Rare B Decays at the NLL Level

Tobias Hurth   CERN

Collaboration:

Francesca Borzumati, KEK
Christoph Greub, Bern
Matthias Steinhauser, Hamburg
Daniel Wyler, Zürich

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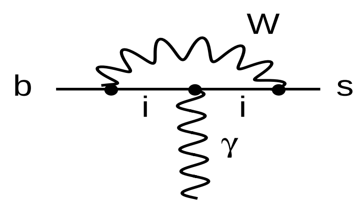
Introduction

- Rare B decays like $B \to X_{s,d}\,\gamma$ or $B \to X_s\, l^+ l^-$ directly probe the SM at the quantum level.



$\Rightarrow$ High sensitivity to nonstandard contributions
(2HDM : charged Higgs; MSSM : chargino, gluino....)
$\Rightarrow$ Sensitivity to SM parameters (V_{ts})

- Heavy Mass Expansion

$$\Gamma(B \to X_s\gamma) = \Gamma(b \to s\gamma) + \Delta^{nonpert.}$$

- Experiment:

$$BR(B \to X_s\gamma) = (3.15 \pm 0.35 \pm 0.32 \pm 0.26) \times 10^{-4}$$

(CLEO, 98')

$$BR(B \to X_s\gamma) = (3.11 \pm 0.80 \pm 0.72) \times 10^{-4}$$

(ALEPH, 98')

$$BR(B \to X_s\gamma) = (3.34 \pm 0.50 \pm 0.37 \pm 0.28) \times 10^{-4}$$

(Belle, Osaka 00')

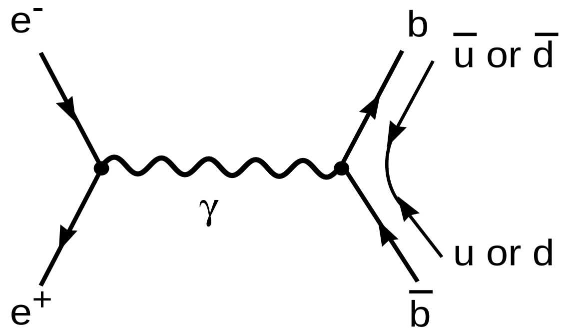
- More precise measurements are expected in the near future (B-factories, CLEO III): $\pm 10\%$.

- Theory: NLL QCD

$$BR(B \to X_s\gamma) = (3.32 \pm 0.22 \pm 0.26) \times 10^{-4}$$

The $B \to X_s\gamma$ decay already plays an important role in restricting the parameter space of extensions of the SM.

Inclusive versus Exclusive



$$e^+e^- \to Y(4S) \to B^+B^-, B^0\bar{B}^0$$

- e^+e^--machines :

  CLEO, B factories (Babar, Belle)   (≥ 1999)

low non B background ↔ low statistics

- Hadronic machines :

  HeraB (≥ 2000), Tevatron II (≥ 2001),
  LHCB, BTeV (≥ 2005)

large quark production cross sections ↔ large background

- Inclusive Modes (i.e. $B \to X_s\gamma$, $B \to X_s l^+ l^-$)
  - theoretically clean (laboratory for perturbative QCD)
  - experimentally more challenging
- Exclusive Modes (i.e. $B \to K^*\gamma$, $B \to K^* l^+ l^-$)
  - large theoretical uncertainties from hadronic form factors
  - cleaner experimental signals (especially at hadronic machines)

Focus on inclusive modes !

Photon Energy Spectrum

- Bremstrahlung $b \to s\gamma\, gluon$

- Nonperturbative Fermi motion
. of the b quark within the B meson.

- Background suppression($b \to cqq' + \gamma, \ldots$)
  $\to$ lower energy cut necessary

$$2.2GeV \leq E_\gamma \leq E_\gamma^{max} = 2.7GeV \quad (1998:\ E_\gamma^{cut} = 2.1GeV)$$

- Model dependence

$$R_{E_\gamma \geq 2.2} = 0.87 \pm 0.06$$

based on Ali-Greub model (1991)

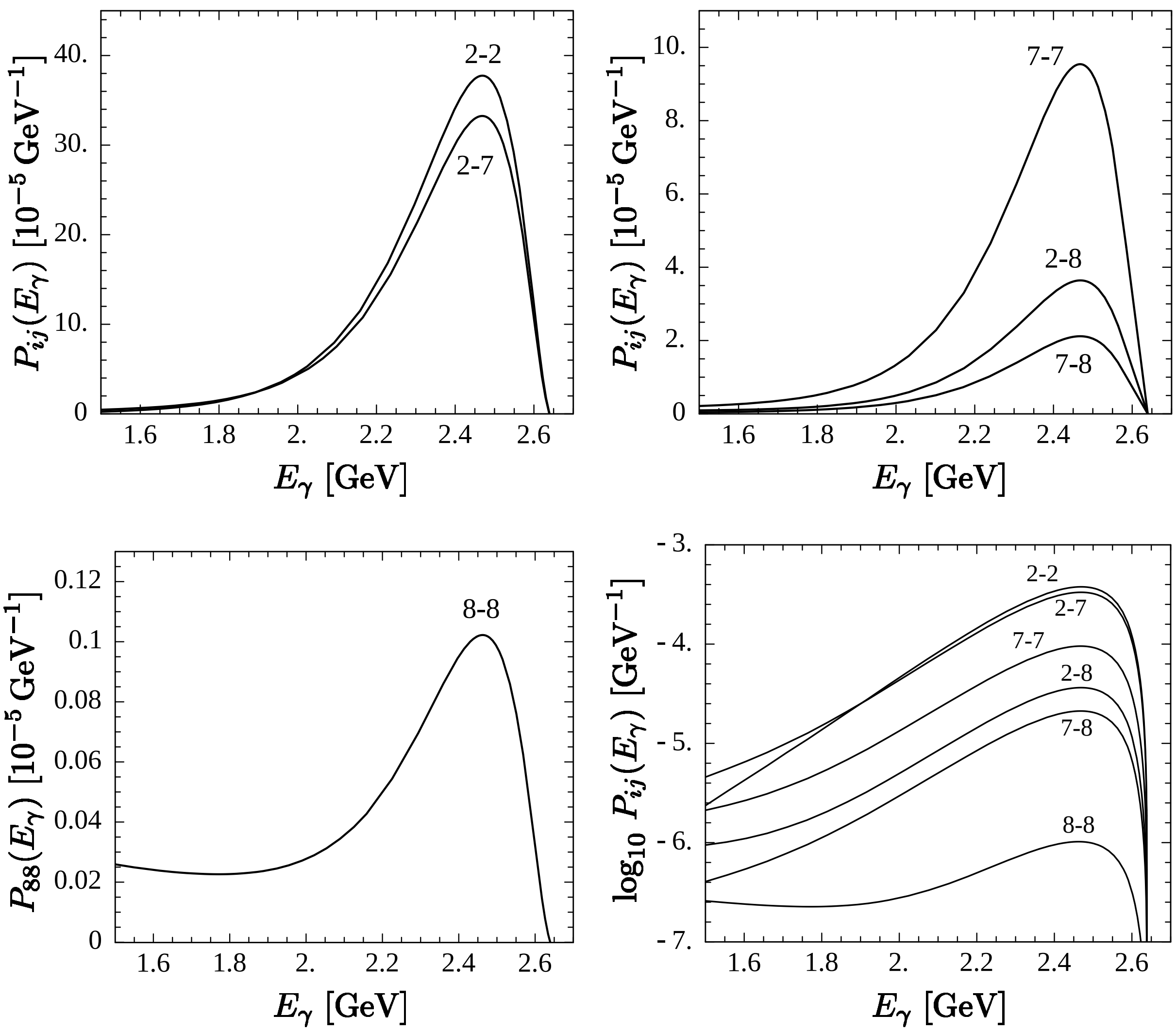
$$R_{E_\gamma \geq 2.2} = 0.78 + 0.09 - 0.11$$

based on HQET; Kagan-Neubert (1998)

- However:

  The shape of the photon spectrum is not sensitive to physics beyond the SM.

- Shape of the photon spectrum



Different components of the photon spectrum in $B \to X_s\gamma$ decays (Kagan, Neubert 1998).

Future Aims

- One should directly compare theory and experiment using the same energy cuts.

- Determine the parameters of these models by using the high precision measurements of the photon spectrum.

- Change of the energy cut $E_\gamma > 2.1 GeV \to E_\gamma > 1.6 GeV$ would remove the strong model dependence, but intermediate ψ background would come into the play.

Nonperturbative Corrections

Heavy Mass Expansion:

$$\Gamma(B \to X_s\gamma) = \Gamma(b \to s\gamma) + \Delta^{nonpert.}$$

- $\Delta^{nonpert.}$ scaling with $1/m_b^2$:
  Well below 10% (1%!) (Falk et al., Ali et al.)

  $$BR(B \to X_s\gamma)/BR^{exp.}_{semilept.} = R_{quark}\,(1 - \Delta^{m_b^2}_{semilept.} + \Delta^{m_b^2}_{bs\gamma})$$

- $\Delta^{nonpert.}$ scaling with $1/m_c^2$:
  Can be analyzed in a model-independent way (Voloshin, Khodjamiriam et al., Ligeti et al., Grant et al., Buchalla et al.):

$$\Delta^{m_c^2} = +3\%$$

$\Rightarrow$ We can focus on the dominant perturbative partonic decay rate $\Gamma(b \to s\gamma)$!

$$R_{quark}(\delta) = \frac{\Gamma[b \to s\gamma] + \Gamma[b \to s\gamma gluon]_\delta}{\Gamma[b \to X_c e\bar{\nu}_e]},$$

$$E_\gamma > (1-\delta)E_\gamma^{max} = (1-\delta)m_b/2$$

Nonperturbative Corrections

- If $B \to X_s\gamma$ is due to the operator O_7 only:

$$T(q) = i \int d^4x \, < B|TO_7^+(x)\, O_7(0)|B > \exp(iqx)$$

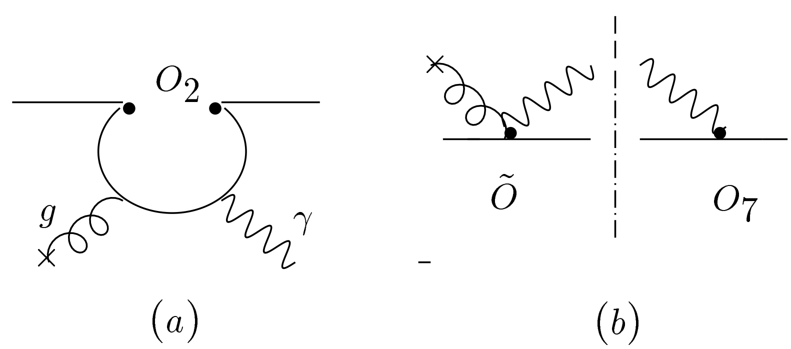
OPE for $TO_7^+(x)\, O_7(0)$ and HQE:

$$\Gamma^{(O_7,O_7)}_{B\to X_s\gamma} = \frac{\alpha G_F^2 m_b^5}{32\pi^4} |V_{tb}V_{ts}|^2 \, C_7^2(m_b) \left(1 + \frac{\delta^{NP}}{m_b^2}\right)$$

$$\delta^{NP} = \tfrac{1}{2}\lambda_1 - \tfrac{9}{2}\lambda_2$$

$\Rightarrow \quad BR(B \to X_s\gamma)$: $\sim 1\%$

- Interference terms with the operator O_2:



$$\frac{\Gamma^{(\tilde{O},O_7)}_{B\to X_s\gamma}}{\Gamma^{LL}_{b\to s\gamma}} = -\frac{1}{9}\frac{C_2}{C_7}\frac{\lambda_2}{m_c^2} \simeq +0.03 \quad .$$

Expansion parameter is $m_b\Lambda_{QCD}/m_c^2 \approx 0.6$
However: higher order terms are indeed suppressed !

- Systematic analysis of terms like $\Gamma^{(O_2,O_2)}{}_{B\to X_s\gamma}$ is missing !

MSSM

- Global fit to electroweak data: SM $\leftrightarrow$ MSSM
- Susy contributions decouple

- Precise mechanism of necessary soft Susy breaking is unknown $\rightarrow$ proliferation of free parameters (uMSSM)

- In the MSSM two kinds of new contributions to flavour violation
  - CKM induced contributions from H^+, χ^+ exchanges
  - flavour mixing in the sfermion mass matrix

$\Rightarrow$ Supersymmetric flavour problem

Dynamics of flavour $\leftrightarrow$ Mechanism of SUSY breaking

Experimental question !

$\Rightarrow$ Model-independent analysis provides important hints on the more fundamental theory of soft SUSY breaking

Model-independent Analysis

- Gabbiani et al. (1996), Hagelin et al. (1994)
  - gluino- and photino-mediated Susy contributions to FCNC phenomena
  - single-mass-insertion method
  - no QCD corrections included

  $\rightarrow$ order-of-magnitude bounds on soft parameters

- Borzumati, Greub, H., Wyler (1999)

  Gluino-mediated contributions to $b \rightarrow s\gamma$
  - complete LL-QCD corrections included (new operators induced !)
  - general analysis beyond one-mass insertion
  - new experimental bounds on $b \rightarrow s\gamma$
  - SM contribution included

- Greub et al. (work in progress)

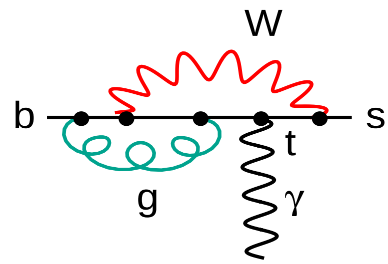
  Interference effects with chargino contribution

  Complete NLL-QCD corrections in the uMSSM

QCD Corrections in Rare B Decays

- Rate enhancement by QCD effects by more than two times within the SM

$\alpha_s(M_W) Log(\frac{m_b^2}{M_W^2})$



$\longrightarrow$ Resummation of Logs necessary:

LL	Leading Logs	$G_F\,(\alpha_s\,Log\,)^N$	$(N = 0, 1..)$
NLL	Next-to-Leading Logs	$G_F\,\alpha_s\,(\alpha_s\,Log\,)^N$	

- Appropriate theoretical framework

$H_{eff}(b \to s\gamma) = -\frac{4G_F}{\sqrt{2}}\,V_{bt}V_{st}^*\,\sum_{i=1}^{8} C_i(\mu)\,\hat{O}_i(\mu)$

$\hat{O}_1 - \hat{O}_6$	vectorial four-quark operators (Type II)
$\hat{O}_7, \hat{O}_8$	dipole operators

- Within SM all contributions $\sim G_F\;V_{tb}V_{ts}^*$

$\longrightarrow$ LL, NLL ordering reflects the actual size of the contributions

- In the MSSM different sources of flavour violation, various couplings

$\longrightarrow$ LL, NLL ordering ?

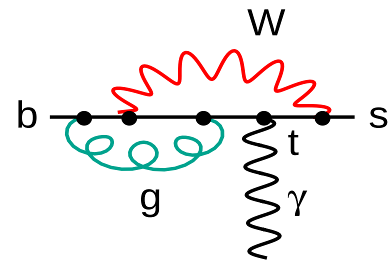
Particularly gluino-quark-squark coupling $\sim g_s$

$\longrightarrow$ O_7, O_8 type operators $\sim \alpha_s$
. New four-quark operators $\sim \alpha_s^2$

Formally:

LL: $\alpha_s(\alpha_s Log)^N$, NLL: $\alpha_s\alpha_s(\alpha_s Log)^N$

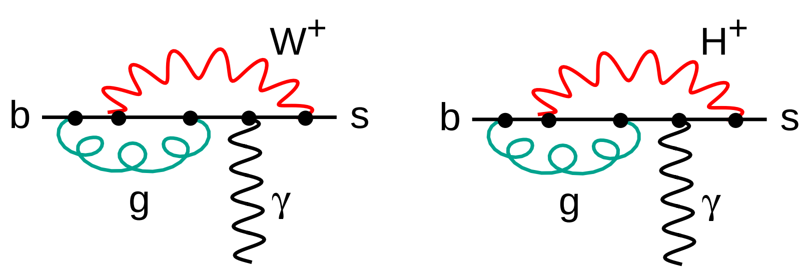
NLL Contributions to $B \to X_s\gamma$



- NLL calculation within SM
  - NLL corrections to matrix elements
    -two-loop virtual corrections (Greub, H., Wyler)
    -one-loop Bremsstrahlung (Ali, Greub; Pott)
    $\Rightarrow$ $\sim +25\%$ shift of the central value
    $\Rightarrow$ scale dependence reduced: $\sim 22\% \to \sim 5\%$
  - NLL corrections to Wilson coefficients
    -two-loop matching conditions (Adel, Yao; Greub, H.)
    -three-loop anomalous dimens. (Chetyrkin, Misiak, Münz)
    $\Rightarrow$ $\sim -4\%$ shift of the central value
  - Two-loop electroweak corrections
    (Czarnecki, Marciano; Strumia; Haisch, Gambino)
    $\Rightarrow$ less than 2% in the branching ratio

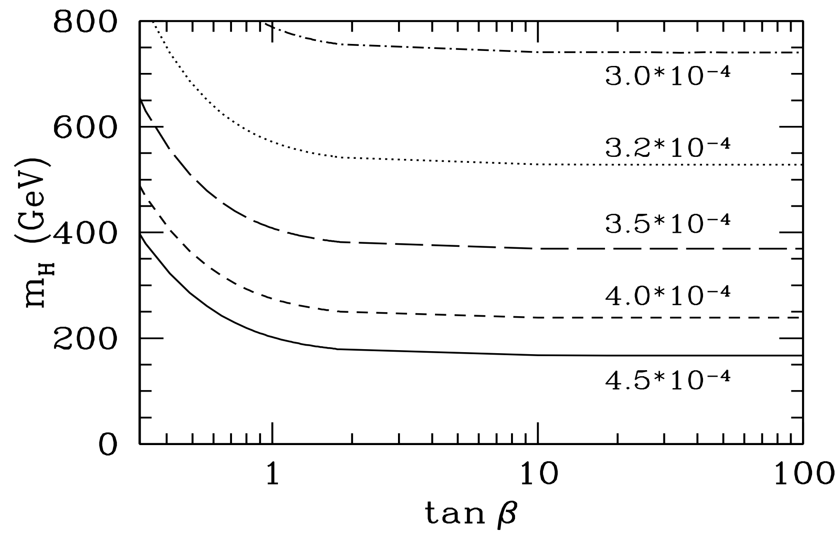
$$BR(B \to X_s\gamma) = (3.32 \pm 0.22 \pm 0.26) \times 10^{-4}$$

- NLL Calculation within 2HDM
  (Ciuchini,Degrassi,Gambino,Guidice; Greub, Borzumati)



$$C_{NLL}(M_W) = C_{NLL}{}^{SM}(M_W) + C_{NLL}{}^{NEW}(M_W)$$

• 2HDM 'II' (→ MSSM): $m_{H^+} \geq 165 GeV$



⇒ Bounds are rather sensitive to QCD and electroweak corrections

Multi Higgs Doublet Models

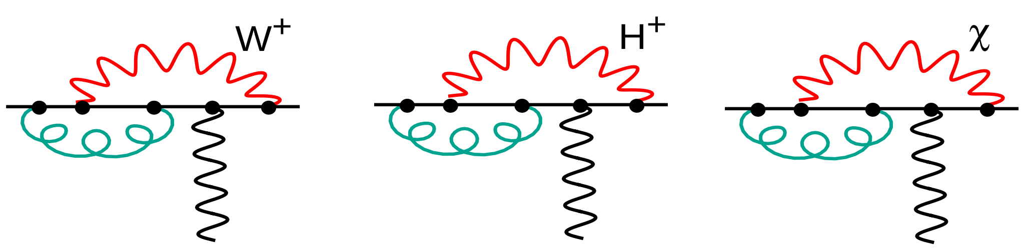
⇒ The truncation of the perturbative series at the NLL level is often not appropriate.

• Susy scenario including chargino contribution (Ciuchini, Degrassi, Gambino, Guidice)

a) Minimal flavour violation b) Heavy gluino limit:

• $\mu_{\tilde{g}} \sim O(m_{\tilde{g}}, m_{\tilde{q}}, m_{\tilde{t}_1}) \gg \mu_W \sim O(m_W, m_{H^+}, m_t, m_\chi, m_{\tilde{t}_2})$

• NLL QCD corrections up to first order in $\mu_W/\mu_{\tilde{g}}$.



$$C_{NLL}(M_W) = C_{NLL}{}^{SM}(M_W) + C_{NLL}{}^{H^+}(M_W) + C_{NLL}{}^{\chi}(M_W)$$

NLL QCD contributions in uMSSM

- Complete NLL:

  $W\, g \quad H^+ g \quad \chi^+ g \quad \chi^0 g \quad \tilde{g}\, g$ (Bobeth, Misiak, Urban)

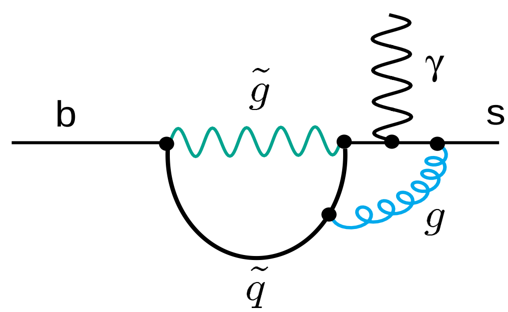
  $W\, \tilde{g} \quad H^+ \tilde{g} \quad \chi^+ \tilde{g} \quad \chi^0 \tilde{g} \quad \tilde{g}\, \tilde{g}$

  Minimal flavour violation, heavy gluino:

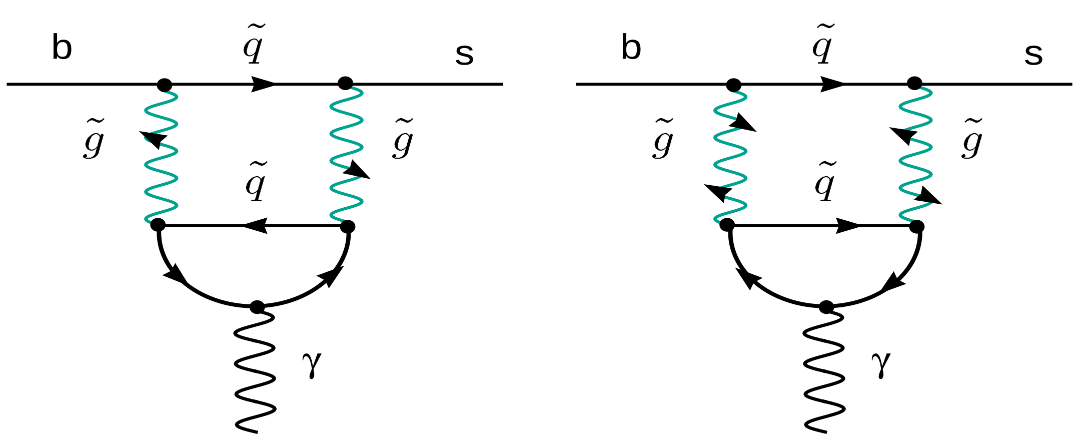
  $m_{\tilde{g}} = 700 GeV, m_{H^+} = 100 GeV, m_{\chi^+} = 140 GeV, tan\beta = 3$

  $\Rightarrow -17\%$ corrections due to gluonic SUSY contributions

- Gluonic Parts ($\tilde{g}\, g$)



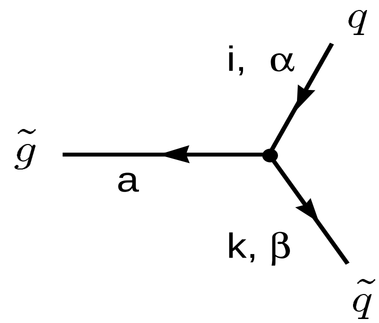
- Two-Gluino Parts ($\tilde{g}\, \tilde{g}$)



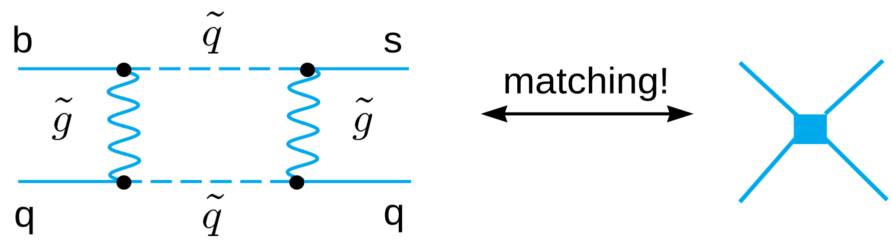
Gluino Contribution to $B \to X_s\gamma$

- Gluino-quark-squark coupling

$$-ig_s T^a_{\beta\alpha}(\Gamma^{ki}_{QL} P_L - \Gamma^{ki}_{QR} P_R)$$



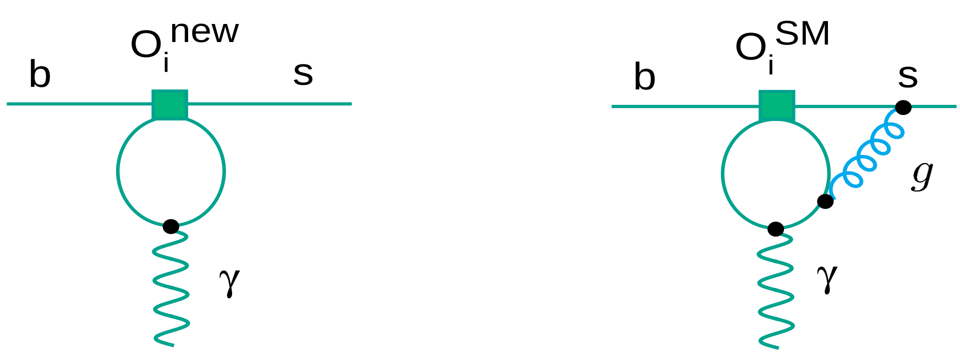
- New operators



$\Rightarrow$ scalar or tensorial $\hat{O}_i^{NEW}$ ("Type I") of order α_s^2

Also magnetic operators with $m_{\tilde{g}}$ instead of m_b

- Mixing at the one-loop level



$$Type\,I \times \alpha_1 \leq Type\,II \times \alpha_2$$

LL NLL

Ordering of the QCD Perturbative Expansion

- Formally:

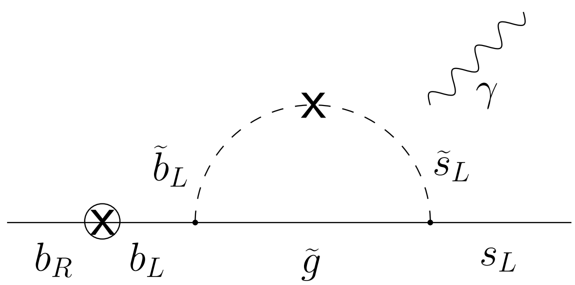
$$\text{LL: } \alpha_s(\alpha_s Log)^N, \qquad \text{NLL: } \alpha_s\alpha_s(\alpha_s Log)^N$$

- Canonical Form of anomalous dimension matrix assured by suitable definition of operators:

$$\gamma_{ji,\tilde g}(\mu) = \frac{\alpha_s(\mu)}{4\pi}\,\gamma^0_{ji,\tilde g} + \frac{\alpha_s^2(\mu)}{(4\pi)^2}\,\gamma^1_{ji,\tilde g} + \ldots.. ,$$

$$\mathcal{H}^{\tilde g}_{eff} = \sum_i C_{i,\tilde g}(\mu)\mathcal{O}_{i,\tilde g}(\mu) + \sum_i\sum_q C^q_{i,\tilde g}(\mu)\mathcal{O}^q_{i,\tilde g}(\mu)\,.$$

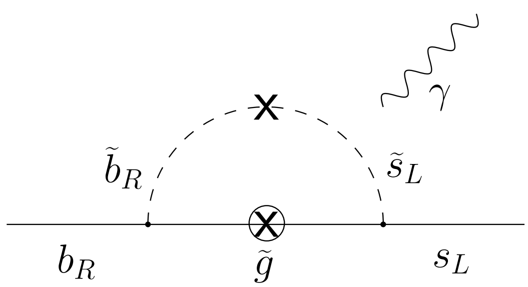
- Magnetic operators, with chirality violation signalled by the presence of the b-quark mass m_b:

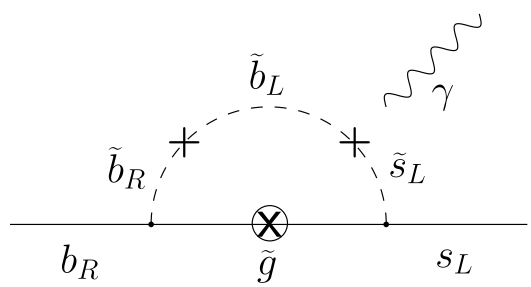


$$\mathcal{O}_{7b,\tilde g} = \; e g_s^2(\mu)\overline{m}_b(\mu)(\bar s\sigma^{\mu\nu}P_R b)F_{\mu\nu}, \qquad \mathcal{O}'_{7b,\tilde g} = \; e g_s^2(\mu)\overline{m}_b(\mu)(\bar s\sigma^{\mu\nu}P_L b)F_{\mu\nu},$$

$$\mathcal{O}_{8b,\tilde g} = \; g_s(\mu) g_s^2(\mu)\overline{m}_b(\mu)(\bar s\sigma^{\mu\nu}T^a P_R b)G^a_{\mu\nu}, \qquad \mathcal{O}'_{8b,\tilde g} = \; g_s(\mu) g_s^2(\mu)\overline{m}_b(\mu)(\bar s\sigma^{\mu\nu}T^a P_L b)G^a_{\mu}$$

- Magnetic operators in which the chirality-violating parameter is the gluino mass $m_{\tilde g}$:





$$\mathcal{O}_{7\tilde g,\tilde g} = \; e g_s^2(\mu)(\bar s\sigma^{\mu\nu}P_R b)\,F_{\mu\nu}\,, \qquad \mathcal{O}'_{7\tilde g,\tilde g} = \; e g_s^2(\mu)\,(\bar s\sigma^{\mu\nu}P_L b)F_{\mu\nu},$$

$$\mathcal{O}_{8\tilde g,\tilde g} = \; g_s(\mu) g_s^2(\mu)(\bar s\sigma^{\mu\nu}T^a P_R b)G^a_{\mu\nu}, \qquad \mathcal{O}'_{8\tilde g,\tilde g} = \; g_s(\mu) g_s^2(\mu)(\bar s\sigma^{\mu\nu}T^a P_L b)G^a_{\mu\nu}.$$

• Four-quark operators with vector Lorentz structure (Type II):

$$
\begin{aligned}
\mathcal{O}^{q}_{11,\tilde{g}} &= g_s^4(\mu)(\bar{s}\gamma_\mu P_L b)(\bar{q}\gamma^\mu P_L q), & \mathcal{O}^{q\prime}_{11,\tilde{g}} &= g_s^4(\mu)(\bar{s}\gamma_\mu P_R b)(\bar{q}\gamma^\mu P_R q),\\
\mathcal{O}^{q}_{12,\tilde{g}} &= g_s^4(\mu)(\bar{s}_\alpha\gamma_\mu P_L b_\beta)(\bar{q}_\beta\gamma^\mu P_L q_\alpha), & \mathcal{O}^{q\prime}_{12,\tilde{g}} &= g_s^4(\mu)(\bar{s}_\alpha\gamma_\mu P_R b_\beta)(\bar{q}_\beta\gamma^\mu P_R q_\alpha),\\
\mathcal{O}^{q}_{13,\tilde{g}} &= g_s^4(\mu)(\bar{s}\gamma_\mu P_L b)(\bar{q}\gamma^\mu P_R q), & \mathcal{O}^{q\prime}_{13,\tilde{g}} &= g_s^4(\mu)(\bar{s}\gamma_\mu P_R b)(\bar{q}\gamma^\mu P_L q),\\
\mathcal{O}^{q}_{14,\tilde{g}} &= g_s^4(\mu)(\bar{s}_\alpha\gamma_\mu P_L b_\beta)(\bar{q}_\beta\gamma^\mu P_R q_\alpha), & \mathcal{O}^{q\prime}_{14,\tilde{g}} &= g_s^4(\mu)(\bar{s}_\alpha\gamma_\mu P_R b_\beta)(\bar{q}_\beta\gamma^\mu P_L q_\alpha),
\end{aligned}
$$

• Four-quark operators with scalar and tensor Lorentz structure (Type I):

$$
\begin{aligned}
\mathcal{O}^{q}_{15,\tilde{g}} &= g_s^4(\mu)(\bar{s}P_R b)(\bar{q}P_R q), & \mathcal{O}^{q\prime}_{15,\tilde{g}} &= g_s^4(\mu)(\bar{s}P_L b)(\bar{q}P_L q),\\
\mathcal{O}^{q}_{16,\tilde{g}} &= g_s^4(\mu)(\bar{s}_\alpha P_R b_\beta)(\bar{q}_\beta P_R q_\alpha), & \mathcal{O}^{q\prime}_{16,\tilde{g}} &= g_s^4(\mu)(\bar{s}_\alpha P_L b_\beta)(\bar{q}_\beta P_L q_\alpha),\\
\mathcal{O}^{q}_{17,\tilde{g}} &= g_s^4(\mu)(\bar{s}P_R b)(\bar{q}P_L q), & \mathcal{O}^{q\prime}_{17,\tilde{g}} &= g_s^4(\mu)(\bar{s}P_L b)(\bar{q}P_R q),\\
\mathcal{O}^{q}_{18,\tilde{g}} &= g_s^4(\mu)(\bar{s}_\alpha P_R b_\beta)(\bar{q}_\beta P_L q_\alpha), & \mathcal{O}^{q\prime}_{18,\tilde{g}} &= g_s^4(\mu)(\bar{s}_\alpha P_L b_\beta)(\bar{q}_\beta P_R q_\alpha),\\
\mathcal{O}^{q}_{19,\tilde{g}} &= g_s^4(\mu)(\bar{s}\sigma_{\mu\nu} P_R b)(\bar{q}\sigma^{\mu\nu} P_R q), & \mathcal{O}^{q\prime}_{19,\tilde{g}} &= g_s^4(\mu)(\bar{s}\sigma_{\mu\nu} P_L b)(\bar{q}\sigma^{\mu\nu} P_L q),\\
\mathcal{O}^{q}_{20,\tilde{g}} &= g_s^4(\mu)(\bar{s}_\alpha\sigma_{\mu\nu} P_R b_\beta)(\bar{q}_\beta\sigma^{\mu\nu} P_R q_\alpha), & \mathcal{O}^{q\prime}_{20,\tilde{g}} &= g_s^4(\mu)(\bar{s}_\alpha\sigma_{\mu\nu} P_L b_\beta)(\bar{q}_\beta\sigma^{\mu\nu} P_L q_\alpha),
\end{aligned}
\quad (1
$$

• Magnetic operators, with chirality violation signalled by the presence of the c-quark mass m_c :

$$
\begin{aligned}
\mathcal{O}_{7c,\tilde{g}} &= e g_s^2(\mu)\overline{m}_c(\mu)(\bar{s}\sigma^{\mu\nu}P_R b)F_{\mu\nu}, & \mathcal{O}'_{7c,\tilde{g}} &= e g_s^2(\mu)\overline{m}_c(\mu)(\bar{s}\sigma^{\mu\nu}P_L b)F_{\mu\nu},\\
\mathcal{O}_{8c,\tilde{g}} &= g_s(\mu) g_s^2(\mu)\overline{m}_c(\mu)(\bar{s}\sigma^{\mu\nu}T^a P_R b)G^a_{\mu\nu}, & \mathcal{O}'_{8c,\tilde{g}} &= g_s(\mu) g_s^2(\mu)\overline{m}_c(\mu)(\bar{s}\sigma^{\mu\nu}T^a P_L b)G^a_{\mu
\end{aligned}
$$

LL-Analysis introduces already the complete operator basis !

Sfermion Mass Matrix

- Gluino-quark-squark coupling in mass eigenstates $\tilde{q}_i \, (i = 1, .., 6)$

$$-i g_s T^a_{\beta\alpha} (\Gamma^{ki}_{QL} P_L - \Gamma^{ki}_{QR} P_R)$$

- Super KM basis $\tilde{q}_{L,R\,j}$ $(j = 1, 2, 3)$
  - quark mass matrices are diagonal !
  - squarks are rotated 'parallel' to their fermionic superpartners !
  - in general not mass eigenstates: $\tilde{q}_{L,R} = \Gamma^{+}_{QL,R} \tilde{q}_i$

Sfermion mass matrix in uMSSM in $\tilde{q}_{L,R}$ basis:

$$\mathcal{M}^2_D = (F/D)^D_{6\times 6} + \begin{pmatrix} m^2_{Q,LL} & m^2_{D,LR} \\ m^2_{D,RL} & m^2_{D,RR} \end{pmatrix}$$

$$\mathcal{M}^2_U = (F/D)^U_{6\times 6} + \begin{pmatrix} m^2_{Q,LL} & m^2_{U,LR} \\ m^2_{U,RL} & m^2_{U,RR} \end{pmatrix}$$

from F, D terms	from soft breaking
3×3 diagonal submatrices	m^2_i not diagonal

All neutral gaugino couplings are flavour diagonal!

$\Rightarrow$ FCNC are induced by off-diagonal (off-generational) terms in $m^2_{LL}, m^2_{RR}, m^2_{LR}$

Numerical Analysis

- One-mass insertion approximation

$$\delta_{LR,\,23} = m^2_{LR,\,23}/\tilde{m}^2$$

Sfermion propagator can be expanded as a series in terms of δ, where $\tilde{m}^2$ is an average mass.

  - $x := m^2_{\tilde{g}}/m^2_{\tilde{q}},\ \ m_{\tilde{q}} = 500GeV$
  - $\delta_{LR,23}$
  - $\delta_{LL,23}$

- Input parameters:

$tan\beta = 2$, $m_b = 3GeV$, $m_t = 175GeV, m_Z = 91.18GeV$, $sin^2(\theta_W) = 0.2316, m_{\tilde{q}} = 500GeV$

- Direct comparison (only gluino): $\delta_{LR,23},\ 10^{-2}$

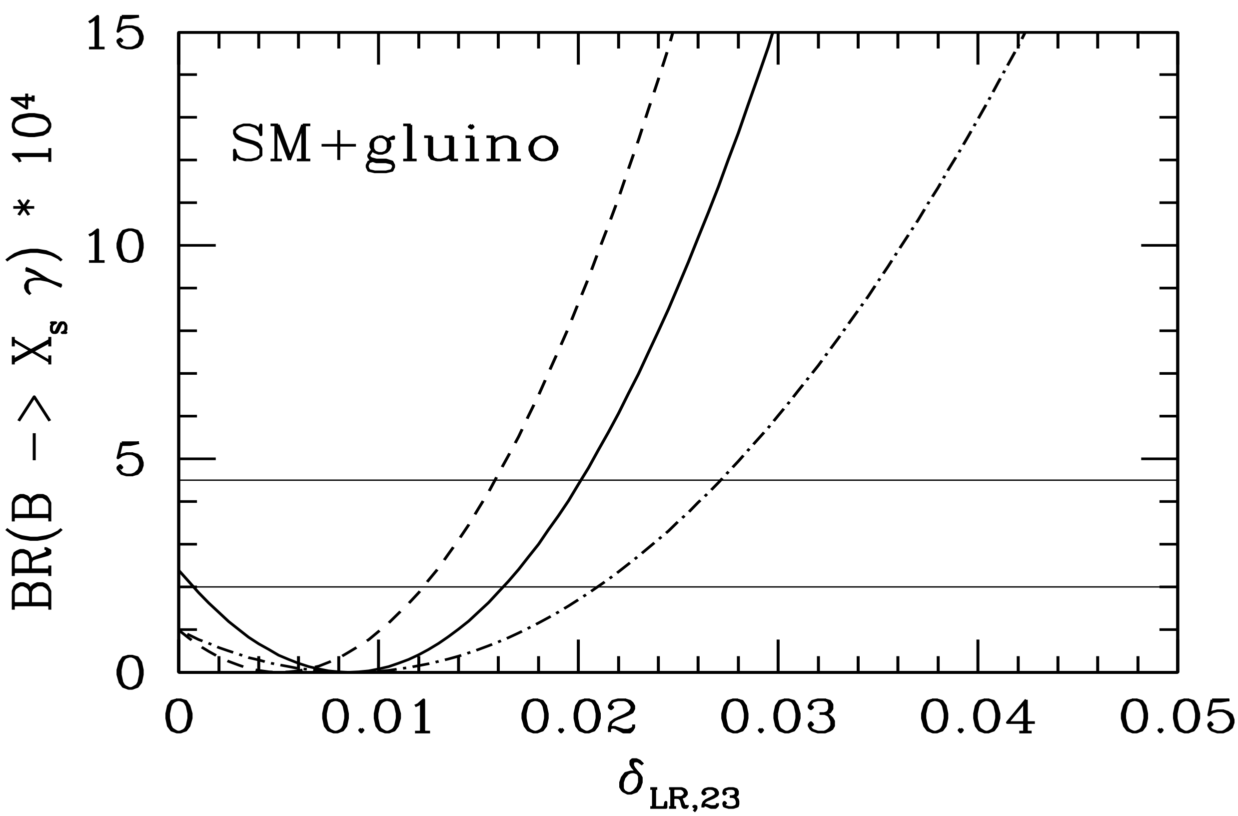
x	Gab. et al.	no QCD	with QCD
0.3	1.8	1.9	1.2
1.0	2.3	2.2	1.7
4.0	4.2	4.3	3.6

- $\alpha_s \leftrightarrow G_F$

- $\Gamma^{kb}_{DL}\,\Gamma^{*\,ks}_{DL},\ \Gamma^{kb}_{DR}\,\Gamma^{*,ks}_{DL} \leftrightarrow V_{tb}\,V^*_{ts}$

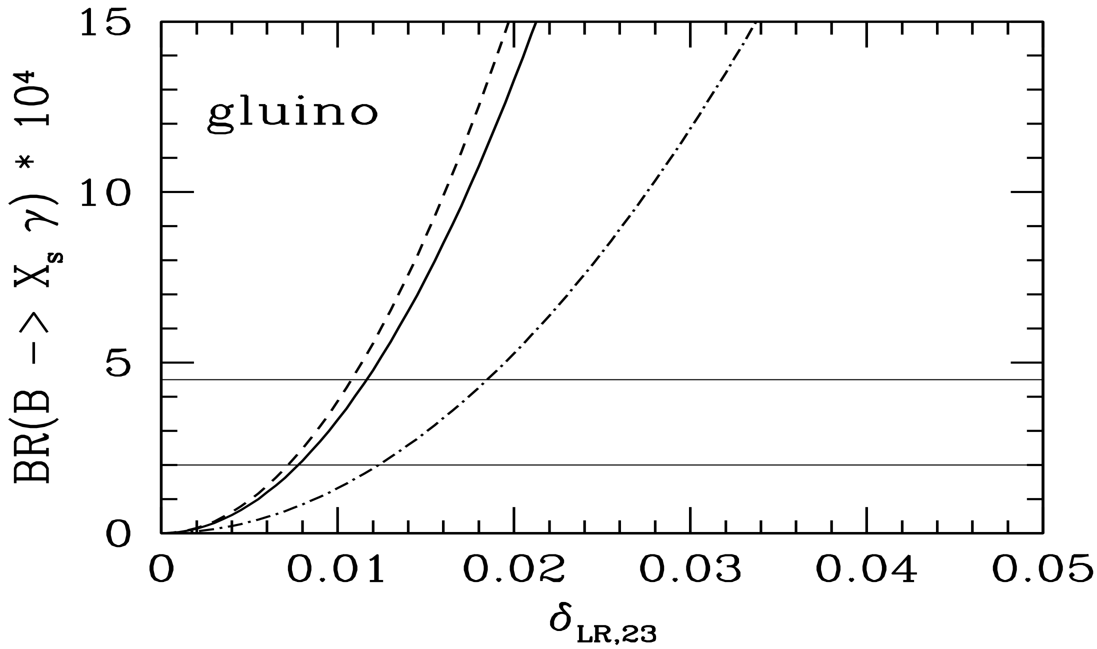
Bounds on the Sfermion Mass Matrix

$\delta_{LR,23}$



The size of LL QCD corrections to $BR(B \to X_s\gamma)$
for $x = 0.3$ as a function of $\delta_{LR,23}$.

Solid line:
includes LL QCD corrections systematically.
Dashed line:
no QCD corrections to the Wilson coefficients;
explicit α_s factor taken at scale $\mu = m_b$.
Dashed-dotted line:
no QCD corrections to the Wilson coefficients;
explicit α_s factor taken at scale $\mu = m_W$.



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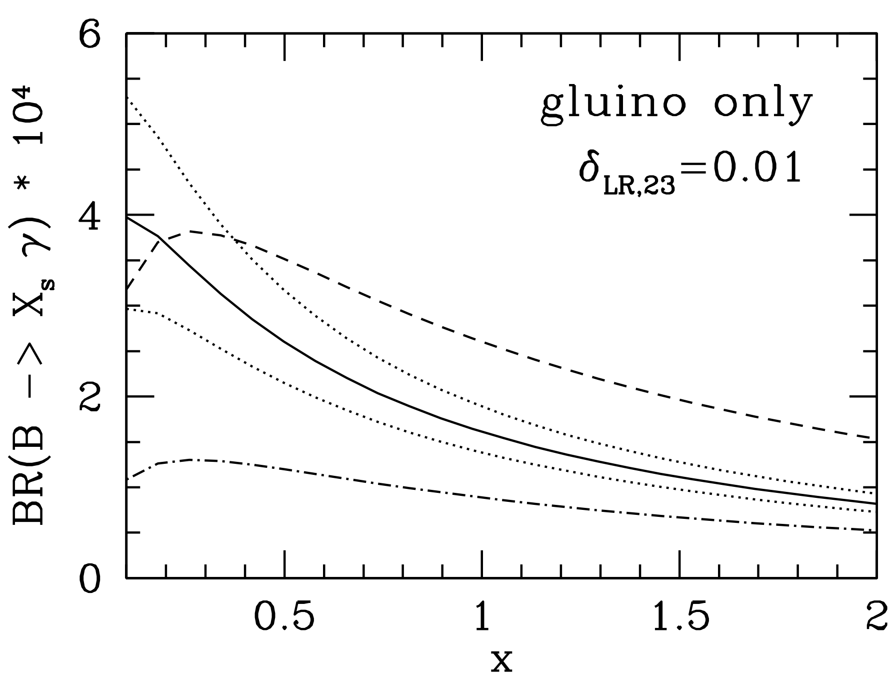
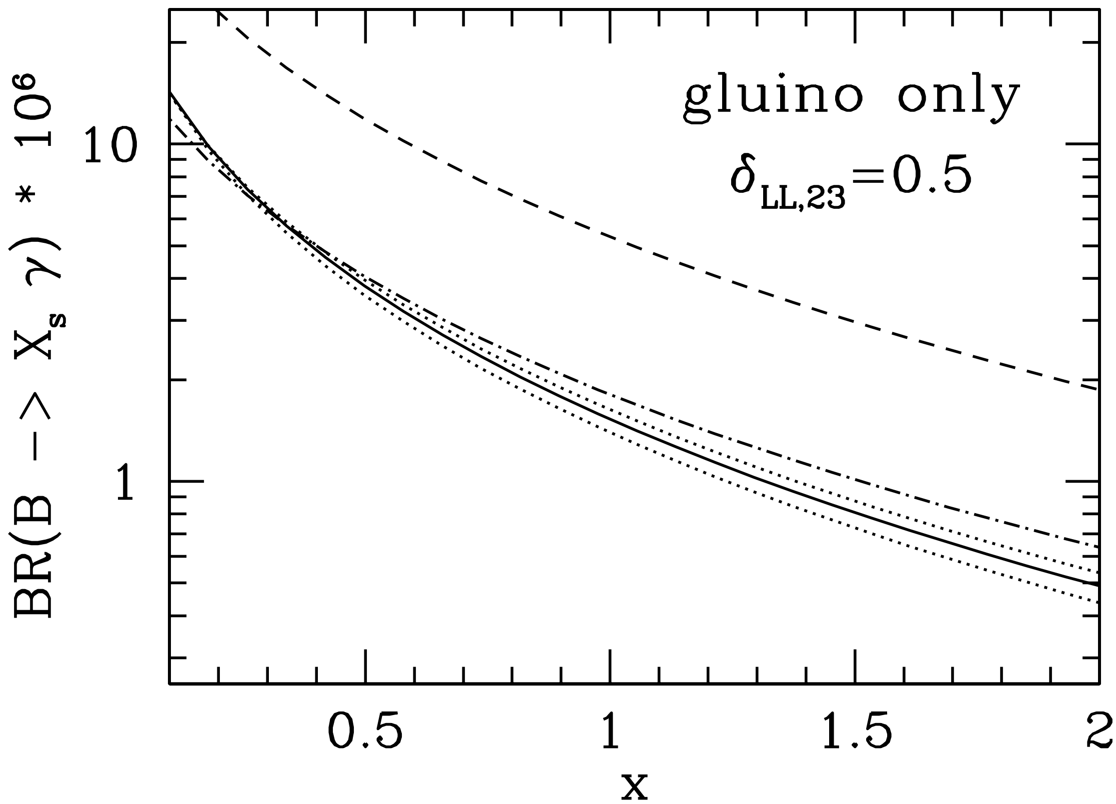


$BR(B \to X_s\gamma)$ as a function of $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$; $\delta_{LR,23} = 0.01$; $m_{\tilde{q}} = 500$ GeV.

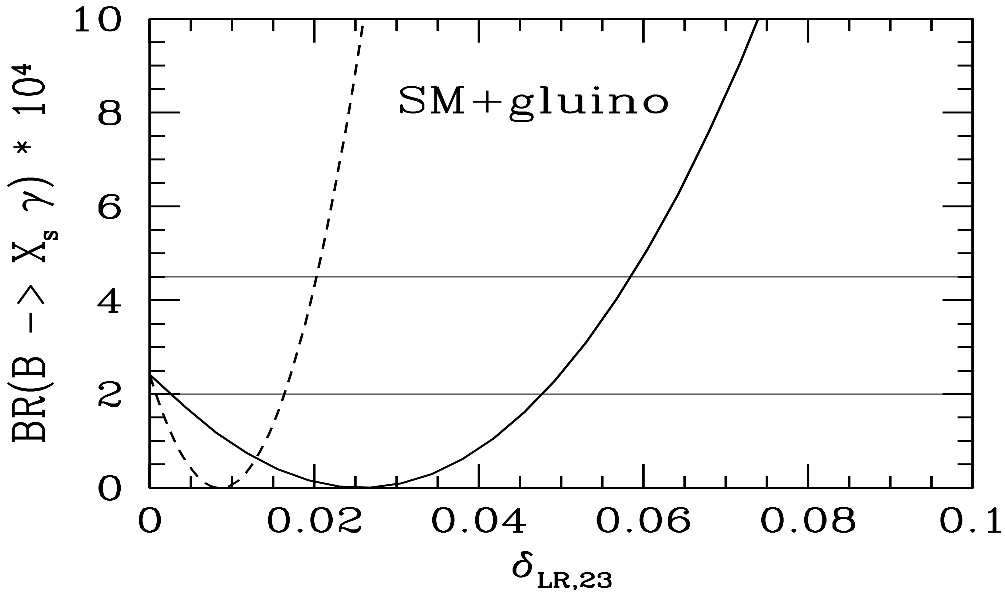
$\delta_{LL,23}$



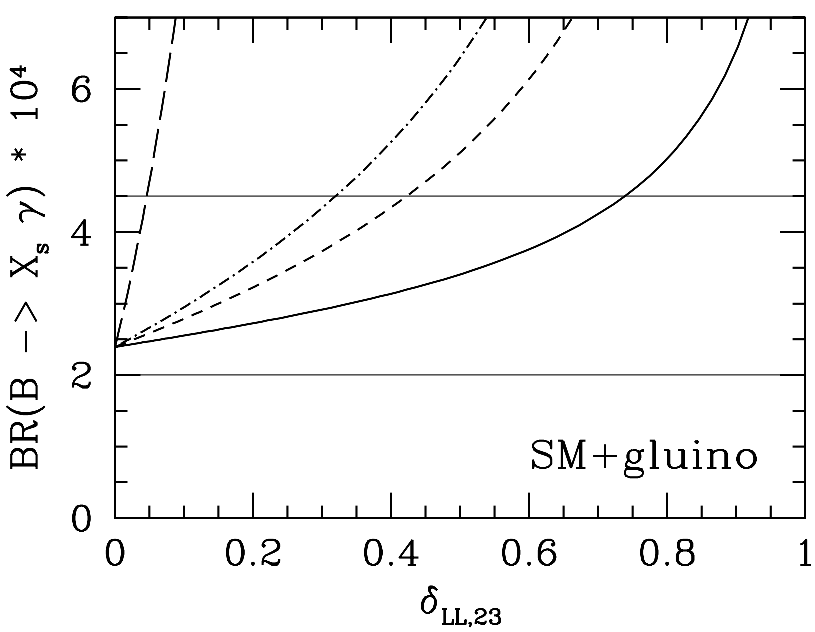
$BR(B \to X_s\gamma)$ as a function of $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$,
only $\delta_{LL,23}$ is non-vanishing and fixed to the value 0.5.

The solid line shows the branching ratio at the LO in QCD, for $\mu_b = 4.8\,\text{GeV}$.
The two dotted lines indicate the range of variation of the branching ratio when μ_b spans the interval 2.4–9.6 GeV.
Dashed line: no QCD corrections to the Wilson coefficients; explicit α_s factor taken at scale $\mu = m_b$.
Dashed-dotted line: no QCD corrections to the Wilson coeffs.; explicit α_s factor taken at scale $\mu = m_W$.

Interference Effects



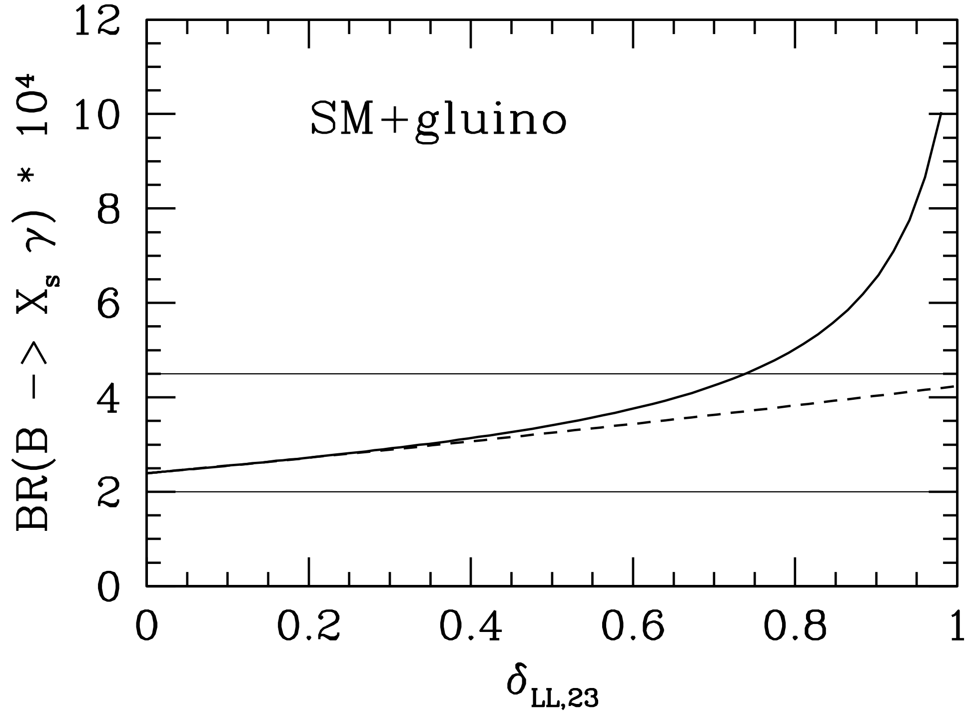
$BR(B \to X_s\gamma)$ as a function of $\delta_{LR;23}$ including interference effects with a chain $\delta_{LL;23}\delta_{LR;33}$ in solid line.

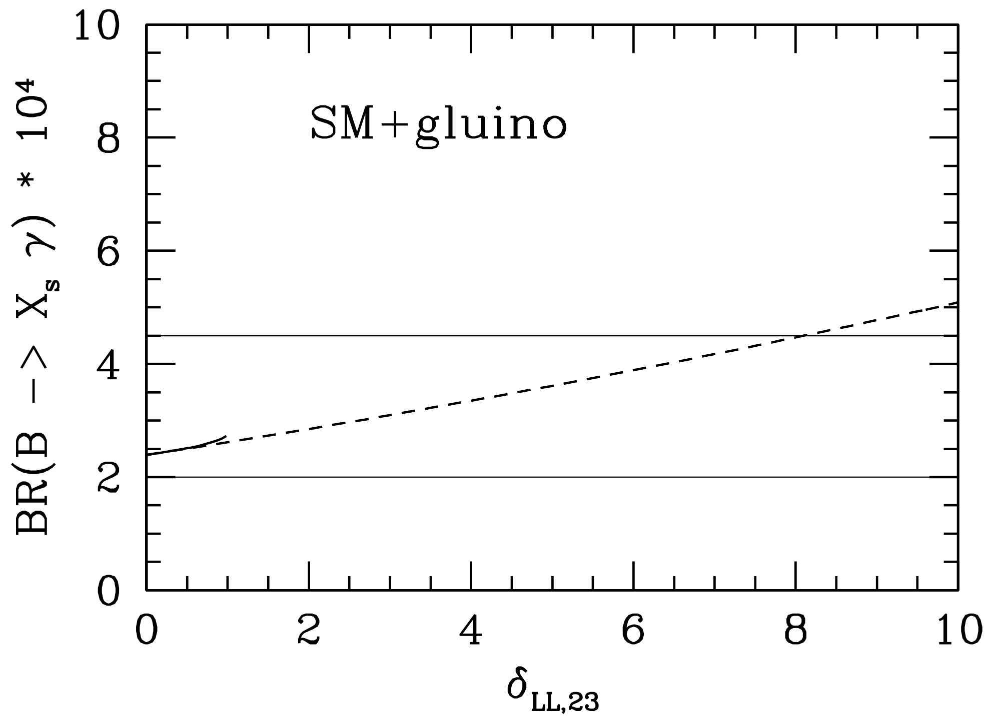


$BR(B \to X_s\gamma)$ vs. $\delta_{LL,23}$, when $\delta_{LL,23}$ and $\delta_{LR,33}$ are the only sources of chiral-flavour violation.

$\delta_{LR,33}$: 0 (solid line), 0.006 (short-dashed line), 0.01 (dot- dashed line), 0.1 (long-dashed line); $x = 0.3$ and $m_{\tilde{q}} = 500\,\text{GeV}$.

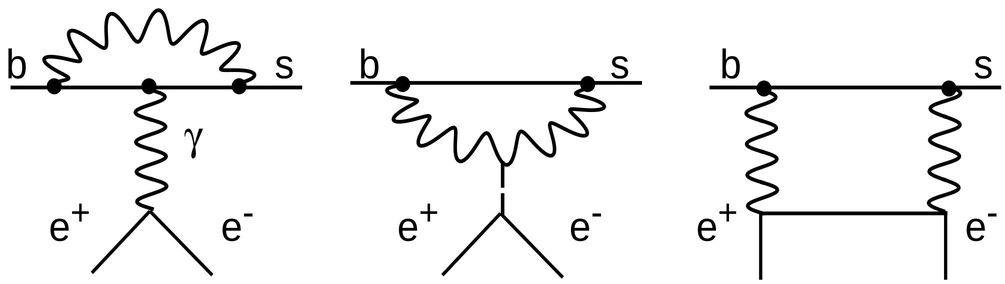
Mass-Insertion Approximation



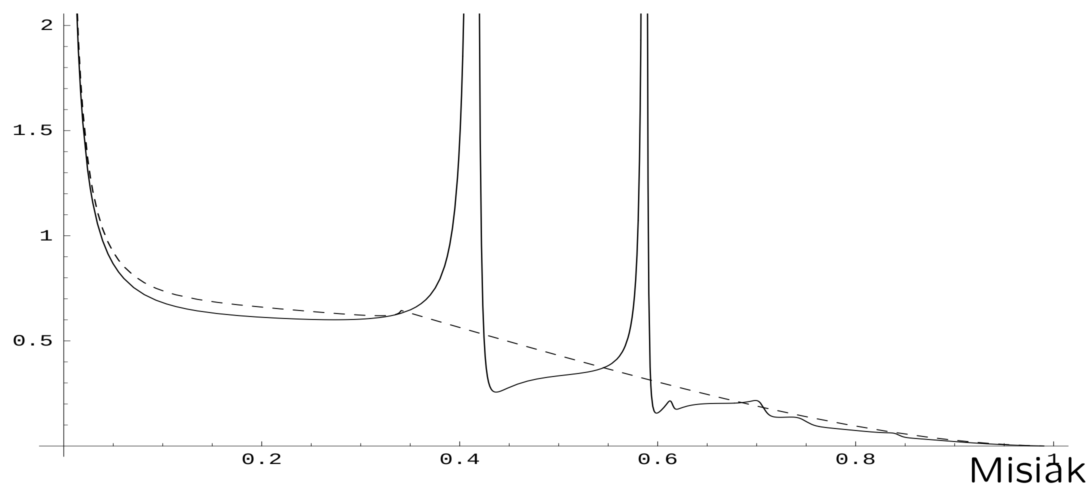


Mass insertion approximation (dashed line) vs. exact result (solid line) as a function of $\delta_{LL;23}$, $x = 0.3$ and $x = 4$.

The Decay $B \to X_s l^+ l^-$



- Nonperturbative contributions:
  - power corrections in $1/m_b^2$ and $1/m_c^2$ as in $B \to X_s \gamma$
  - on-shell $c\bar{c}$ resonances
    $\Rightarrow$ kinematical cuts in the dilepton mass spectrum:
    $0.05 < (m_{l^+l^-}/m_b)^2 < 0.25$



- Dominating short distance contribution:
  - Two additional operators induced:

$$\mathcal{O}_9 = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell), \quad \mathcal{O}_{10} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell).$$

  - $\mathcal{O}_2$ mixes into $\mathcal{O}_9$ at one loop:
    large logarithm $L = \log(m_b/M_W)$ without the exchange of gluons

$$\Rightarrow (LL)\, G_F\, (\alpha_s)^{-1} (\alpha_s L)^N; \quad (NLL)\, G_F\, (\alpha_s L)^N$$

– The LL term is accidentially quite small
$\Rightarrow$ only NNLL term leads to $\pm 10\%$ accuracy

* Wilson coefficients known to NNLL (Bobeth et al.)

$$C_9(\mu) = \frac{1}{\alpha_s(\mu)} C_9^{(-1)}(\mu) + C_9^{(0)} + \alpha_s(\mu) C_9^{(1)} + \ldots$$

* 2-loop matrix elements of the four-quark operators (NNLL) are unknown.

- Experiment:
$BR(B \to X_s \mu^+ \mu^-) < 5.8 \; 10^{-5}$ (CLEO)

- Theory:

$$BR(B \to X_s l^+ l^-)_{Cut:\ \hat{s} \in [0.05, 0.25]} =$$

$$= BR(B \to X_c e \overline{\nu}) \int_{Cut} d\hat{s} \left[R_{quark}^{l^+ l^-}(\hat{s}) + \delta_{1/m_b^2} R(\hat{s}) + \delta_{1/m_c^2} R(\hat{s}) \right]$$

$$= 0.104 \left[(1.36 \pm 0.18 \,(\text{only scale})\,) + 0.06 - 0.02 \right] 10^{-5}$$

$$= (1.46 \pm 0.19 \,(\text{only scale})\,) \, 10^{-6}$$

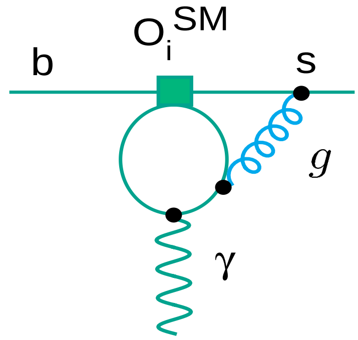
13 % perturbative uncertainty !

- Sensitivity to new Physics:
If $C_7^{eff}(m_b)$ changes sign: $BR_{\,cut}$: 1.5 $10^{-6} \to 3 \; 10^{-6}$

- Comparision with exclusive modes:
$BR(B \to K^* \mu^+ \mu^-) = 1.9 \, 10^{-6}$; $\Delta BR = (^{+26}_{-17}, \pm 6, ^{+6}_{-4}, ^{-0.7}_{+0.4}, \pm 2)\%$

Technical Details of the NLL QCD Calculation

2-loop matrix elements of the four-quark operators
(Greub, H., Wyler: $B \to X_s\gamma$ on-shell photon ($q^2 = 0$)):



- After Feynman parametrization

$$\Rightarrow \int_0^1 dxdydudv \, \frac{[x(1-x)]^{1-\epsilon}\, y^{\epsilon-1}\, [1-v]^{\epsilon}\, v}{C^{2\epsilon}} \, Poly(x,y,v,u)$$

$$C = m_b^2 v(1-v)u - m_c^2/(x(1-x))(1-v)y + i\delta$$

- Use Mellin Barnes representation of the propagator for C

$$\frac{1}{(k^2-M^2)^\lambda} = \frac{1}{(k^2)^\lambda}\frac{1}{\Gamma(\lambda)}\frac{1}{2\pi i}\int_\gamma (-\frac{M^2}{k^2})^s \Gamma(\lambda+s)\Gamma(-s)ds$$

($\lambda > 0$, γ parallel to the imaginary axis)

$$k^2 \leftrightarrow m_b^2 v(1-v)u \quad ; \quad M^2 \leftrightarrow \frac{m_c^2}{x(1-x)}(1-v)y \quad .$$

$$c_0 + \sum_{n,m} c_{nm}\, z^n \log^m z \quad , \quad z = \frac{m_c^2}{m_b^2}$$

- We retained all terms up to $m = 3$; truncating at $n = 2$ differs only by 1%

- No naked $log(m_c^2/m_b^2)$ which would diverge for $m_c = 0$. (general proof: Sterman et al.)

For the case $B \to X_s l^+ l^-$ the calculation has to be generalized to off-shell photons ($q^2 \neq 0$) !!

Model-independent Analysis of $B \to X_s l^+ l^-$ and $B \to X_s \gamma$

Conservative analysis:

Global fit to the Wilson coefficients C_7, C_9, C_{10}

(Ali et al., Hewett et al.)

- $\Gamma(B \to X_s \gamma)$
- $d\Gamma(B \to X_s l^+ l^-) \,/\, d\hat{s}$
  Invariant dilepton mass distribution
- $A(s) = \int_{-1}^{1} dcos\theta \; d^2\Gamma(B \to X_s l^+ l^-) \,/\, dsdcos\theta \;\; sgn(cos\theta)$
  Forward-Backward Charge Asymmetry

$\Rightarrow$ Determines magnitude + sign of C_7, C_8, C_{10}

(kinematical distributions: high statistics necessary!)

Precision test of new physics !

$B \to X_d\gamma$

- CKM suppression by the factor $|V_{td}|^2/|V_{ts}|^2$ in the SM may not be true in extended models.

- Effective Hamiltonian as in $B \to X_s\gamma$ up to $s \to d$ and $\lambda_u = V_{ub}V_{ud}^*$ relative to λ_t and λ_c not small.

- Power corrections in $1/m_b^2$ and in $1/m_c^2$ are the same as in $B \to X_s\gamma$ (besides CKM factors).
  But: long distance contributions from the u-quark loops are important. Can only modeled at present. No spurious enhancement of the form $ln(m_u/\mu)$ (Ricciardi; Ali, Braun).

- Much of the theoretical improvements carried out in the context of $B \to X_s\gamma$ can straightforwardly adapted for the decay $B \to X_d\gamma$:
  - reduction of scale dependence μ_b: $LL \to NLL$ 10 %
  - for fixed value of the CKM parameters:
    $\Delta BR(B \to X_d\gamma)/BR(B \to X_d\gamma) = \pm(6 - 10)\%$.

Theoretical uncertainty in $R(d\gamma/s\gamma)$ even smaller:

$$R(d\gamma/s\gamma) \equiv \frac{\mathcal{B}(B\to X_d\gamma)}{\mathcal{B}(B\to X_s\gamma)}$$

$\Rightarrow$ Future measurements of $R(d\gamma/s\gamma)$ will have a large impact on the CKM phenomenology !

- Theory:

  $-0.1 \le \rho \le 0.4,\ 0.2 \le \eta \le 0.46$
  + parametric dependences:

  $$6.0 \times 10^{-6} \le BR(B \to X_d\gamma) \le 2.6 \times 10^{-5}$$

  $$0.017 \le R(d\gamma/s\gamma) \le 0.074\ .$$

- Experiment: (90% CL)

  $BR(B^0 \to \rho^0\gamma) < 1.7 \cdot 10^{-5}$
  $BR(B^+ \to \rho^+\gamma) =< 1.1 \cdot 10^{-5}$
  $BR(B \to \rho\gamma)/BR(B \to K^*\gamma) < 0.32$
  $\to |V_{td}/V_{ts}| < 0.72$.

  (CLEO II, PRL 84 2000).

  $BR(B^0 \to \rho^0\gamma) =< 0.56 \cdot 10^{-5}$
  $BR(B^+ \to \rho^+\gamma) =< 2.27 \cdot 10^{-5}$
  $BR(B \to \rho\gamma)/BR(B \to K^*\gamma) < 0.28$

- Inclusive Rare B decays are
  - a laboratory for perturbative QCD and
  - a powerful tool for constraining new physics
- The constraints on the parameter space of various models are sensitive to NLL QCD corrections.
- The complete NLL QCD analysis is also necessary in order to ensure the validity of the RG-improved perturbation theory in the model under consideration.
- Status:
  - NLL prediction for $B \to X_s \gamma$:
    *good agreement with the CLEO, ALEPH and also with the new Belle measurement.
    *reduction of the model dependence necessary (shape of the photon spectrum $\leftrightarrow$ nonperturbative background)
    *matching conditions in the uMSSM missing
  - Inclusive $B \to X_s l^+ l^-$ decay:
    *equally sensitive to new physics as exclusive mode
    *can be predicted more precisely for small values of the dilepton mass ($\sim$ 10% instead of $\sim$ (20 – 30)%)
    *perturbative uncertainty of 13% in the NNLL prediction (two-loop matrix elements of the four quark operators).
  - Inclusive $B \to X_d \gamma$ decay:
    *large impact on CKM phenomenology
    *long distance contributions (up-quark)