Perturbative Quantization of Gauge and Gravity Theories

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hep-th/9904026

Analytic Quantization of Gravity

In this talk will present a way of quantizing gravity theories in the context of perturbation theory, with following properties:

1) Unitarity manifest.

2) Gauge invariance and Lorentz covariance of building blocks are manifest.

3) No gauge fixing of the general coordinate invariance.

4) Makes explicit connection of gauge and gravity theories in the context of perturbation theory.

5) Provides a powerful way for addressing the ultraviolet properties of quantum gravity.

The formalism is not based on using gravity Hamiltonians or Lagrangians but makes direct use of analytic properties of the S-matrix plus tree-level string relations between gravity and gauge theories.

Non-renormalizability of Quantum Gravity

Traditionally, perturbative gravity may be described in terms of Feynman rules, derived from a path integral.

Power counting suggests that field theories of gravity are not renormalizable and therefore not fundamental quantum theories.

Non-renormalizability of Einstein gravity confirmed by explicit two-loop calculations. — Goroff and Sagnotti (1986); van de Ven (1992)

Supergravity better but still appears to be non-renormalizable.

Various authors concluded that supergravity would diverge at three loops. Deser, Kay and Stelle; Kallosh; Howe and Stelle; Green, Schwarz, Brink; and many others

Although believable, arguments are based on power counting and not on direct calculations.

Possible loophole: Coefficient of divergences might vanish due to a hidden symmetry.

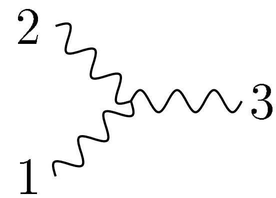
We now have the tools to investigate this!

However first we need to completely reformulate perturbative quantum gravity.

Problem with conventional perturbative gravity

Traditionally we start with a path integral and generate Feynman rules.

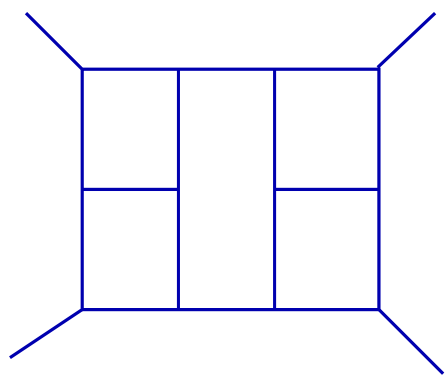
Unfortunately, e.g. the three vertex, is a mess:



$$G_{3\mu\alpha,\nu\beta,\sigma\gamma}(k_1,k_2,k_3) =$$

$$\begin{aligned}
&\mathrm{sym}\Big[-\tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}) - \tfrac{1}{2}P_6(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma}) + \tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\gamma}) \\
&\quad + P_6(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\sigma}\eta_{\beta\gamma}) + 2P_3(k_{1\nu}k_{1\gamma}\eta_{\mu\alpha}\eta_{\beta\sigma}) - P_3(k_{1\beta}k_{2\mu}\eta_{\alpha\nu}\eta_{\sigma\gamma}) \\
&\quad + P_3(k_{1\sigma}k_{2\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + P_6(k_{1\sigma}k_{1\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + 2P_6(k_{1\nu}k_{2\gamma}\eta_{\beta\mu}\eta_{\alpha\sigma}) \\
&\quad + 2P_3(k_{1\nu}k_{2\mu}\eta_{\beta\sigma}\eta_{\gamma\alpha}) - 2P_3(k_1\cdot k_2\eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\mu})\Big]
\end{aligned}$$

Background field gauges or superspace do not help enough.



The internal vertices in the background field method are ordinary vertices. $\sim 10^{30}$ terms in diagram!

We want to be able to calculate diagrams such as the above!

String Theory Intuition

Basic string theory fact:

$$\text{closed string} \sim (\text{left-mover open string}) \times (\text{right-mover open string})$$

In the field theory or infinite string tension limit this should imply

$$\text{gravity} \sim (\text{gauge theory}) \times (\text{gauge theory})$$

We will argue that this is a key property of perturbative gravity.

1) How do we make this precise?

2) Can we use this to quantize gravity?

3) What can this teach us about (super) gravity?

4) How does this relate to the Einstein-Hilbert Lagrangian?

In this talk we will address these issues.

Kawai-Lewellen-Tye Tree-Level Relations

At tree-level KLT have given a complete description of the relationship between closed string amplitudes and open string amplitudes.

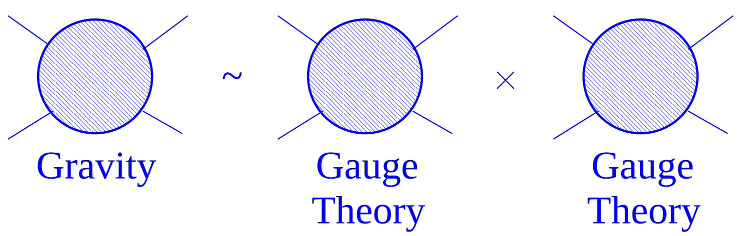
In the field theory limit ($\alpha' \to 0$)

$$M_4^{\text{tree}}(1,2,3,4) = s_{12} A_4^{\text{tree}}(1,2,3,4)\, A_4^{\text{tree}}(1,2,4,3)\,,$$

$$M_5^{\text{tree}}(1,2,3,4,5) = s_{12}s_{34} A_5^{\text{tree}}(1,2,3,4,5) A_5^{\text{tree}}(2,1,4,3,5) + s_{13}s_{24} A_5^{\text{tree}}(1,3,2,4,5)\, A_5^{\text{tree}}(3,1,4,2,5)$$

where we have stripped all coupling constants. M_n is gravity amplitude and A_n is color stripped gauge theory amplitude.

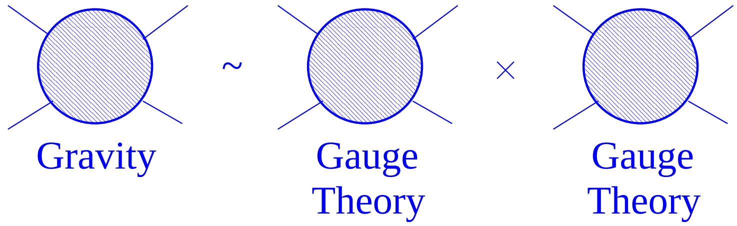
$$s_{ij} = (k_i + k_j)^2$$



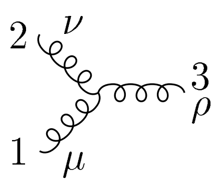
These relations hold for any external states.

'Left' Lorentz and spinor indices contract with 'left' ones and 'right' contract with 'right' ones.

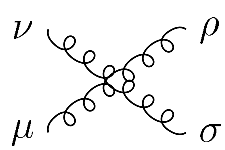
Gravity in Terms of Gauge Theory



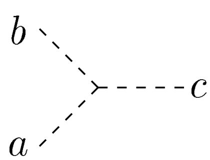
Using this we can describe gravity with color stripped gauge theory Feynman rules: Z.B., Abilio De Freitas and Henry Wong



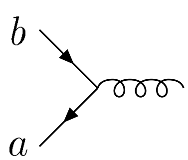
$$= \frac{i}{\sqrt{2}}(k_1 - k_2)^\rho \eta^{\mu\nu} + \text{cyclic}$$



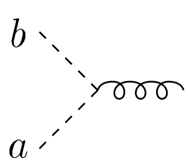
$$= i\eta^{\mu\rho}\eta^{\nu\sigma} - \frac{i}{2}\left(\eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}\right)$$



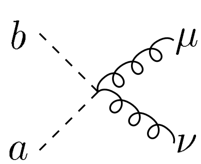
$$= f^{abc}$$



$$= i\delta^{ab}\gamma^\mu$$



$$= i\delta^{ab}(k_1 - k_2)^\mu$$



$$= \frac{i}{2}\delta^{ab}\eta^{\mu\nu}$$

This contains all information on graviton scattering.

Vector polarizations

$$\varepsilon_\mu^+(k;q) = \frac{\langle q^- |\, \gamma_\mu \,|k^-\rangle}{\sqrt{2}\,\langle q\, k\rangle}, \qquad \varepsilon_\mu^-(k,q) = \frac{\langle q^+ |\, \gamma_\mu \,|k^+\rangle}{\sqrt{2}\,[k\, q]}$$

All required properties of polarization vectors satisfied:

$$\varepsilon_i^2 = 0\,, \qquad k \cdot \varepsilon(k,q) = 0\,, \qquad \varepsilon^+ \cdot \varepsilon^- = -1$$

Notation

$$\langle j\, l\rangle = \langle k_{j\,-} | k_{l+}\rangle = \sqrt{2k_j \cdot k_l}\; e^{i\phi}$$
$$[j\, l] = \langle k_{j\,+} | k_{l\,-}\rangle = -\sqrt{2k_j \cdot k_l}\; e^{-i\phi}$$

Changes in reference momentum q are equivalent to gauge transformations.

Graviton polarization vectors are the squares of these!

$$\varepsilon_{\mu\nu}^{++} = \varepsilon_\mu^+ \varepsilon_\nu^+\,, \qquad\qquad 2 = 1 + 1$$

Leads to compact expressions for amplitudes.

Gravity Tree Amplitudes from Gauge Theory

We obtain amplitudes in any minimally coupled theory of gravity to matter:

$$M_4^{\rm tree}(1_h^-, 2_h^-, 3_h^+, 4_h^+) = \left(\frac{\kappa}{2}\right)^2 s_{12} A_4^{\rm tree}(1_g^-, 2_g^-, 3_g^+, 4_g^+) \times A_4^{\rm tree}(1_g^-, 2_g^-, 4_g^+, 3_g^+)$$

$$= \left(\frac{\kappa}{2}\right)^2 s_{12} \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle} \times \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,4\rangle\,\langle 4\,3\rangle\,\langle 3\,1\rangle}$$

$$M_4^{\rm tree}(1_g^-, 2_g^-, 3_g^+, 4_h^+) = g\frac{\kappa}{2} s_{12} A_4^{\rm tree}(1_g^-, 2_g^-, 3_g^+, 4_g^+) \times A_4^{\rm tree}(1_s^-, 2_s^-, 4_s^+, 3_g^+)$$

$$= g\frac{\kappa}{2} \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle} \times f^{a_1 a_2 a_3} \frac{[4\,3]\,\langle 3\,2\rangle}{\langle 2\,4\rangle}$$

$$M_4^{\rm tree}(1_g^-, 2_{\bar q}^-, 3_q^+, 4_h^+) = g\frac{\kappa}{2} s_{12} A_4^{\rm tree}(1_g^-, 2_{\bar q}^-, 3_q^+, 4_g^+) \times A_4^{\rm tree}(1_s^-, 2_s^-, 4_s^+, 3_g^+)$$

$$= g\frac{\kappa}{2} \frac{\langle 1\,2\rangle^3\,\langle 1\,3\rangle}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle} \times T^{a_1} \frac{[4\,3]\,\langle 3\,2\rangle}{\langle 2\,4\rangle}$$

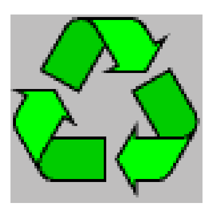
$$M_4^{\rm tree}(1_g^-, 2_g^-, 3_g^+, 4_g^+) = g^2 s_{12} A_4^{\rm tree}(1_g^-, 2_g^-, 3_g^+, 4_g^+) \times A_4^{\rm tree}(1_s, 2_s, 4_s, 3_s)$$

$$= g^2\ {\rm Tr}[T^{a_1} T^{a_2} T^{a_3} T^{a_4}] \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle} + \text{perms}$$

Other tree amplitudes with fermions, gluons, gluinos, scalars, gravitinos are just as simple to obtain!

We can recycle known gauge theory tree amplitudes into gravity amplitudes!

We still need to quantize the theory, i.e. include loops.

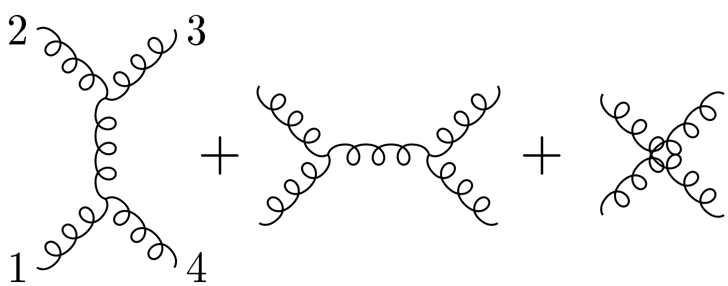


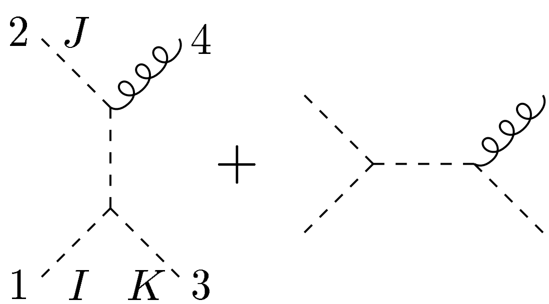
Example

Z.B. and Abilio De Freitas and Henry Wong

$$M_4^{\rm tree}(1_g^-, 2_g^-, 3_g^+, 4_h^+) = g\frac{\kappa}{2} s_{12} A_4^{\rm tree}(1_g^-, 2_g^-, 3_g^+, 4_g^+) \times A_4^{\rm tree}(1_s^I, 2_s^J, 4_g^+, 3_s^K)$$

$$= g\frac{\kappa}{2}\frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle} \times f^{IJK}\,\frac{[4\,3]\,\langle 3\,2\rangle}{\langle 2\,4\rangle}$$





Quantization via Unitarity

Bern, Dixon, Kosower

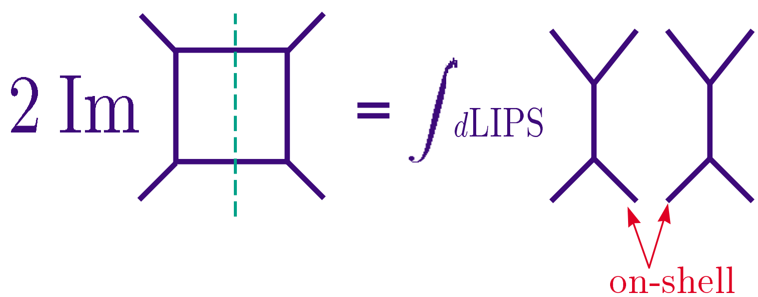
Basic property: The scattering matrix is unitary.

$$S^\dagger S = 1$$

We will use this well known property of the S-matrix to obtain all quantum corrections.

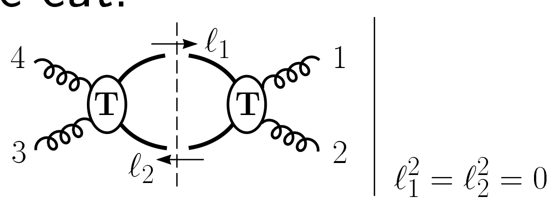
Take $S = 1 + i\,T$

$$2\ \mathrm{Im}\ T = T^\dagger T$$



Harder to calculate = Easier to calculate

To maintain gauge invariance, sum over all Feynman diagrams on either side of the cut.



From unitarity we can obtain the imaginary parts of loop amplitudes from tree amplitudes.

To obtain the complete quantum S-matrix we also need real parts, especially rational functions.

Generic form of a loop amplitude:

$$\begin{aligned} A &\sim \ln(-s - i\epsilon) + \text{rational} + \text{other logs} \\ &\sim \ln(s) - i\pi + \text{rational} + \text{other logs} \end{aligned}$$

The $i\pi$ term is fixed by unitarity and the $\ln(s)$ can be reconstructed from this.

However rational terms seemingly can't be reconstructed. Well known that dispersion relations have subtraction ambiguities.

Problem seems basic. Consider complex function

$$a(\ln(s) - i\pi) + b$$

You can get a from imaginary part but not b.

But in fact we can get around this problem if we use analytic properties as a functions of dimension!

Analytic Properties for $D \neq 4$

Consider: $$|A_4^{\text{1-loop}}(1^+, 2^+, 3^+, 4^+)| = \frac{1}{48\pi^2}$$

Has no imaginary part! How do we construct real rational parts from nothing?

Magic Trick: Continue the amplitude to $D = 4 - 2\epsilon$ dimensions.

From dimensional analysis in massless theories:

$$\begin{aligned} A^{D=4-2\epsilon} &\sim \int d^{4-2\epsilon}p \ \cdots \\ &\sim \sum_i (s_i)^{-\epsilon} \times \text{rational}_i + \cdots \\ &\sim \sum_i \text{rational}_i (1 - \epsilon \ln s_i) + \cdots \end{aligned}$$

Thus:

$$\text{rational} = \sum_i \text{rational}_i$$

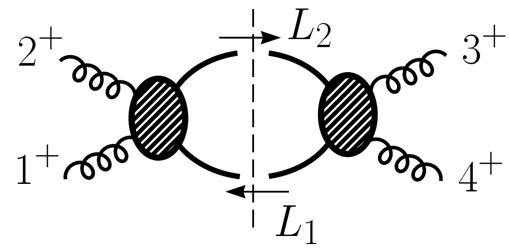
From $\mathcal{O}(\epsilon)$ branch cuts can reconstruct $\mathcal{O}(\epsilon^0)$ rational terms.

Amazingly this allows us to perturbatively quantize gravity theories without reference to a Lagrangian or Hamiltonian.

Proven technology – state of the art QCD calculations. See, e.g., Z.B., L. Dixon, D.A. Kosower, Ann. Rev. Nucl. Part. Sci. 46:109 (1996) [hep-ph/9602280].

Four-Point Identical Helicity QCD Example

Four identical helicity gluons with a scalar in the loop.



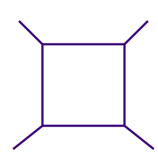
$$A^{\rm tree}(1^+, 2^+, L_1^s, L_2^s) = i\frac{[1\,2]}{\langle 1\,2\rangle}\frac{\mu^2}{(L_1 - k_1)^2}$$

where $L_1 = \ell_1 + \mu$ and $D = 4 - 2\epsilon$.

$$A_4^{\rm tree}(-L_1, 1^+, 2^+, L_2) \times A_4^{\rm tree}(-L_2, 3^+, 4^+, L_1)$$
$$= -\mu^4\frac{[1\,2]}{\langle 1\,2\rangle}\frac{[3\,4]}{\langle 3\,4\rangle}\frac{1}{[(\ell_1 - k_1)^2 - \mu^2]}\frac{1}{[(\ell_2 - k_3)^2 - \mu^2]}$$

Applying this equation at one-loop we have

$$\mathcal{A}_4^{\text{1-loop}}(1,2,3,4)|_{s\text{-cut}} =$$
$$\int \frac{d^D L_1}{(2\pi)^D}\frac{i}{L_2^2}\mathcal{A}_4^{\rm tree}(-L_2, 3^+, 4^+, L_1)\frac{i}{L_1^2}\mathcal{A}_4^{\rm tree}(-L_1, 1^+, 2^+, L_2)\Big|_{L_1^2 = L_2^2 = 0}$$
$$= -\frac{[1\,2]}{\langle 1\,2\rangle}\frac{[3\,4]}{\langle 3\,4\rangle}\int \frac{d^D L_1}{(2\pi)^D}\frac{\mu^4}{p^2(L_1 - k_1)^2(L_1 - k_1 - k_2)^2(L_1 + k_4)^2}$$
$$= \frac{i}{(4\pi)^2}\frac{1}{6}\frac{[1\,2]}{\langle 1\,2\rangle}\frac{[3\,4]}{\langle 3\,4\rangle} + \mathcal{O}(\epsilon)$$

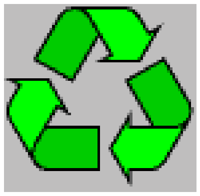


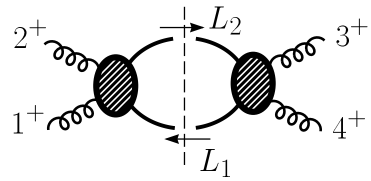
The t-channel cut is similar.

Gravity Cuts

Z.B., Dixon, Perelstein and Rozowsky

How do we calculate gravity loop amplitudes?





Two-particle sewing equation:

$$
\begin{aligned}
&M_4^{\rm tree}(-L_1^s, 1^+, 2^+, L_2^s) \times M_4^{\rm tree}(-L_2, 3^+, 4^+, L_1) \\
&= s^2 \big(A_4^{\rm tree}(-L_1, 1^+, 2^+, L_2) \times A_4^{\rm tree}(L_2, 3^+, 4^+, -L_1)\big) \\
&\quad \times \big(A_4^{\rm tree}(L_2, 1^+, 2^+, -L_1) \times A_4^{\rm tree}(L_1, 3^+, 4^+, -L_2)\big)
\end{aligned}
$$

Easy to evaluate using known QCD results.

$$
\begin{aligned}
&M_4^{\rm tree}(-L_1, 1^+, 2^+, L_2) \times M_4^{\rm tree}(-L_2, 3^+, 4^+, L_1) \\
&= \mu^8 \frac{[1\,2]^2\,[3\,4]^2}{\langle 1\,2\rangle^2 \langle 3\,4\rangle^2} \Big[\frac{1}{(\ell_1 - k_1)^2 - \mu^2} + \frac{1}{(\ell_1 - k_2)^2 - \mu^2}\Big] \\
&\qquad \times \Big[\frac{1}{(\ell_2 - k_3)^2 - \mu^2} + \frac{1}{(\ell_2 - k_4)^2 - \mu^2}\Big]
\end{aligned}
$$

Get scalar box integrals with power of μ^8 in the numerator.

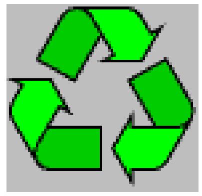
The t- and u-channel formulas are similar. Final result:

$$
M_4^{\rm oneloop}(1^+, 2^+, 3^+, 4^+) = \frac{i\kappa^4}{(4\pi)^2} \Big(\frac{st}{\langle 1\,2\rangle \langle 2\,3\rangle \langle 3\,4\rangle \langle 4\,1\rangle}\Big)^2 \frac{(s^2 + st + t^2)}{960}\,.
$$

where $s = (k_1 + k_2)^2$ and $t = (k_1 + k_4)^2$

$N = 8$ Supergravity Cuts

How do we calculate $N = 8$ supergravity amplitudes?



Two-particle sewing equation:

$$\sum_{N=8 \text{ states}} M_4^{\text{tree}}(-\ell_1, 1, 2, \ell_2) \times M_4^{\text{tree}}(-\ell_2, 3, 4, \ell_1)$$
$$= s^2 \sum_{N=4 \text{ states}} \left(A_4^{\text{tree}}(-\ell_1, 1, 2, \ell_2) \times A_4^{\text{tree}}(\ell_2, 3, 4, -\ell_1)\right)$$
$$\times \sum_{N=4 \text{ states}} \left(A_4^{\text{tree}}(\ell_2, 1, 2, -\ell_1) \times A_4^{\text{tree}}(\ell_1, 3, 4, -\ell_2)\right)$$

Easy to evaluate using known $N = 4$ Yang-Mills results.

$$\sum_{N=8 \text{ states}} M_4^{\text{tree}}(-\ell_1, 1, 2, \ell_2) \times M_4^{\text{tree}}(-\ell_2, 3, 4, \ell_1)$$
$$= stu M_4^{\text{tree}}(1, 2, 3, 4) \left[\frac{1}{(\ell_1 - k_1)^2} + \frac{1}{(\ell_1 - k_2)^2}\right]$$
$$\times \left[\frac{1}{(\ell_2 - k_3)^2} + \frac{1}{(\ell_2 - k_4)^2}\right]$$

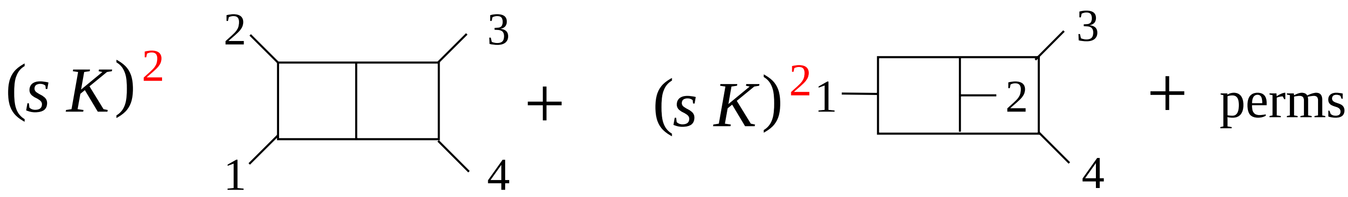
The t- and u-channel formulas are similar.

This is all you need to iterate two-particle cuts to *all* loop orders!

Algebra is amazingly simple.

Two-Loop $N = 8$ SUGRA

From 2 and 3 particle cuts we obtain exact two-loop result:



where $K = stA_4^{\rm tree}$. The two-loop divergences are:

$$\mathcal{M}_4^{\text{2-loop},\ D=7-2\epsilon}|_{\rm pole} = \frac{1}{2\epsilon\ (4\pi)^7}\frac{\pi}{3}(s^2+t^2+u^2)\ stuM_4^{\rm tree},$$

$$\mathcal{M}_4^{\text{2-loop},\ D=9-2\epsilon}|_{\rm pole} = \frac{1}{4\epsilon\ (4\pi)^9}\frac{-13\pi}{9072}(s^2+t^2+u^2)^2\ stuM_4^{\rm tree},$$

$$\mathcal{M}_4^{\text{2-loop},\ D=11-2\epsilon}|_{\rm pole} = \frac{1}{48\epsilon\ (4\pi)^{11}}\frac{\pi}{5791500}(438(s^6+t^6+u^6) - 53s^2t^2u^2)\ stuM_4^{\rm tree}.$$

Counterterms are derivatives of

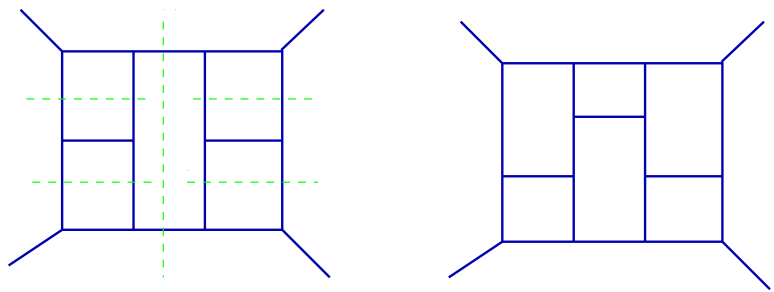
$$t_8t_8R^4 \equiv t_8^{\mu_1\mu_2\cdots\mu_8}\,t_8^{\nu_1\nu_2\cdots\nu_8}\,R_{\mu_1\mu_2\nu_1\nu_2}\,R_{\mu_3\mu_4\nu_3\nu_4}\,R_{\mu_5\mu_6\nu_5\nu_6}\,R_{\mu_7\mu_8\nu_7\nu_8}$$

For $D = 5, 6$ the amplitude is finite contrary to expectations from Howe and Stelle superspace power counting arguments.

Concrete example where previous superspace arguments point to divergence which is actually not present. Also shows that $D = 11$ supergravity is divergent! (Also see paper by Deser and Seminara.)

Power Counting Beyond Two Loops.

The two-particle cut sewing equation iterate to *all* loop orders!



Power counting this subclass of contributions suggests the following simple finiteness formula

$$L < \frac{10}{D-2}$$

This formula indicates finiteness when previous superspace arguments did not, e.g. $D = 4$, $L = 3$ and $D = 5, 6$, $L = 2$.

For $N = 8$ sugra, the first $D = 4$ counterterm encountered in the two-particle cuts occurs at 5 loops not 3 loops.

These results recently confirmed by Howe, Petrini and Stelle.

The relationship between gravity and gauge theory provides new information on the divergence properties of gravity.

As another example, this relationship is also used by Dunbar, Julia, Seminara and Trigiante (hep-th/9911158) to help in their study of counterterms in less than maximal sugra.

Lagrangians

Z.B. and Aaron Grant

Consider the Lagrangians

$$\mathcal{L}_{\rm gravity} = \sqrt{g}\, R\,, \qquad\qquad \mathcal{L}_{\rm YM} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}$$

In Lagrangian or Hamiltonian not obvious how gravity is 'square' of gauge theory.

For S matrix it is obvious:

$$M_4^{\rm tree}(1,2,3,4) = s_{12} A_4^{\rm tree}(1,2,3,4)\, A_4^{\rm tree}(1,2,4,3)$$

We must find the right gauge choices and field variables.

Claim: To 'factorize' the Lorentz indices of gravitons in Einstein-Hilbert Lagrangian to all orders take:

$$g_{\mu\nu} = \exp[\sqrt{2/(D-2)}\phi]\, \exp[h_{\mu\nu}]\,,$$

$$\phi \to -\sqrt{\frac{2}{D-2}}\,(\phi + h^\mu{}_\mu)$$

where ϕ is a dilaton. (In many cases dilaton decouples.) We have checked the claim through $\mathcal{O}(h^6_{\mu\nu})$

Three-Point Correspondence

Factorization is necessary but not sufficient. We want a more explicit gravity $\sim$ (gauge theory)^2 correspondence.

What Yang-Mills gauge do we want to match to?

Three-vertex is simplest in non-linear Gervais-Neveu gauge.

$$L_{\rm GF} \sim {\rm Tr}(\partial \cdot A + A^2)$$

The color-stripped 3 vertex is

$$V^{\rm GN}_{\mu\nu\rho}(k_1, k_2, k_3) = k_1^\rho \eta^{\mu\nu} + k_2^\mu \eta^{\nu\rho} + k_3^\nu \eta^{\rho\mu}$$

By performing further field redefinitions and by choosing non-linear gravity gauge we can get a match.

$$L_3 = \kappa[\frac{1}{2} h_{\mu\nu} h_{\rho\sigma,\mu\nu} h_{\rho\sigma} + h_{\nu\mu} h_{\rho\mu,\sigma} h_{\rho\sigma,\nu}]$$

which generates the vertex

$$V^{\mu\nu\rho}_{\text{gravity}\,\alpha\beta\gamma}(k_1, k_2, k_3) = \kappa[V^{\mu\nu\rho}_{\rm GN}(k_1, k_2, k_3) \times V^{\alpha\beta\gamma}_{\rm GN}(k_1, k_2, k_3) + V^{\nu\mu\rho}_{\rm GN}(k_2, k_1, k_3) \times V^{\beta\alpha\gamma}_{\rm GN}(k_2, k_1, k_3)]$$

This gravity 3 vertex which follows from the Einstein Lagrangian manifestly exhibits the KLT factorization!

Future

1. General investigation of divergence properties of gravity theories.

2. Deeper relation and reformulation of gravity???

3. Implication for classical solutions? Can more general classical solutions be made to reflect gravity $\sim$ (gauge theory)^2?

Summary

1. Gravity $\sim$ (gauge theory)^2 important property of perturbative gravity.

2. D-dimensional unitarity is a powerful way to perturbatively quantize the theory.

3. Recycling is good!

4. $N = 8$ sugra is less divergent than previously thought, but it does appear to diverge at 5 loops. (In recent paper, Chalmers suggests it might even be finite to all orders!)

5. $N = 1$, $D = 11$ supergravity diverges at two-loops.

6. We are now directly investigating UV divergence properties of more general gravity theories. 2-loop calculations.

7. The Einstein-Hilbert Lagrangian can be rearranged to reflect 'left'-'right' factorization of amplitudes: $g_{\mu\nu} \sim e^{\phi} e^{h_{\mu\nu}}$.

Gravity $\sim$ (gauge theory)^2 can be exploited to develop a better understanding of quantum gravity.