electron ident.

$pp \rightarrow e^+ \nu(\gamma)$  
$\sqrt{s} = 14$ TeV

See also results of a new calculation of electroweak radiative corrections to $W$ production at the resonance and beyond at hadron colliders by S. Dittmaier and M. Krämer (LHC workshop report, hep-ph/0003275).
IV. The $\mathcal{O}(\alpha)$ corrections to $Z$ boson production

The QED $\mathcal{O}(\alpha)$ corrections to

$$p\bar{p} \rightarrow Z, \gamma^* \rightarrow l^+l^- (l = e, \mu)$$

have been calculated and implemented in the MC program ZGRAD (U.Baur, S.Keller, W.Sakumoto, hep-ph/9707301).

Recently, also the (non-universal) weak one-loop corrections have been calculated and implemented in ZGRAD2 (U.Baur, O.Brein, W.Hollik, C.Schappacher, D.W.).

While (non-universal) weak corrections are small in the $Z$ peak region, they become increasingly important for high parton CM energies $\hat{s}$ due to large Sudakov-like logarithms $\ln(\hat{s}/M_V^2)$.

The impact of QED corrections on the $M(l^+l^-)$ invariant mass distribution and on the $p_T(l)$ spectrum at the Tevatron has been studied with ZGRAD:

- initial-final state interference and ISR contributions are small (after factorizing the collinear singularities into the PDFs)

- FSR dominates and strongly affects the $M(l^+l^-)$ distribution.
The impact of QED corrections on $d\sigma/dM(l^+l^-)$ at the Tevatron with ZGRAD: (from U. Baur et al., Phys. Rev. D57, 199 (1998))

![Graph a) showing $pp\to e^+e^-(\gamma)$ with $\sqrt{s} = 1.8$ TeV. Solid line: with lep. id. requirements, Dash: without lep. id. requirements.](image)

![Graph b) showing $pp\to \mu^+\mu^-(\gamma)$ with $\sqrt{s} = 1.8$ TeV. Solid line: including detector effects, Dash: no detector effects.](image)

Difference in $M_Z$ when comparing approximate (Berends et al.) with full calculation: $\approx 10$ MeV

→ also affects $M_W$ measurement!
A precise measurement of the weak mixing angle $\sin^2 \theta^\text{lept}_{\text{eff}}$ at hadron colliders from the forward-backward asymmetry $A_{FB}$ might be possible.

Prospects for the precision on $\sin^2 \theta^\text{lept}_{\text{eff}}$ extracted from $A_{FB}$ (Z peak): (from LHC report, hep-ph/9611334; J.Rha and E.Ellison, Run II workshop; U.Baur et al., Phys. Rev. D57, 199 (1998); Snowmass 96)

LEP/SLC: 0.00017 (LEPEWWG2000); planned: 0.00012

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<tr>
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<th>$\delta \sin^2 \theta^\text{lept}_{\text{eff}}$ (statistical)</th>
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<tr>
<td>LHC with 100 $fb^{-1}$</td>
<td>0.0002 for $</td>
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<tr>
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<td>0.00014 for $</td>
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<td></td>
<td>0.000039 for full $\eta$ coverage</td>
</tr>
<tr>
<td>Tevatron with 10 $fb^{-1}$</td>
<td>0.00028</td>
</tr>
<tr>
<td>TEV33 (30 $fb^{-1}$)</td>
<td>0.00013 per experiment</td>
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The effects of QED (and QCD) corrections on $A_{FB}(M(ll))$ at the Tevatron and the LHC have been studied with ZGRAD (from U.Baur et al., hep-ph/9707301):

- QED (+QCD) corrections are considerably larger than the expected statistical errors of $\sin^2 \theta^\text{lept}_{\text{eff}}$!

- weak corrections should be included and the effects of higher-order QED need to be studied (U.Baur and T.Stelzer, hep-ph/9910206)
ZGRAD2 predictions for $A_{FB}$ including electroweak corrections at the LHC: (U.Baur et al., in preparation)

→ interesting effects from non-universal electroweak corrections at $M(ll) \sim M_W, 2M_W$ which are the more pronounced the larger the rapidity coverage!

$\Delta A_{FB} = (3-4) \times 10^{-3}$ (per experiment) could be observable at the LHC (100 $fb^{-1}$). In the electron case for large rapidity coverage for one of the electrons the effect is of $\mathcal{O}(10^{-3})$.

A detailed study of electroweak $\mathcal{O}(\alpha)$ corrections to $p\bar{p} \to l^+l^-(l = e, \mu)$ with ZGRAD2 is work in progress!
The impact of electroweak corrections on $d\sigma/dM(e^+e^-)$ and $A_{FB}(M(e^+e^-))$ at high invariant masses at the LHC with ZGRAD2: (U.Baur et al., in preparation)

with separation cuts and lepton identification requirements (ATLAS inspired):

\[ \text{preliminary} \]

- solid: $O(\alpha^3)$ QED/EBA $\quad pp \rightarrow e^+e^-(\gamma)$
- dash: full $O(\alpha^3)$ EWK/EBA $\quad \sqrt{s} = 14 \text{ TeV}$

\[ \frac{d\sigma}{dM(e^+e^-)} \]

\[ m(e^+e^-) \text{ (GeV)} \]

\[ 0.7 \quad 0.8 \quad 0.9 \quad 1.0 \quad 1.1 \]

\[ 500 \quad 1000 \quad 1500 \quad 2000 \]

\[ \text{with cuts and cl. id, MRSR2} \]

- solid: EBA
- dash: $O(\alpha^3)$ QED $\quad pp \rightarrow e^+e^-(\gamma)$
- dots: full $O(\alpha^3)$ EWK $\quad \sqrt{s} = 14 \text{ TeV}$

\[ A_{FB} \]

\[ m(e^+e^-) \text{ (GeV)} \]

\[ 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \]
Summary and Conclusions

In view of future precise measurements at the Tevatron and the LHC it is crucial to fully control QCD and electroweak (EWK) radiative corrections.

As a first step the QED $\mathcal{O}(\alpha)$ corrections to $Z$ boson production and the EWK corrections to resonant $W$ boson production in hadronic collisions have been calculated and implemented in Monte Carlo generators which are publicly available:

ZGRAD: QED $\mathcal{O}(\alpha)$ corrections to $Z$ boson production
WGRAD: EWK $\mathcal{O}(\alpha)$ corrections to resonant $W$ boson production

The impact on interesting observables has been studied, esp. on those which are relevant for the extraction of $M_W$, $M_Z$ and $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ ($A_{FB}$).

In particular, it has been investigated how experimental lepton identification requirements affect the EWK corrections.
However, for the envisioned precision

- the complete $\mathcal{O}(\alpha^3)$ corrections should be included

  Done! (up to QED effects in PDFs)

The updated versions of WGRAD and ZGRAD include all the improvements necessary to be ready for Run II:

- **ZGRAD2**: QED+complete EWK one-loop corrections to $Z$ boson production with proper treatment of higher-order terms around the $Z$ resonance.

- **WGRAD2**: complete EWK $\mathcal{O}(\alpha)$ corrections (photonic and non-photonic) to $W$ boson production (non-resonant contribution and tuned comparison with the calculation by S.Dittmaier and M.Krämer is work in progress).

- the impact of higher-order corrections needs to be studied and eventually taken into account:
  
  $\rightarrow$ two-photon radiation in $W$ and $Z$ boson production?


  $\rightarrow$ resummation of large logs (QED and EWK Sudakov-like)?

  There is still work to be done!