RADCOR 2000, Carmel, Sept. 11-15, 2000

Theory of ε'/ε

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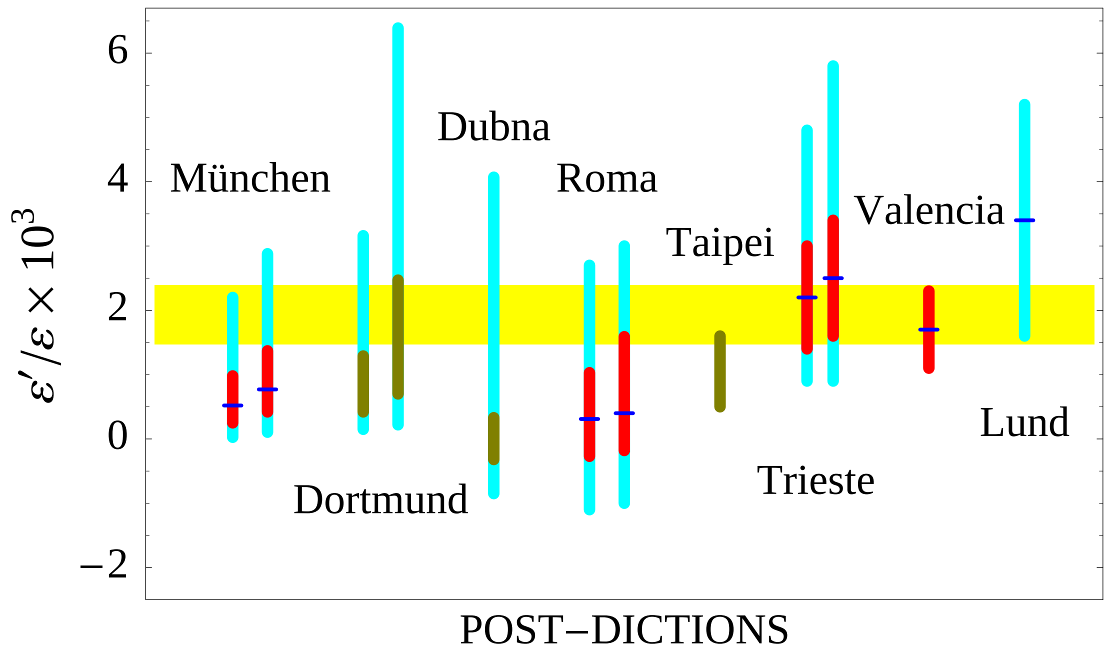
- The $\Delta I = 1/2$ selection rule and ε'/ε
- Going beyond factorization: FSI and more.
- $1/N$, lattice, phenomenological models
- Hadronic matrix elements: a comparative discussion
- Conclusions

POST-DICTIONS

(After February 1999)

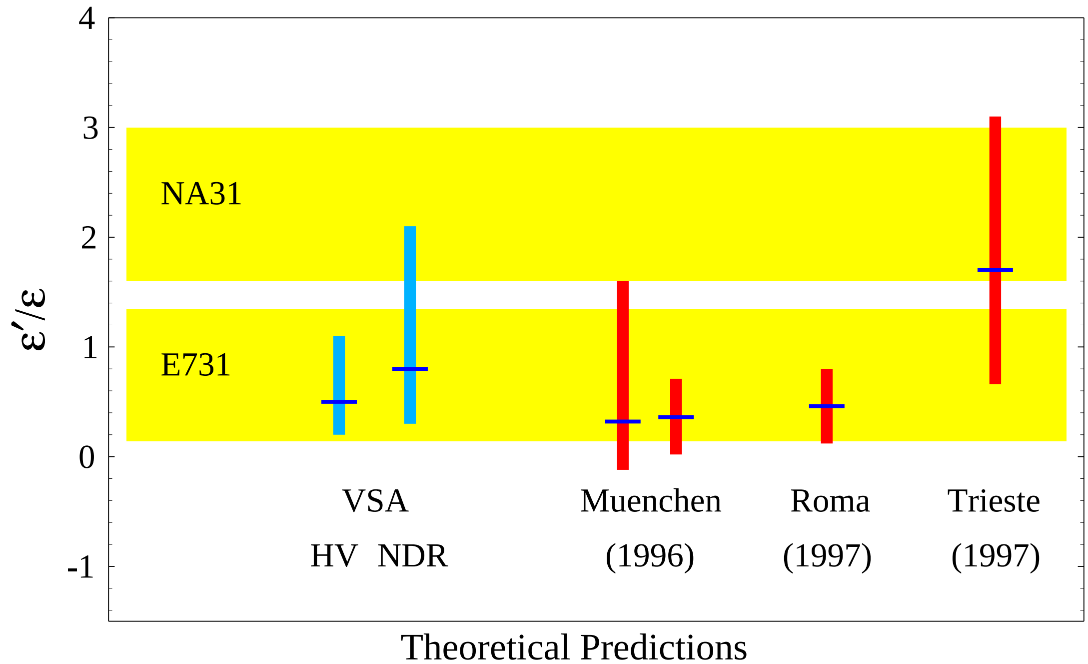


NA31-E731-KTeV-NA48: $(19.3 \pm 4.6) \times 10^{-4}$

Present World Average

PRE-DICTIONS

(Before February 1999)



NA31: $(23 \pm 6.5) \times 10^{-4}$

E731: $(7.4 \pm 6.0) \times 10^{-4}$

Two body Final State Interactions (FSI)

$K \to (\pi\pi)_{I=0}$ FSI attractive $(\delta_0 > 0)$ $\Rightarrow$ enhanced
$K \to (\pi\pi)_{I=2}$ FSI repulsive $(\delta_2 < 0)$ $\Rightarrow$ depleted
[Fermi (1955)]

Qualitatively one should expect ε'/ε larger than that produced by leading $1/N$ (factorization).

Dispersion relation [Mushkelishvili (1953), Omnes (1958)]:

$$M(s+i\epsilon) = P(s)\ \exp\left(\frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\delta(s')}{s'-s-i\epsilon} ds'\right)$$

where $P(s)$ is related to the factorization amplitude.

Solve as: $$A_I(s) = A_I'\ (s - m_\pi^2)\ R_I(s)\ e^{i\delta_I(s)}$$

Recent studies give $R_{0,2}(m_k^2) = 1.4,\ 0.9$.
[Pich and Pallante, (1999)].

Ambiguities in the determination of the derivative of the factorization amplitude $A_I'(s = m_\pi^2)$, using LO chiral perturbation theory [A.J. Buras et al. (2000)]. However, the lower is the subtraction point the smaller are higher order chiral corrections !

The $I = 0$ enhancement may be quantitatively enough for ε'/ε , but is that all ?

Cooking up ε'/ε : Recipe and Ingredients

$$CP\ |K^0\rangle = |\bar{K}^0\rangle$$

$$\begin{aligned} K_1 &= (K^0 + \bar{K}^0)/\sqrt{2} \quad \text{CP even} \quad \to \pi\pi \\ K_2 &= (K^0 - \bar{K}^0)/\sqrt{2} \quad \text{CP odd} \quad \to \pi\pi\pi \end{aligned}$$

$$\begin{aligned} K_S &= (K_1 + \varepsilon K_2)/\sqrt{1+|\varepsilon|^2} \\ K_L &= (K_2 + \varepsilon K_1)/\sqrt{1+|\varepsilon|^2} \end{aligned}$$

$$\varepsilon = \frac{\langle(\pi\pi)_{I=0}|\mathcal{H}_W|K_L\rangle}{\langle(\pi\pi)_{I=0}|\mathcal{H}_W|K_S\rangle}\,,$$

$$\frac{\varepsilon'}{\varepsilon} = \frac{1}{\sqrt{2}}\left\{\frac{\langle(\pi\pi)_{I=2}|\mathcal{H}_W|K_L\rangle}{\langle(\pi\pi)_{I=0}|\mathcal{H}_W|K_L\rangle} - \frac{\langle(\pi\pi)_{I=2}|\mathcal{H}_W|K_S\rangle}{\langle(\pi\pi)_{I=0}|\mathcal{H}_W|K_S\rangle}\right\}$$

The $\Delta I = 1/2$ selection rule in $K \to (\pi\pi)_{I=0,2}$ decays (Gell-Mann and Pais, 1954):

$$\omega \equiv |\mathcal{A}_2|/|\mathcal{A}_0| = 1/22.2$$

Write the $I = 0, 2$ amplitudes (Watson, 1952):

$$\mathcal{A}_I(K \to \pi\pi) = A_I \ \exp\, i\,(\delta_I)$$

δ_I: Final State Interaction Phase

From π-π S-wave scattering lenght (Chell and Olsson, 1993):

$$\begin{array}{lclcl} \delta_0 & \simeq & 34.2^0 \pm 2.2^0 & \cos\delta_0 & \simeq & 0.8 \\ \delta_2 & \simeq & -6.9^0 \pm 0.2^0, & \cos\delta_2 & \simeq & 1.0 \end{array}$$

The rescaling of the "factorized" amplitudes due to FSI does not explain alone the selection rule. Other non-factorizable contributions are needed: are the latter corrections specific to CP-conserving transitions only?

Reproducing the $\Delta I = 1/2$ selection rule is a pre-requirement for any calculation of ε'/ε .

(H)OPE: the Effective Lagrangian

$$\mathcal{L}_{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud}\, V_{us}^* \sum_i [z_i(\mu) + \tau\ y_i(\mu)] Q_i(\mu)$$

$$\tau = -V_{td} V_{ts}^* / V_{ud} V_{us}^*$$

For $\mu < m_c$ $(q = u, d, s)$:

$$\left.\begin{array}{lcl} Q_1 & = & \left(\overline{s}_\alpha u_\beta\right)_{\mathrm{V-A}} \left(\overline{u}_\beta d_\alpha\right)_{\mathrm{V-A}} \\ Q_2 & = & (\overline{s}u)_{\mathrm{V-A}}\, (\overline{u}d)_{\mathrm{V-A}} \end{array}\right\} \quad \text{Current-Current}$$

$$\left.\begin{array}{lcl} Q_{3,5} & = & (\overline{s}d)_{\mathrm{V-A}} \sum_q (\overline{q}q)_{\mathrm{V\mp A}} \\ Q_{4,6} & = & \left(\overline{s}_\alpha d_\beta\right)_{\mathrm{V-A}} \sum_q (\overline{q}_\beta q_\alpha)_{\mathrm{V\mp A}} \end{array}\right\} \quad \text{Gluon ``penguins''}$$

$$\left.\begin{array}{lcl} Q_{7,9} & = & \frac{3}{2}\, (\overline{s}d)_{\mathrm{V-A}} \sum_q \widehat{e}_q\, (\overline{q}q)_{\mathrm{V\pm A}} \\ Q_{8,10} & = & \frac{3}{2} \left(\overline{s}_\alpha d_\beta\right)_{\mathrm{V-A}} \sum_q \widehat{e}_q (\overline{q}_\beta q_\alpha)_{\mathrm{V\pm A}} \end{array}\right\} \quad \text{Electroweak ``penguins''}$$

“Penguins” feel all three quark families in the loop:

they are sensitive to the CP phase.

CP conserving

$$\mathrm{Re}A_0 = \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \frac{1}{\cos\delta_0} \sum_i z_i \ \mathrm{Re}\langle Q_i \rangle_0$$

$$\mathrm{Re}A_2 = \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \frac{1}{\cos\delta_2} \sum_i z_i \ \mathrm{Re}\langle Q_i \rangle_2 + \omega\ \Omega_{\eta+\eta'}\ \mathrm{Re}A_0$$

CP violating

$$\mathrm{Im}A_0 = \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \frac{1}{\cos\delta_0} \sum_i \mathrm{Im}\tau\ y_i \ \mathrm{Re}\langle Q_i \rangle_0$$

$$\mathrm{Im}A_2 = \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \frac{1}{\cos\delta_2} \sum_i \mathrm{Im}\tau\ y_i \ \mathrm{Re}\langle Q_i \rangle_2 + \omega\ \Omega_{\eta+\eta'}\ \mathrm{Im}A_0$$

Isospin breaking $\pi^0 - \eta - \eta'$ mixing (NLO):

$$\Omega_{\mathrm{IB}}^{\pi-\eta-\eta'} \simeq 0.16 \pm 0.05$$

[Ecker,Müller,Neufeld and Pich, 1999]

Complete NLO chiral corrections may make $\Omega_{\mathrm{IB}}^{\mathrm{NLO}}$ as large as -0.7 [Gardner and Valencia, 1999]

Computing Direct CP violation in $K \to \pi\pi$

$$\eta_{00} \equiv \frac{\langle \pi^0 \pi^0 | \mathcal{H}_W | K_L \rangle}{\langle \pi^0 \pi^0 | \mathcal{H}_W | K_S \rangle} \simeq \varepsilon - 2\varepsilon'$$

$$\eta_{+-} \equiv \frac{\langle \pi^+ \pi^- | \mathcal{H}_W | K_L \rangle}{\langle \pi^+ \pi^- | \mathcal{H}_W | K_S \rangle} \simeq \varepsilon + \varepsilon'$$

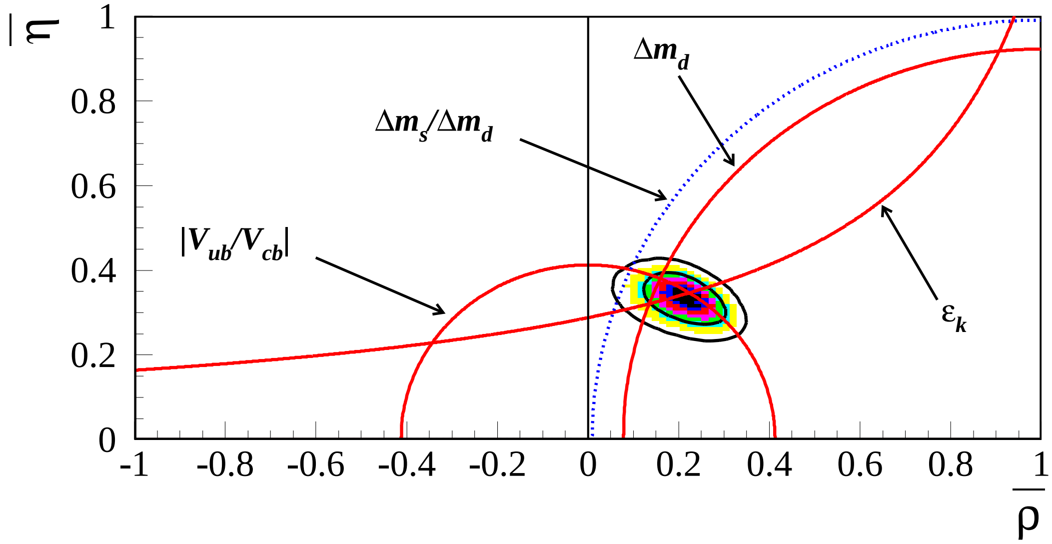
Using the effective $\Delta S = 1$ quark lagrangian:

$$\boxed{\frac{\varepsilon'}{\varepsilon} = e^{i\phi} \frac{G_F \omega}{2|\epsilon| \,\mathrm{Re}\, A_0} \,\mathrm{Im}\, \lambda_t \;\; \left[\Pi_0 - \frac{1}{\omega} \Pi_2 \right]}$$

$$\begin{aligned} \Pi_0 &= \frac{1}{\cos\delta_0} \sum_i y_i \,\mathrm{Re}\langle Q_i \rangle_0 \,(1 - \Omega_{\eta+\eta'}) \\ \Pi_2 &= \frac{1}{\cos\delta_2} \sum_i y_i \,\mathrm{Re}\langle Q_i \rangle_2 \end{aligned}$$

$$\phi = \frac{\pi}{2} + \delta_2 - \delta_0 - \theta_\varepsilon = (0 \pm 4)^0$$

$$\text{Im}\lambda_t \simeq \eta\ |V_{us}||V_{cb}|^2$$



Thanks to F. Parodi, 1999

$$\hat{B}_K = 1.0 \pm 0.2: \quad \text{Im}\,\lambda_t = (1.21 \pm 0.12) \times 10^{-4}$$

Munich:

$$\hat{B}_K = 0.80 \pm 0.15: \quad \text{Im}\,\lambda_t = (1.33 \pm 0.14) \times 10^{-4}$$

Calculation of four-quark matrix elements

The ideal approach

A: Consistent definition of renormalized operators: correct scheme and scale matching with short-distance.

B: Self-contained calculation of all hadronic matrix elements (including B_K).

C: It reproduces simultaneously the $\Delta I = 1/2$ selection rule and ε'/ε .

VSA: $\langle\pi^+\pi^-|Q_6|K^0\rangle = 2\langle\pi^-|\overline{u}\gamma_5 d|0\rangle\langle\pi^+|\overline{s}u|K^0\rangle$
$- 2\langle\pi^+\pi^-|\overline{d}d|0\rangle\langle 0|\overline{s}\gamma_5 d|K^0\rangle$
$+ 2\left[\langle 0|\overline{s}s|0\rangle - \langle 0|\overline{d}d|0\rangle\right]\langle\pi^+\pi^-|\overline{s}\gamma_5 d|K^0\rangle$

Generalized Factorization: Effective Wilson coefficients, matched with factorized matrix elements at the scale μ_F (H-Y Cheng, 1999).

Phenomenological $1/N$: Fix some of the matrix elements by fitting the $\Delta I = 1/2$ rule and vary others around the $1/N$ values (München).

Chiral Quark Model: All matrix elements at $O(p^4)$ in terms of $\langle\overline{q}q\rangle$, $\langle\frac{\alpha_s}{\pi}GG\rangle$, M, phenomenologically fixed via the $\Delta I = 1/2$ rule (Trieste).

Phenomenological NJL: Chiral loops up to $O(p^6)$ and fit to the $\Delta I = 1/2$ rule. It includes scalar, vector and axial-vector resonances (Dubna).

1/N: Chiral loops regularized via cutoff, partial $O(p^4)$ (Dortmund).

1/N and NJL: It includes scalar, vector and axial-vector resonances, good scale stability (Bijnens and Prades, 1999-2000).

1/N and QCD Sum Rules: $\hat{B}_K$ at the NLO in $1/N$ in the chiral limit: consistent NLO matching (Peris and De Rafael, 2000).

Lattice: $K \to \pi$ matrix elements of four-quark operators. Use chiral symmetry to obtain $K \to \pi\pi$ (Roma, RBC).

Linear σ-model: $m_\sigma = 500 - 900$ MeV: ε'/ε and A_0 cannot be reproduced simultaneously (Keum et al., Harada et al., Bloch et al., 1999).

Cut-off people: beware of dimension eight operators!

$$\langle \mathcal{Q}_n^{(6)} \rangle_\mu^{\overline{\rm MS}} = \langle \mathcal{Q}_n^{(6)} \rangle_\mu^{\rm cut-off} + \sum_i d_i \langle \mathcal{Q}_i^{(6)} \rangle_\mu + \sum_i \mathcal{C}_i^{(8)} \langle O_i^{(8)} \rangle$$

[Cirigliano, Donoghue and Golowich, 2000]

Isospin violation: $\Delta I = 5/2$ transitions

$$\epsilon' = -\frac{i}{\sqrt{2}} e^{i(\chi_2 - \chi_0)} \, \omega \, \frac{\mathcal{I}m A_0}{\mathcal{R}e A_0} \left[1 - \frac{1}{\omega} \frac{\mathcal{I}m A_2}{\mathcal{I}m A_0} + (\Delta\omega_{5/2} - \Omega_{\rm IB})^{\rm EM+STR} \right]$$

[STR: Gardner and Valencia; Ecker et al.; Maltman and Wolfe.
EM: Cirigliano, Donoghue and Golowich, 1999-2000]

Compute $K \to \pi\pi$ directly on the lattice

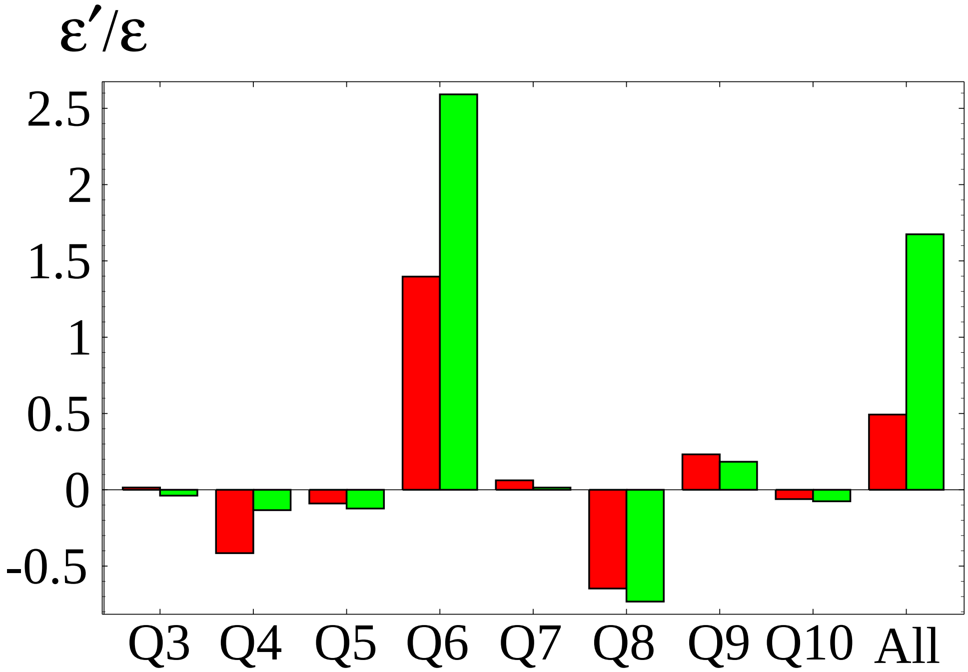
In finite volume a simple formula relates the transition amplitude to the physical decay rate. It overcomes the Maiani-Testa *no-go theorem* (1990).

[Lellouch and Lüscher, 2000]

Is final state dynamics accounted for in quenched calculations ?

ε'/ε: a (Penguin) Comparative Anatomy

|München|Trieste| $(\times 10^3)$



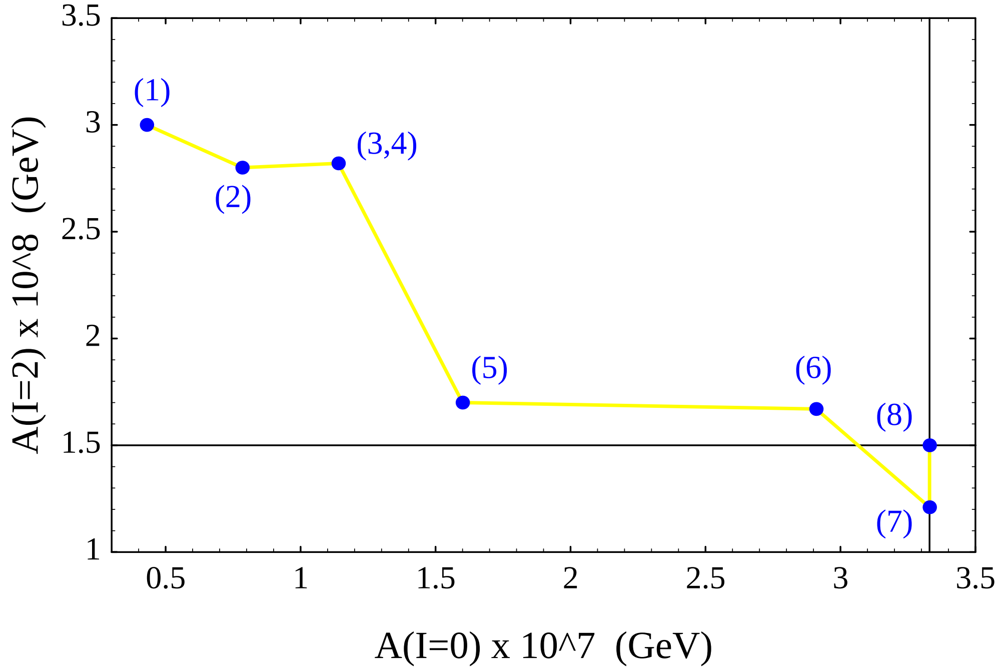
- $\langle Q_{9,10} \rangle_2 = \frac{3}{2} \langle Q_1 \rangle_2$: from $\Delta I = 1/2$ rule

- $\langle Q_8 \rangle_2$ moderately smaller than VSA

- Largest deviations: $\langle Q_6 \rangle$ and $\langle Q_4 \rangle$!

$$|\mathcal{A}_0|/|\mathcal{A}_2| = 22.2$$

Anatomy of the $\Delta I = 1/2$ rule in the χQM



$1 \to 4$: Perturbative QCD and factorization

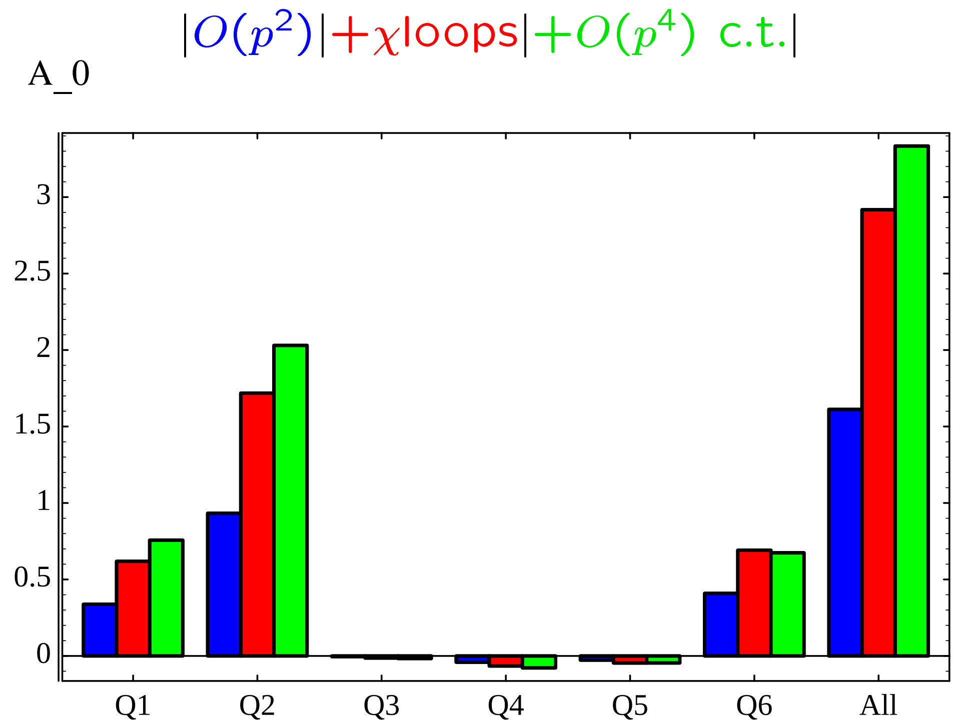
$4 \to 5$: Non-factorizable $\langle \alpha_s GG/\pi \rangle$ corrections (LO)

$5 \to 7$: Chiral loops and $O(p^4)$ counterterms

$7 \to 8$: Isospin breaking $(\pi - \eta - \eta')$

Final state interactions alone are not enough to account for the $\Delta I = 1/2$ rule.

Penguins and $\Delta I = 1/2$ rule in the χQM

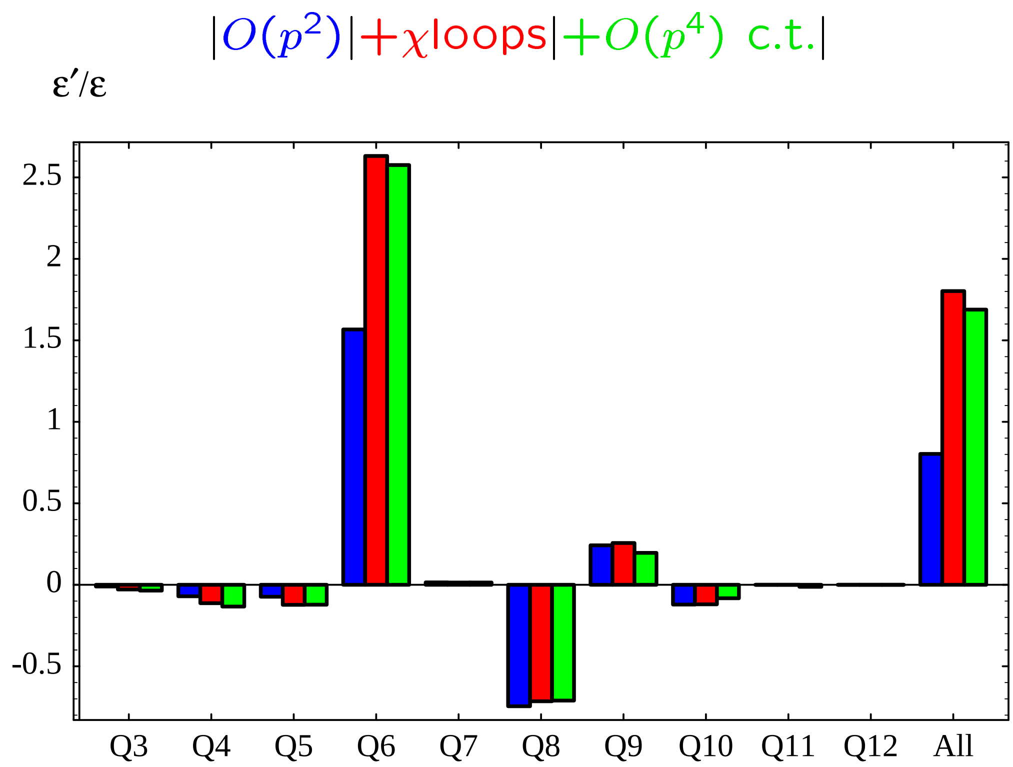


The Q_6 contribution to $A(K \to \pi\pi)_{I=0}$ (in GeV $\times 10^7$) is about 20% of the total $[O(p^4/N)]$.

A_2 is reduced to its experimental value by non-factorizable $\langle GG \rangle$ corrections $[O(p^2/N)]$.

How does this information feed into the determination of the whole set of $\Delta S = 1$ (and 2) matrix elements ?

Anatomy of ε'/ε in the χQM at $O(p^4)$



At $O(p^2)$ the pattern of hadronic matrix elements does not differ much from leading order $1/N$.

Chiral corrections enhance $\langle Q\rangle_{I=0} / \langle Q\rangle_{I=2}$: $B_6/B_8^{(2)} \approx 2$ (non-trivial consequence of the $\Delta I = 1/2$ fit)

$1/N$ approaches beyond LO (Dortmund group, Bijnens and Prades) confirm the $\langle Q_6\rangle$ enhancement.

Role of NLO order chiral corrections and unquenching in lattice calculations ?

Conclusions

- $I = 2$ amplitudes: (semi-)phenomenological approaches which fit the $\Delta I = 1/2$ selection rule in $K \to \pi\pi$ decays, generally agree in the pattern and size of the $\Delta S = 1$ hadronic matrix elements with the existing $1/N$ and lattice calculations.

- $I = 0$ amplitudes: the $\Delta I = 1/2$ rule forces upon us large deviations from factorization: $B-$factors of $O(10)$ for $\langle Q_{1,2}\rangle_0$ (lattice calculations presently suffer from large sistematic uncertainties).

- In the χQM calculation, non-factorizable contributions, "normalized" by the fit of the CP conserving amplitudes, enhance the $I = 0$ matrix elements and deplete the $I = 2$ amplitudes. such that $B_6/B_8^{(2)} \approx 2$. Similar results from $1/N$ and dispersive approaches. FSI are most relevant for the enhancement of the $I = 0$ components (gluonic penguins).

- Lattice: promising work in progress
  - Domain Wall Fermions (control of chiral symmetry),
  - Direct calculation of $K \to \pi\pi$ in finite volume.

- Theoretical error: further work needed on
  - the matching of long-distance and short-distance components (cut-off reg. $\to$ higher dim. operators).
  - the calculation of NLO isospin violation effects (EM + STR)
  - the determination of $\text{Im}(V_{ts}^* V_{td})$.
    From B-physics : B-factories and hadronic colliders (soon).
    From K-physics : $K_L \to \pi^0 \nu\bar{\nu}$ (eventually).

Experiments have stimulated very promising theoretical efforts which may lead us in a reasonably short time to address longstanding problems of strong interacting QCD.