Leading $\mathcal{O}(G_F^2 m_t^4)$ Corrections to Precision Observables in the MSSM

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Carmel, 09/00

based on collaboration with *G. Weiglein*

1. Motivation
2. Techniques for $\mathcal{O}(G_F^2 m_t^4)$ calculations
3. Results for $\Delta\rho$, M_W, $\sin^2\theta_{\text{eff}}$
4. Conclusions

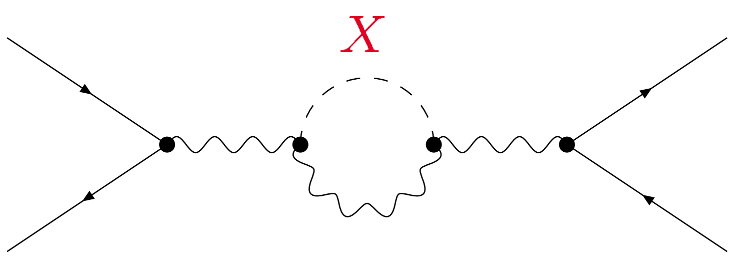
Comparison of electro-weak precision observables with theory:

EW Precision data:		Theory:
$\Delta\rho, \Delta r, M_W, \sin^2\theta_{\text{eff}}$	$\leftrightarrow$	SM, MSSM , ...

$\Downarrow$

Test of theory at quantum level:

Sensitivity to loop corrections



$\Downarrow$

Very high accuracy of measurements and theoretical predictions needed

- Which model fits better?
- Does the prediction of a model contradict the experimental data?

(MSSM)

Superpartners for Standard Model particles

$$\left[u,d,c,s,t,b\right]_{L,R} \quad \left[e,\mu,\tau\right]_{L,R} \quad \left[\nu_{e,\mu,\tau}\right]_L \qquad \text{Spin } \tfrac{1}{2}$$

$$\left[\tilde{u},\tilde{d},\tilde{c},\tilde{s},\tilde{t},\tilde{b}\right]_{L,R} \quad \left[\tilde{e},\tilde{\mu},\tilde{\tau}\right]_{L,R} \quad \left[\tilde{\nu}_{e,\mu,\tau}\right]_L \qquad \text{Spin } 0$$

$$g \quad \underbrace{W^\pm, H^\pm} \quad \underbrace{\gamma, Z, H_1^0, H_2^0} \qquad \text{Spin 1 / Spin 0}$$

$$\tilde{g} \quad \tilde{\chi}^\pm_{1,2} \quad \tilde{\chi}^0_{1,2,3,4} \qquad \text{Spin } \frac{1}{2}$$

Enlarged Higgs sector :
Two Higgs doublets, physical states:
$h^0, H^0, A^0, H^\pm$

MSSM: Enlarged Higgs sector:

Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1^0 + i\chi_1^0)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix}$$

Higgs potential:

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) + \frac{g'^2 + g^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g^2}{2} |H_1 \bar{H}_2|^2$$

Physical states: $h^0, H^0, A^0, H^\pm$

Goldstones: $G^0, G^\pm$

Input parameters:

$$\tan\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2 (\tan\beta + \cot\beta)$$

ρ measures the relative strength between
neutral current interaction and
charged current interaction .

$$\rho = \frac{1}{1 - \Delta\rho} \qquad \Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2}$$

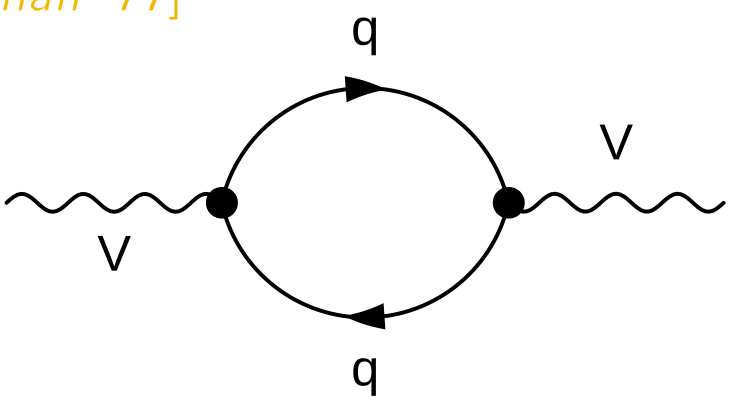
(leading, process independent terms)

$\Delta\rho$ gives the main contribution to EW observables:

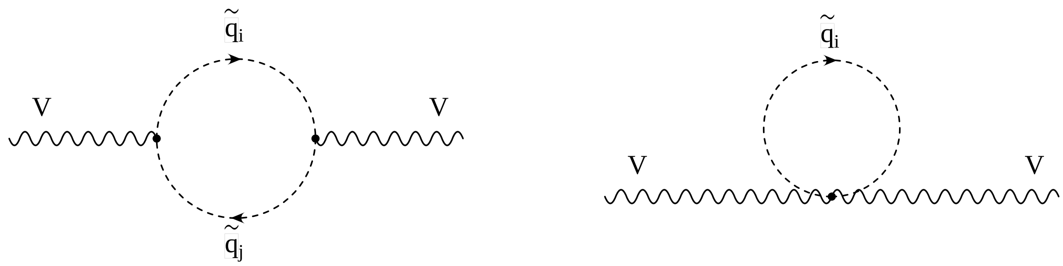
$$\Delta M_W \approx \frac{M_W}{2} \frac{c_w^2}{c_w^2 - s_w^2} \Delta\rho,$$

$$\Delta \sin^2\theta_W^{\rm eff} \approx \frac{c_w^2 s_w^2}{c_w^2 - s_w^2} \Delta\rho$$

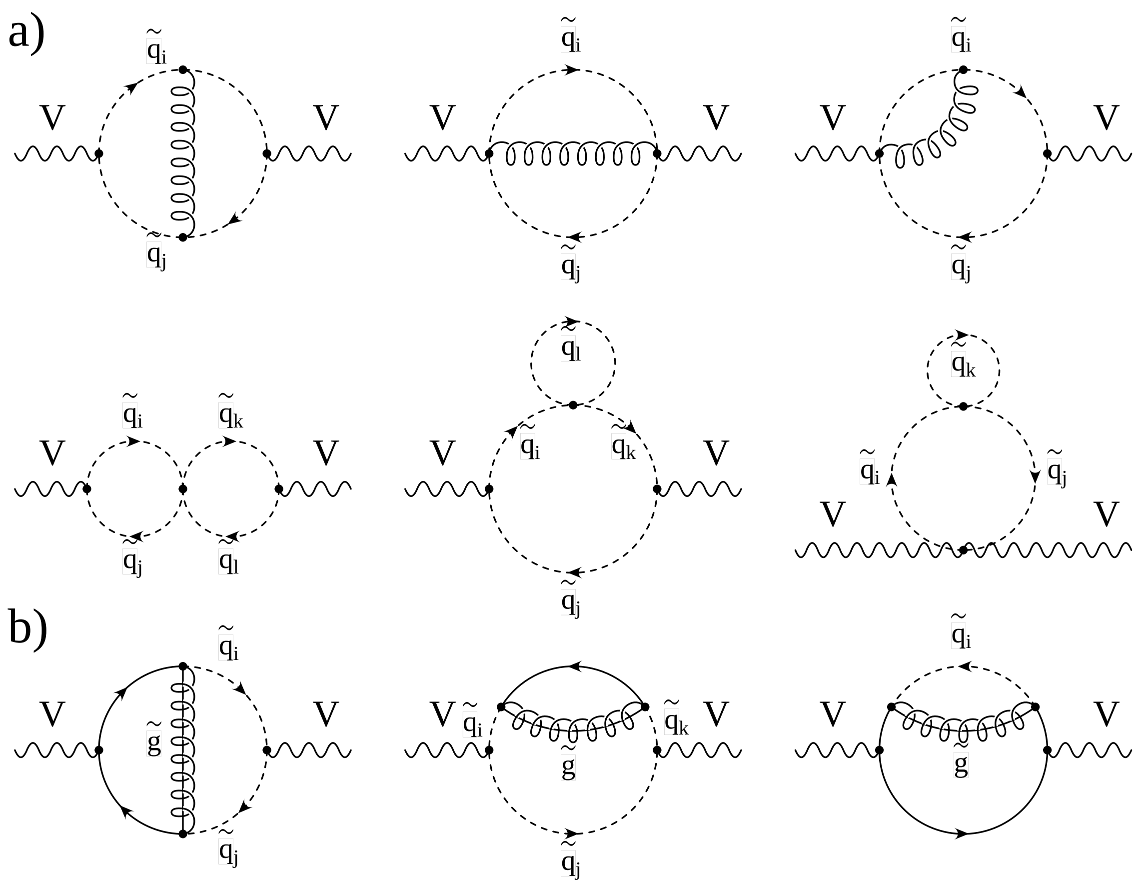
- SM: leading one-loop terms $\mathcal{O}(G_F\, m_t^2)$:
  [*M. Veltman '77*]



- SM: leading terms up to $\mathcal{O}(G_F^2\, m_t^4)$
  [*J. van der Bij, F. Hoogeveen '87*]
  [*R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci, A. Vicere '92*]
  [*J. Fleischer, O.V. Tarasov, F. Jegerlehner '93*]
- MSSM: Leading one-loop corrections from $\tilde{t}/\tilde{b}$-sector → F
  [*R. Barbieri, L. Maiani '83*]
  [*M. Drees, K. Hagiwara '90*]
  [*P. Chankowski et al. '94*]
- MSSM: leading $\mathcal{O}(\alpha\alpha_s)$ corrections → F
  [*A. Djouadi, P. Gambino, S. H., W. Hollik, C. Jünger, G. Weiglein '97*]
  [*S. H, W. Hollik, G. Weiglein '98*]



Two-loop diagrams for $\Delta\rho^{\mathrm{SUSY}}$:



- First SUSY contributions of $\mathcal{O}(m_t^4)$

  Same size as $\mathcal{O}(\alpha\alpha_s)$ corrections?

- Up to now:
  SM result used with $M_H^{\text{SM}} \to m_h^{\text{SUSY}}$

  Correct?

Problems with two-loop calculations:

- Large number of topologies and diagrams (up to $\mathcal{O}(1000)$)
- Complicated tensor structure possible

$\Rightarrow$ Extensive use of computer algebra programs

Our procedure:

- Automatic generation of Feynman diagrams and amplitudes: *FeynArts*

  [*J. Küblbeck, A. Denner, M. Böhm '90*]

  [*H. Eck '95*]

  [*T. Hahn '98, '00*]

  $\to$ MSSM model file complete
  (some $\tilde{f}^{\,4}$ couplings missing, no CTs)

duction to scalar integrals: *TwoCalc*
(works for two-loop self-energies)
[*G. Weiglein '92*]
[*G. Weiglein, R. Scharf, M. Böhm '94*]

- Further evaluation: insertion of integrals, expansion in $\delta = \frac{1}{2}(4 - D)$
  $\to$ algebraical check: cancellation of divergencies

- Result:
  - algebraic *Mathematica* code
  - Fortran code

- Numerical evaluation:
  $\to$ numbers, plots etc

SUSY decouples from SM for $M_{\text{SUSY}} \to \infty$

[*T. Applequist and J. Carazzone '75*]

[*A. Dobado, M. Herrero and S. Piñaranda '98, '99, '00*]

[*A. Djouadi, P. Gambino, S.H., W. Hollik, C. Jünger, G. Weiglein '97*]

$\Rightarrow$ leading terms for decoupled squarks
Concentrate on Higgs bosons and top quark

- extract a prefactor $\mathcal{O}(m_t^4/M_W^4)$
  set $M_W \to 0$

- calculate two-loop diagrams $\to$ F
  one-loop CT diagrams $\to$ F
  CT insertion diagrams $\to$ F

- Needed for cancellation of divergencies:

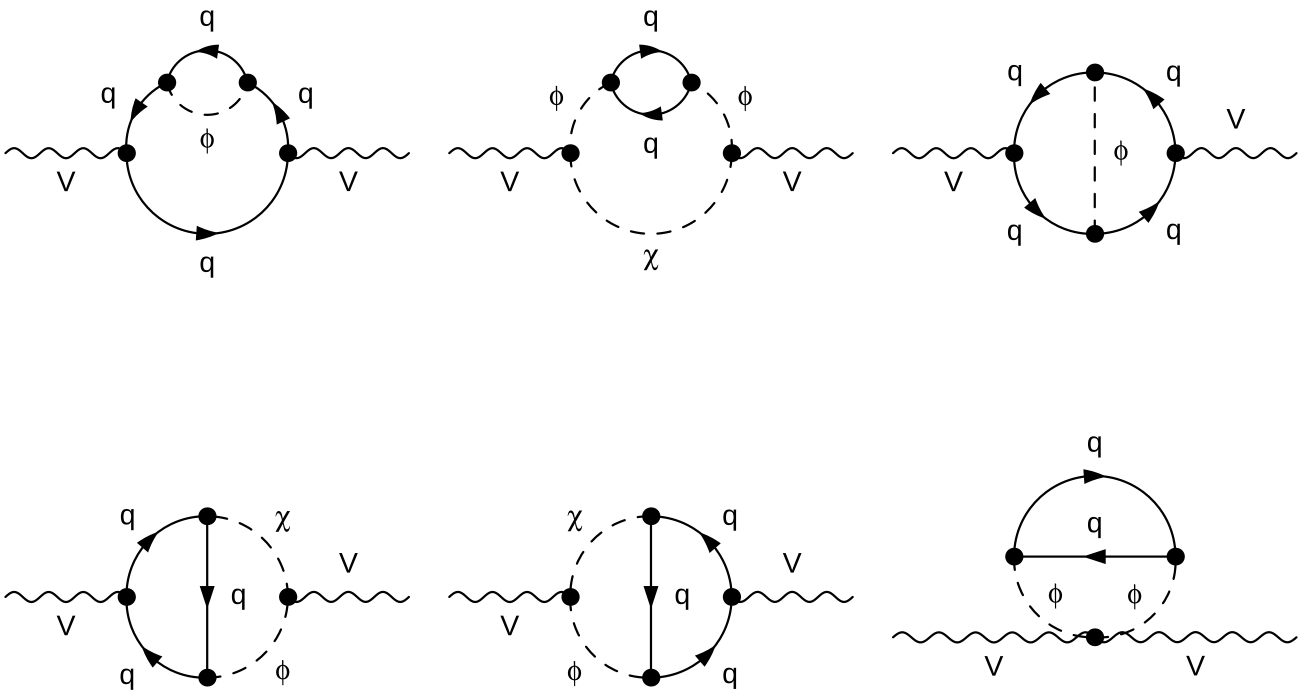
  $M_W = M_Z c_w \to 0$ everywhere

  Tree-level relations for MSSM Higgs masses:
  $m_h = M_G = M_{G^\pm} = 0$

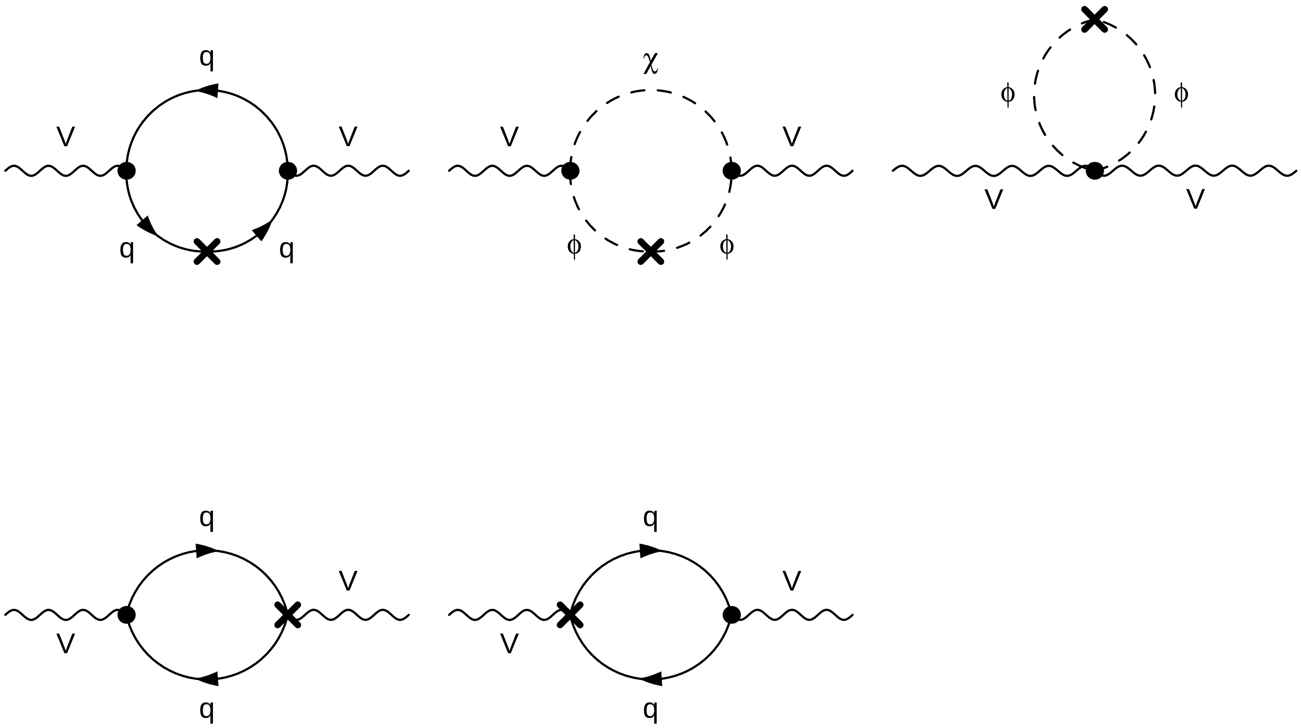
  $m_H = M_{H^\pm} = M_A$

- Additional check:

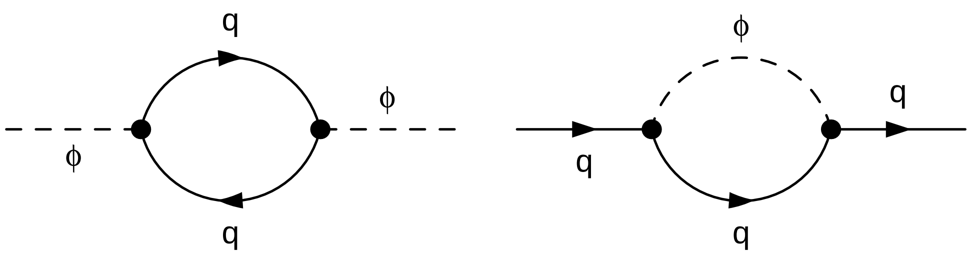
  SM result recalculated
  $\Rightarrow$ perfect agreement



$$V = Z, W^{\pm}$$
$$q = t, b$$
$$\phi, \chi = h^0, H^0, A^0, H^{\pm}, G^0, G^{\pm}$$



CT insertions:



Finite result for $\Delta\rho^{\text{SUSY}}$ in terms of
$s_\beta = \tan\beta/\sqrt{1+\tan^2\beta}$ and $a \equiv m_t^2/M_A^2$:

$$\Delta\rho_1^{\text{SUSY}} = 3\,\frac{G_F^2}{128\,\pi^4}\,m_t^4\,\frac{1-s_\beta^2}{s_\beta^2\,a^2}\times$$
$$\Bigg\{\text{Li}_2\left((1-\Delta)/2\right)\frac{8}{\Delta}\Lambda$$
$$-2\,\text{Li}_2\left(1-\frac{1}{a}\right)\left[5-14a+6a^2\right]$$
$$+\log^2(a)\left[1+\frac{2}{\Delta}\Lambda\right]-\log(a)\left[2-20\,a\right]$$
$$-\log^2\left(\frac{1-\Delta}{2}\right)\frac{4}{\Delta}\Lambda$$
$$+\log\left(\frac{1-\Delta}{1+\Delta}\right)\Delta(1-2\,a)$$
$$-\log\left(|1/a-1|\right)\,(a-1)^2$$
$$+\pi^2\left[\frac{2\Delta}{-3+12\,a}\Lambda+\frac{1}{3}-2\,a^2\frac{s_\beta^2}{1-s_\beta^2}\right]$$
$$-17a+19\frac{a^2}{1-s_\beta^2}\Bigg\}$$

with $\Lambda = 3-13a+11a^2$ and $\Delta = \sqrt{1-4a}$

$$\Delta\rho_1^{\mathrm{SUSY}} = 3\,\frac{G_F^2}{128\,\pi^4}\,m_t^4 \times$$
$$\Bigg\{ 19 - 2\pi^2$$
$$- \frac{1-s_\beta^2}{s_\beta^2}\Bigg[\left(\log^2 a + \frac{\pi^2}{3}\right)$$
$$\left(8a + 32a^2 + 132a^3 + 532a^4\right)$$
$$+ \log(a)\frac{1}{30}\Bigg(560a + 2825a^2$$
$$+ 11394a^3 + 45072a^4\Bigg)$$
$$- \frac{1}{1800}\Big(2800a + 66025a^2 + 300438a^3$$
$$+ 1265984a^4\Big) + \mathcal{O}\left(a^5\right)\Bigg]\Bigg\}$$

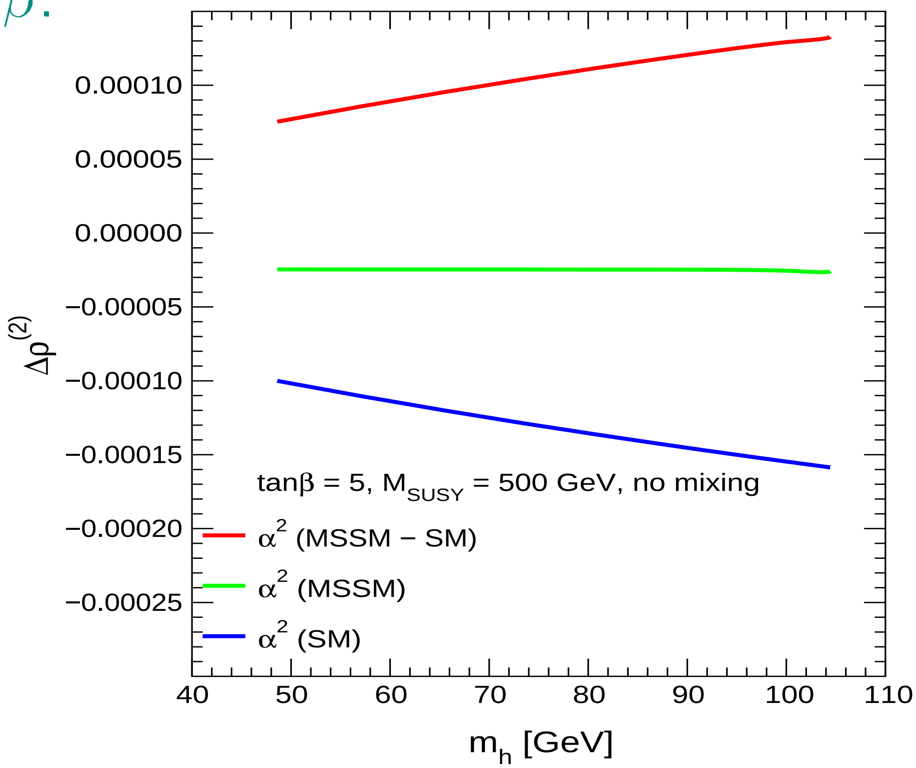
Check:

For $M_A \to \infty$ SM result obtained with $M_H^{\mathrm{SM}} \to 0$

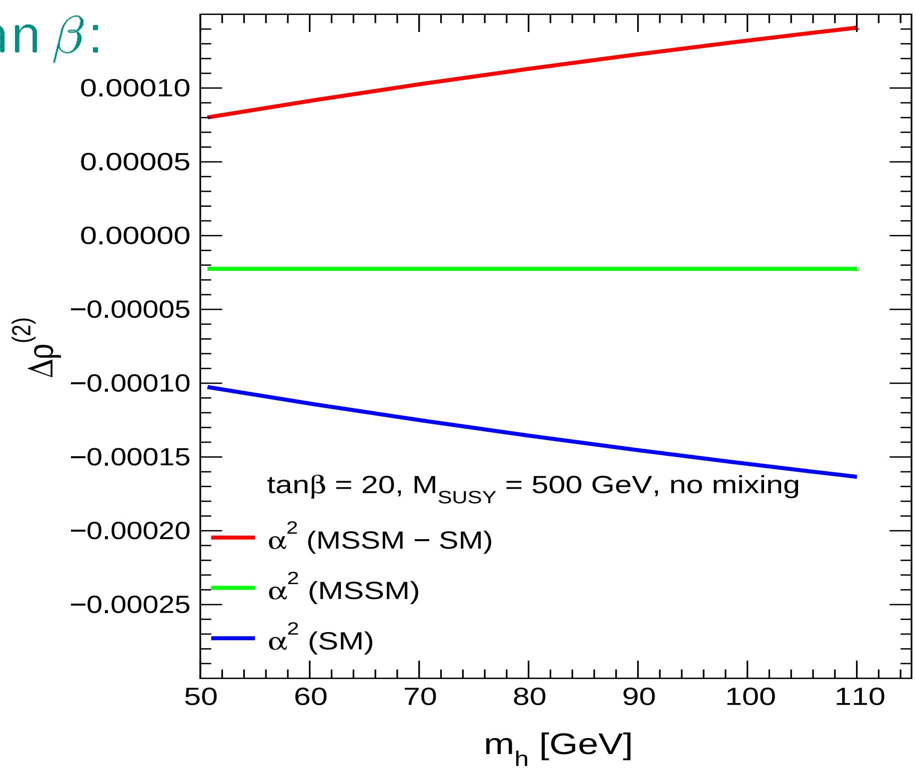
Comparison: MSSM , SM ($M_H^{\rm SM} \to m_h^{\rm SUSY}$) ,
MSSM - SM (effektive change)

M_A varied:

low $\tan\beta$:



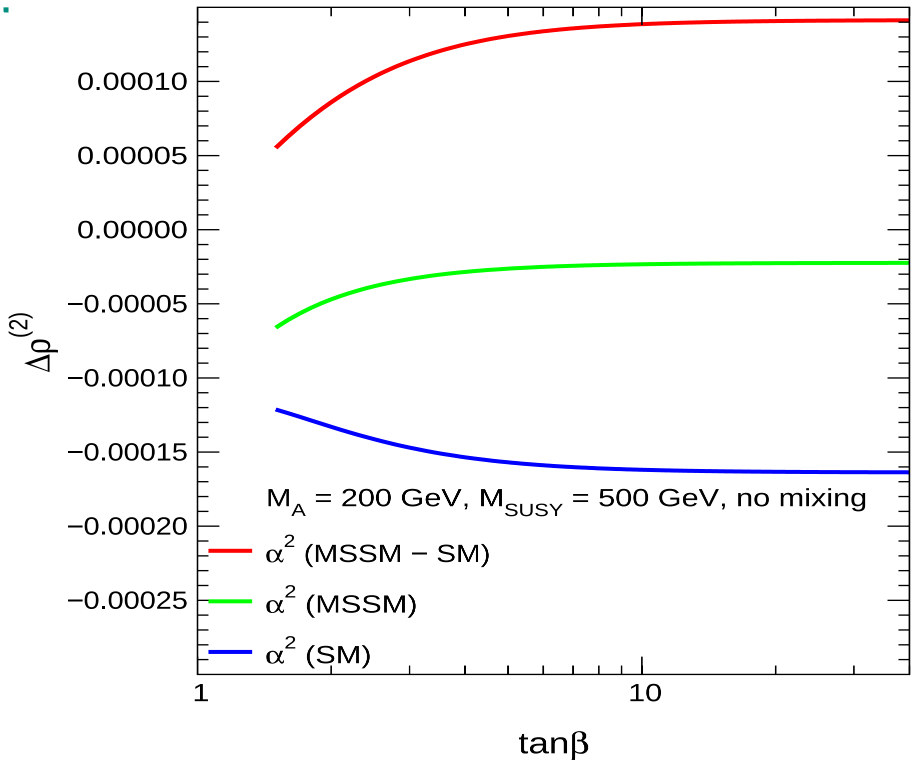
high $\tan\beta$:



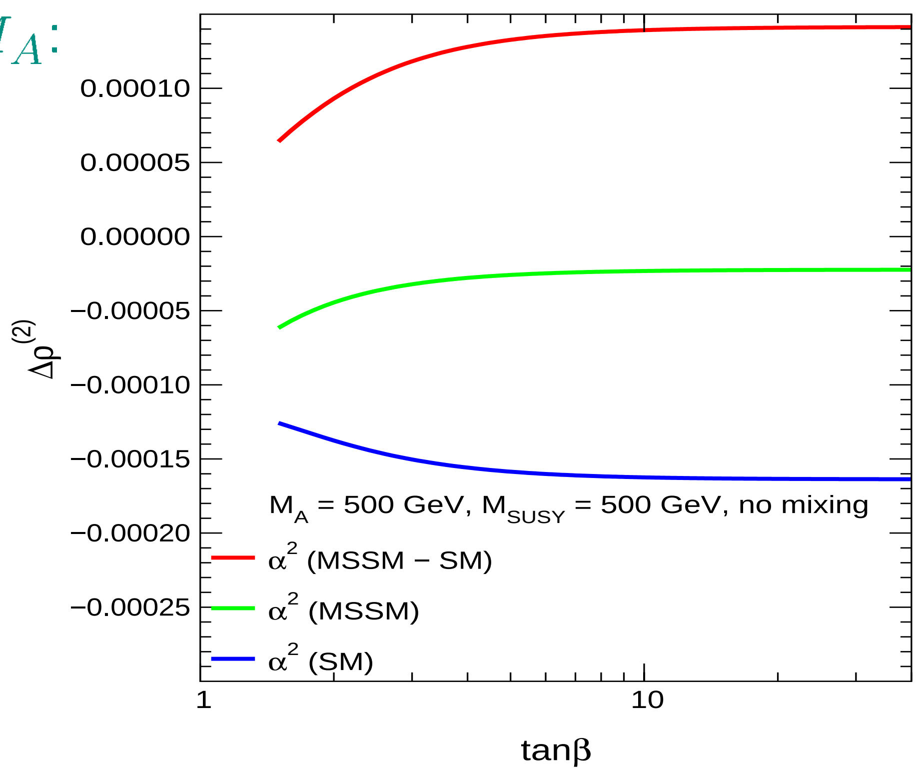
Comparison: MSSM , SM ($M_H^{\mathrm{SM}} \to m_h^{\mathrm{SUSY}}$) ,
MSSM - SM (effektive change)

$\tan\beta$ varied:

low M_A:



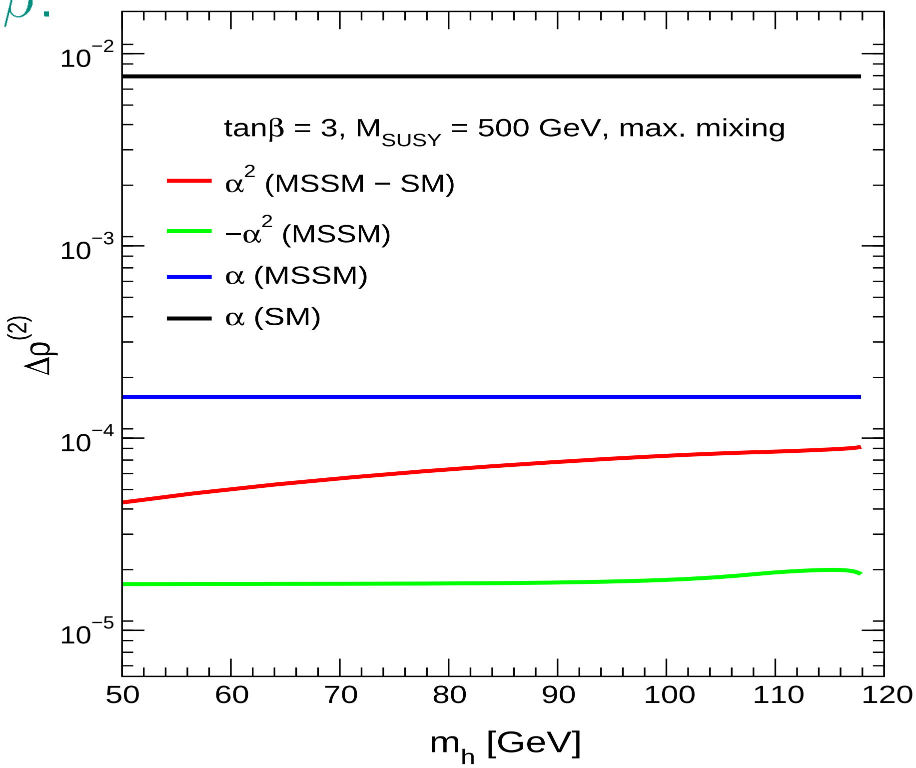
high M_A:



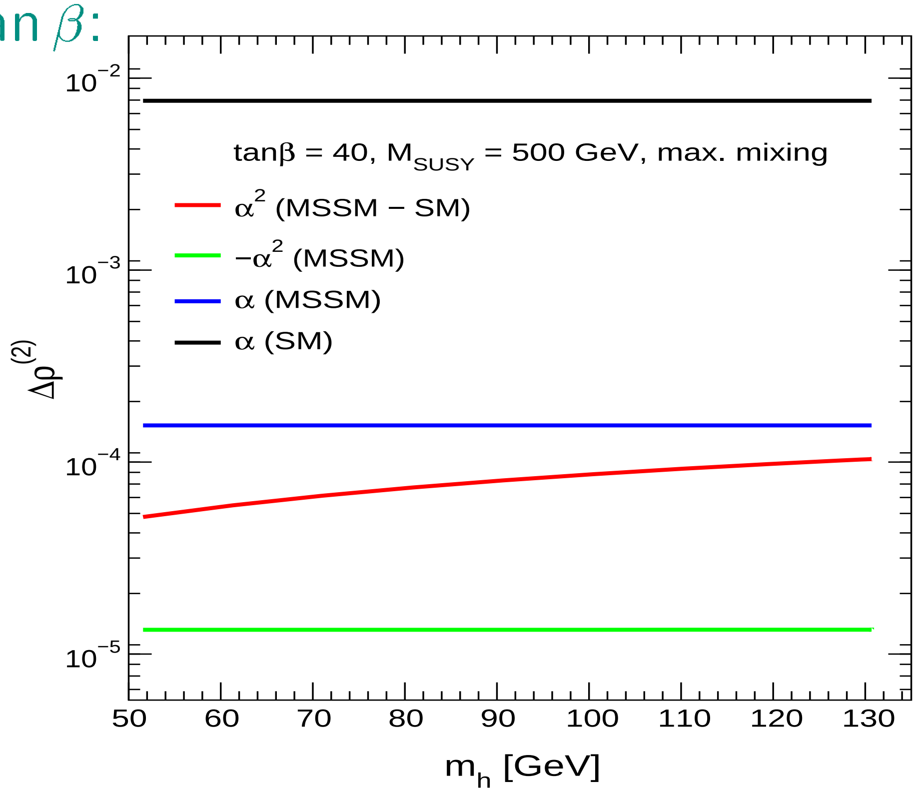
Comparison: one-loop (SM, MSSM),
two-loop (-MSSM , MSSM - SM)

MA varied:

low tan β:

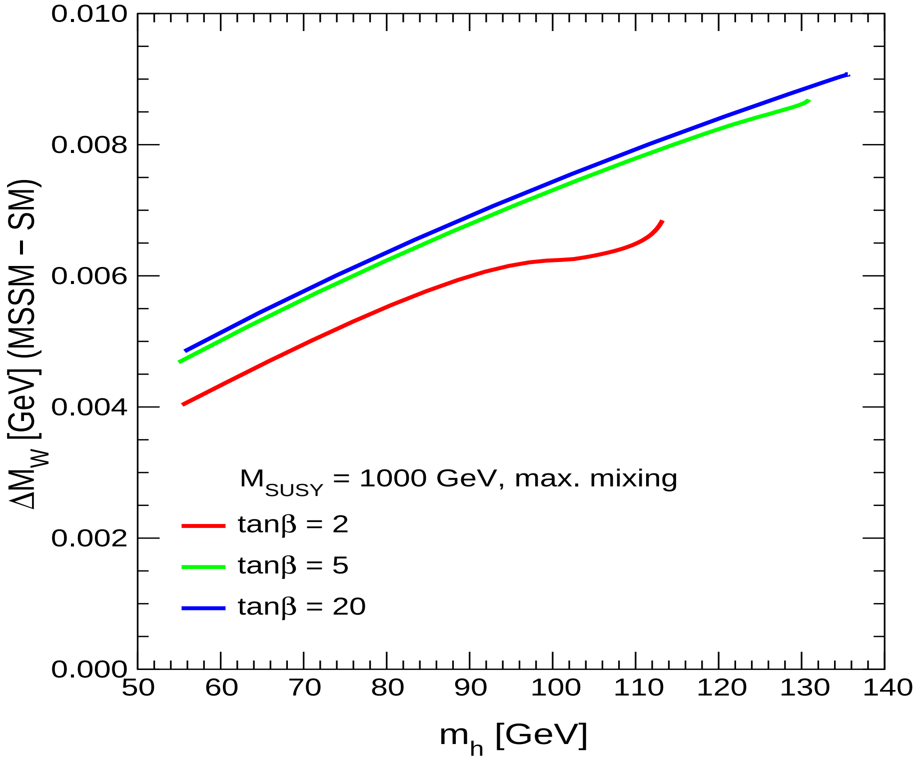


high tan β:

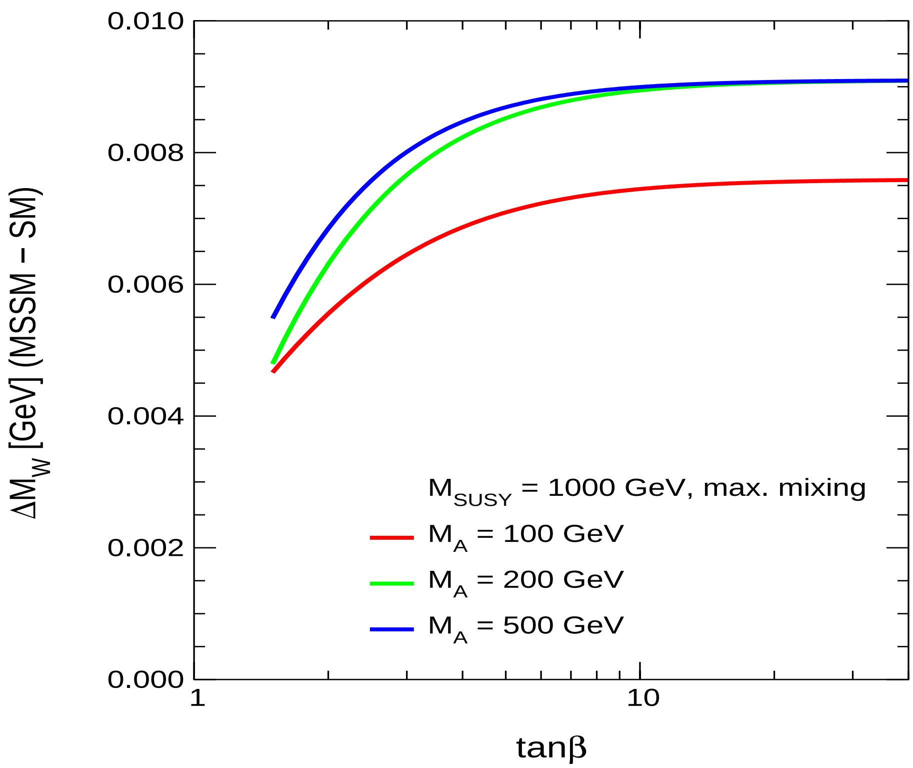


effective change: MSSM - SM

M_A varied, $\tan\beta = 2, 5, 20$:

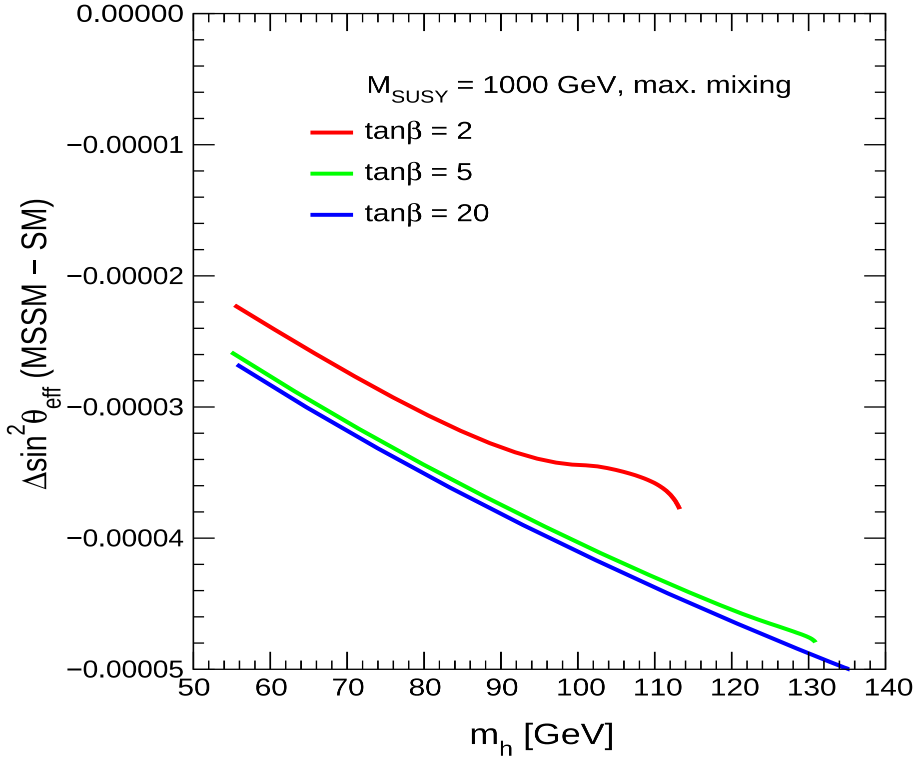


$\tan\beta$ varied, $M_A = 100, 200, 500$:



effective change: MSSM - SM

M_A varied, $\tan\beta = 2, 5, 20$:



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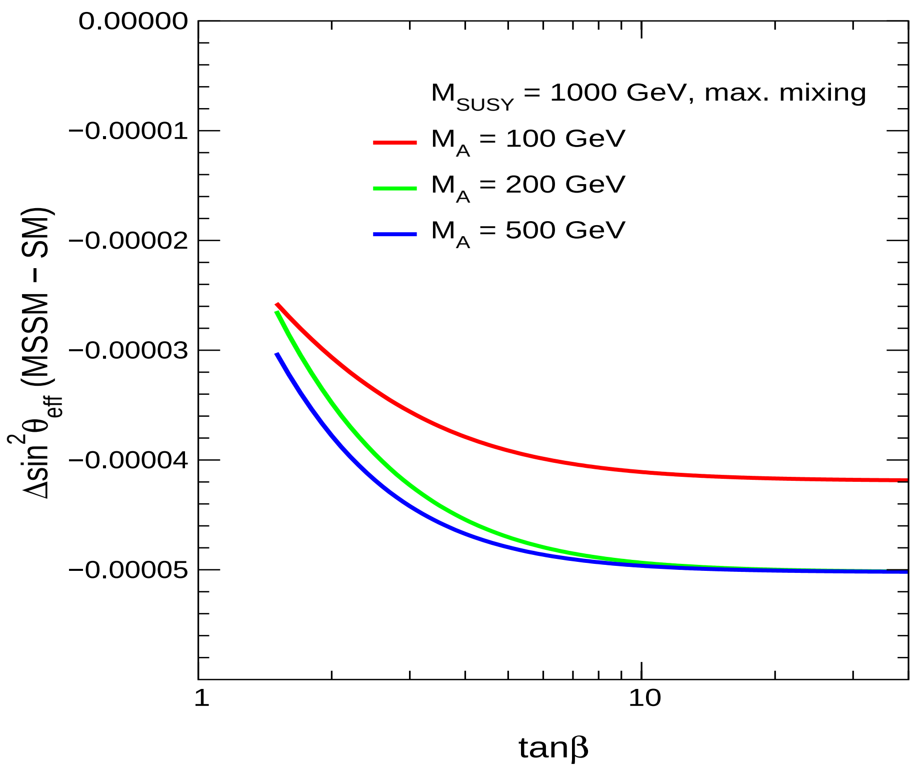

- Calculation of $\Delta\rho^{\text{SUSY}}$ in $\mathcal{O}(G_F^2\, m_t^4)$
  (limit of $M_{\text{SUSY}} \to \infty$)

- Heavy use of computer algebra programs:
  – *FeynArts* (MSSM model file)
  – *TwoCalc*

- Compact formula in terms of
  $\tan\beta$ and $a = m_t^2/M_A^2$
  (SM limit obtained for $M_A \to \infty$)

- Numerical results:
  smaller effect for pure $\mathcal{O}(G_F^2 m_t^4)$ MSSM corrections
  larger result for effective change SM $\to$ MSSM

  – $\Delta\rho^{\text{SUSY}}$: $5 - 10 \times 10^{-5}$

  – ΔM_W^{SUSY}: $\lesssim 10$ MeV

  – $\Delta \sin^2\theta_{\text{eff}}^{\text{SUSY}}$: $\lesssim -5 \times 10^{-5}$