

Carmel

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# The width difference of $B_s$ -mesons

Ulrich Nierste  
Fermilab

Work done in collaboration with  
Martin Beneke, Gerhard Buchalla,  
Christoph Greub and Alexander Lenz.

# Outline

1. Heavy Quark Expansion
2. Lifetime differences
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of  $B_S$ -mesons
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# 1. Heavy Quark Expansion

Voloshin, Shifman; Bigi, Uraltsev, Vainshtein

Optical theorem for total decay rate  $\Gamma$ :

$$\Gamma \propto \text{Im} \langle B | i \int d^4x T \mathcal{H}_{eff}(x) \mathcal{H}_{eff}(0) | B \rangle$$

$\mathcal{H}_{eff}$  is the effective  $|\Delta B| = 1$  hamiltonian.

HQE = Operator product expansion:

$$\Gamma \propto G_F^2 \sum_j m_b^{8-d_j} c_j (\mu/m_b) \underbrace{\langle B | \mathcal{O}_j(\mu) | B \rangle}_{\mathcal{O} \left( \Lambda_{QCD}^{d_j-3} \right)}$$

$c_j$ : Wilson coefficients containing physics from scales  $\geq \mu = \mathcal{O}(m_b)$

$\mathcal{O}_j$ : local operators with dimension  $d_j \geq 3$ .

Effect: Expansion of  $\Gamma$  in  $\Lambda_{QCD}/m_b$  and  $\alpha_s(m_b)$ .

$$\Gamma(\mathbf{B} \rightarrow all) = \Gamma(\mathbf{b} \rightarrow all) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{m_b^2}\right)$$

First term: QCD corrected **parton model**.

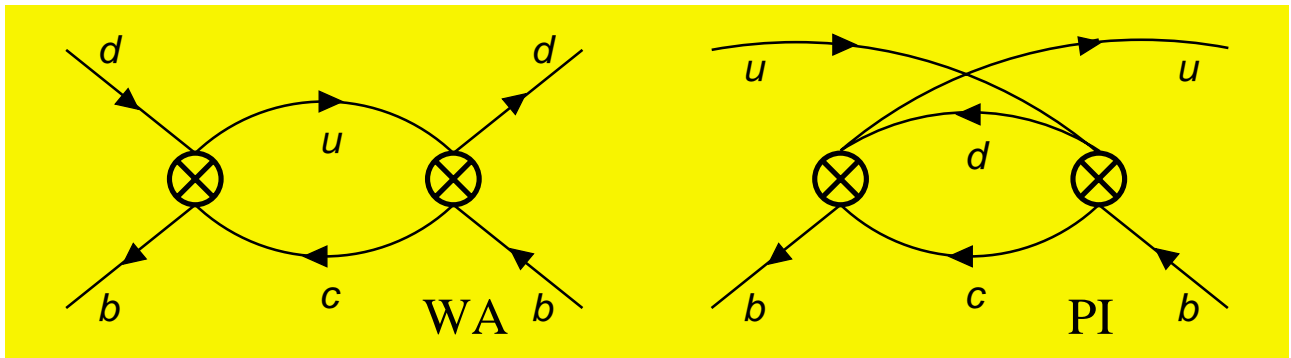
First corrections are  $\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{m_b^2}\right)$  and come from **Fermi motion** of the b-quark and the **chromomagnetic interaction** with the light degrees of freedom.

Validity of HQE  $\Leftrightarrow$  **Quark-hadron duality**

## 2. Lifetime differences

Dominant source of lifetime differences between  $B_d$ ,  $B_s$  and  $B^\pm$  mesons: Participation of the spectator quark in the weak decay. Effect of order  $\mathcal{O}\left(16\pi^2\frac{\Lambda_{\text{QCD}}^3}{m_b^3}\right)$  (Exception:  $\tau(B_s) - \tau(B_d)$  stems from  $SU(3)_F$  breaking in  $\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{m_b^2}\right)$  matrix elements.)

$B_d - B^\pm$  lifetime difference:



Bigi, Shifman, Uraltsev, Vainshtein  
Neubert, Sachrajda

Lifetime difference of  $B_s$  mesons:

$$B_s \sim \bar{b}s \quad \bar{B}_s \sim b\bar{s}$$

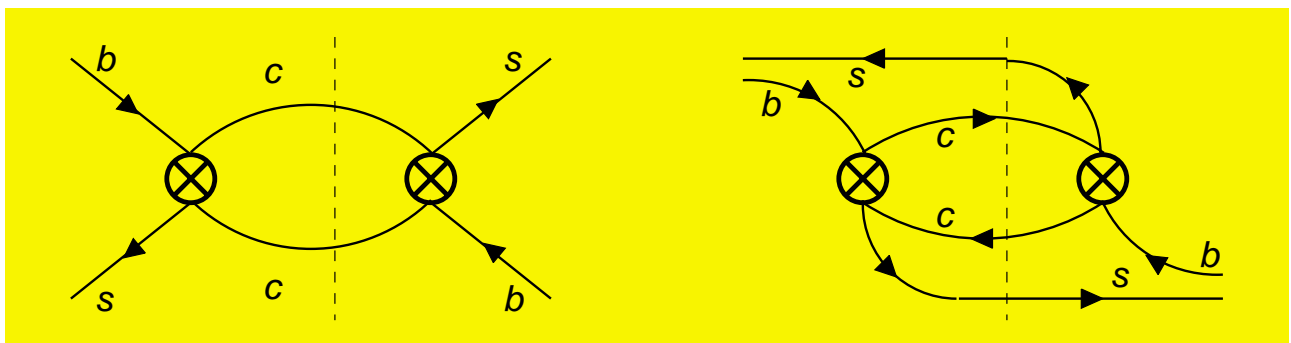
Standard Model: Negligible CP-violation in  $B_s$ - $\bar{B}_s$ -mixing:

$$|B_{L,H}\rangle = \frac{1}{\sqrt{2}} \left[ |B_s\rangle \mp |\bar{B}_s\rangle \right]$$

Width difference

$$\begin{aligned} \Delta\Gamma_{B_s} &\equiv \Gamma_L - \Gamma_H \\ &= -\frac{1}{M_{B_s}} \text{Im} \langle \bar{B}_s | i \int d^4x T \mathcal{H}_{eff}(x) \mathcal{H}_{eff}(0) | B_s \rangle \end{aligned}$$

from final states common to  $B_s$  and  $\bar{B}_s$



Measurement at Tevatron Run-II: Compare average  $B_s$  lifetime  $\tau(B_s)$  measured in  $B_s \rightarrow D_s^- \pi^+$  with  $\tau(B_{s,L}) = 1/\Gamma_L$  measured in  $B_s \rightarrow \psi\phi$  (CP-even component).

Compare:

$$\tau(B^+)/\tau(B_d):$$

insensitive to new physics  $\Rightarrow$  tests HQE

$$\tau(B_s)/\tau(B_d) \simeq 1 \pm \mathcal{O}(1\%) \text{ (in Standard Model):}$$

mildly sensitive to new physics in penguin coefficients

Keum, U.N.

$$\tau(B_{s,L})/\tau(B_{s,H}):$$

New CP-violating physics in  $B_S-\bar{B}_S$ -mixing can suppress  $\Delta\Gamma_{B_s}$  below its SM value.

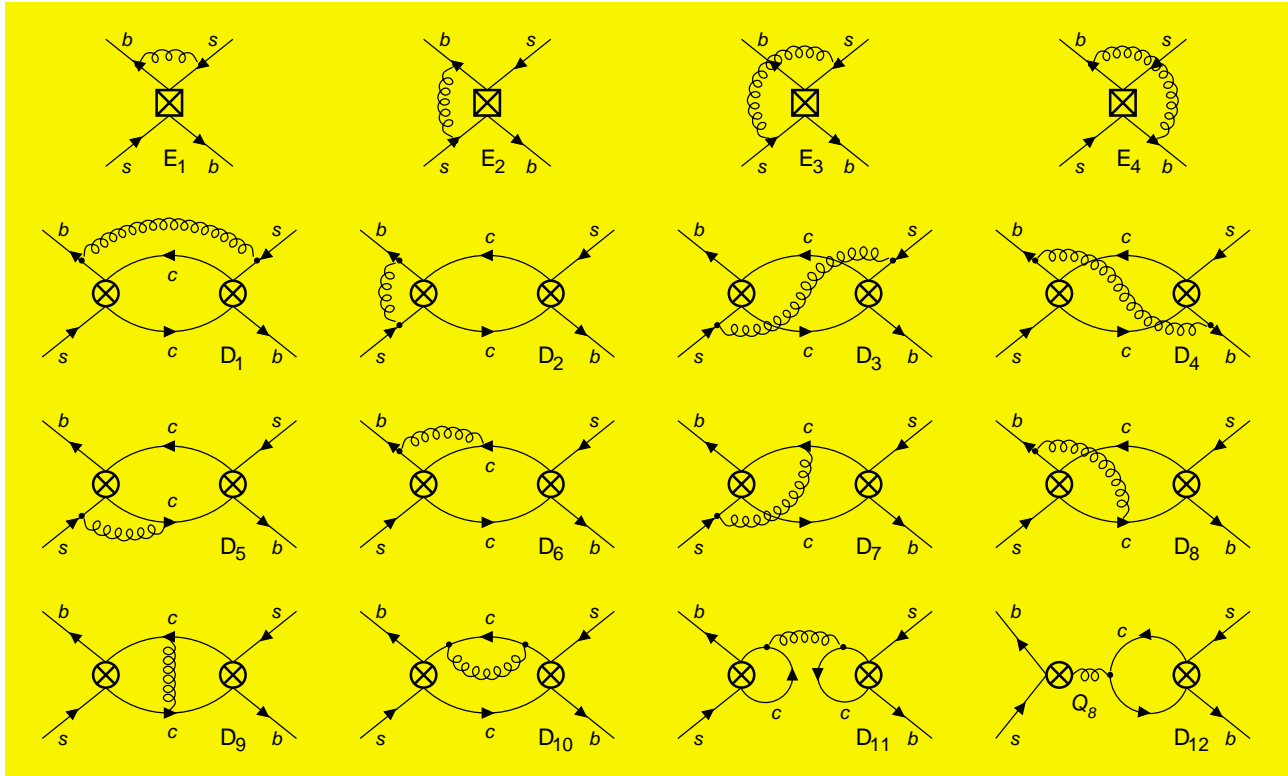
Grossman

## Why calculate lifetime differences to $\mathcal{O}(\alpha_s)$ ?

- to reduce the sizable  $\mu$ -dependence
- consistent use of  $\Lambda_{\overline{MS}}$
- meaningful use of lattice results for hadronic matrix elements like  $\langle \bar{B}_s | \mathcal{O} | B_s \rangle$ ,  $\langle \bar{B}_s | \mathcal{O}_S | B_s \rangle$
- QCD corrections are of order 30%.
- verify infrared safety of the  $c_j$ 's.
- Test of quark-hadron duality:  
Need to go beyond leading logarithmic approximation.



### 3. The width difference of $B_s$ -mesons



Result:

$$\begin{aligned} \text{Im} \langle \bar{B}_s | i \int d^4x T \mathcal{H}_{eff}(x) \mathcal{H}_{eff}(0) | B_s \rangle \\ = -\frac{G_F^2 m_b^2}{12\pi} (V_{cb}^* V_{cs})^2 [F(z) \langle \bar{B}_s | Q | B_s \rangle + F_S(z) \langle \bar{B}_s | Q_S | B_s \rangle] \end{aligned}$$

$F$  and  $F_S$  are IR-safe functions of  $z = m_c^2/m_b^2$ .

IR-singularities cancel via two mechanisms:

- 1 Bloch-Nordsieck cancellations among different cuts of the same diagram
- 2 factorization of IR-singularities, which end up in  $\langle \bar{B}_s | \mathcal{O} | B_s \rangle, \langle \bar{B}_s | \mathcal{O}_S | B_s \rangle$

## Nonperturbative QCD in

$$\begin{aligned}\langle \bar{B}_s | Q | B_s \rangle &= \frac{8}{3} f_{B_s}^2 M_{B_s}^2 B \\ \langle \bar{B}_s | Q_S | B_s \rangle &= -\frac{5}{3} f_{B_s}^2 M_{B_s}^2 \frac{M_{B_s}^2}{(\bar{m}_b + \bar{m}_s)^2} B_S\end{aligned}$$

Include corrections of order  $\Lambda_{\text{QCD}}/m_b$ :

Beneke, Buchalla, Dunietz 1996

$$\left( \frac{\Delta\Gamma}{\Gamma} \right)_{B_s} = \left( \frac{f_{B_s}}{245 \text{ MeV}} \right)^2 [0.008 B + 0.204 B_S - 0.086]$$

for the  $\overline{\text{MS}}$ -scheme at  $\mu = m_b$ .

Quenched lattice QCD:

$$B(\mu = m_b) = 0.80 \pm 0.15 \quad \text{Hashimoto (Lattice '99)}$$

$$B_S(\mu = m_b) = 1.19 \pm 0.20 \quad \text{Yamada et al. (Hiroshima)}$$

$$\left( \frac{\Delta\Gamma}{\Gamma} \right)_{B_s} = \left( \frac{f_{B_s}}{245 \text{ MeV}} \right)^2 (0.162 \pm 0.041 \pm ??? \text{ (latt. syst.)})$$

# Summary

1. Need  $\mathcal{O}(\alpha_s)$  corrections to test the HQE predictions for the lifetime differences of  $B$  mesons.
2. New CP-violation in  $B_s$ -mixing affects  $\Delta\Gamma_{B_s}$ .
3. Next-to-leading QCD-corrections to  $\Delta\Gamma_{B_s}$  are infrared safe and reduce  $\Delta\Gamma_{B_s}$  by 30%.  
 $\Delta\Gamma_{B_s}/\Gamma_{B_s} = (16 \pm 7)\%$ .